

Lecture 6

Revision of lectures 1-5

For each of the following wff do the following:

- (1) Construct a parse tree.
- (2) Construct a truth table.
- (3) Identify the tautologies.
- (4) Identify the contradictions.
- (5) Identify the satisfiable wff.

$$(a) \quad P \rightarrow (Q \rightarrow P).$$

$$(b) \quad (P \leftrightarrow \neg Q) \vee Q.$$

$$(c) \quad (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)).$$

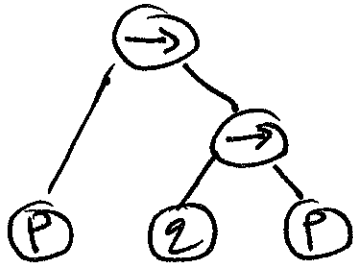
$$(d) \quad (P \rightarrow Q) \rightarrow \neg(Q \rightarrow P).$$

$$(e) \quad (\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P).$$

$$(f) \quad \neg((P \rightarrow Q) \rightarrow P) \rightarrow P.$$

Solutions

(a) Parse tree of $p \rightarrow (q \rightarrow p)$ is



Truth
table

p	q	$p \rightarrow (q \rightarrow p)$
T	T	T
T	F	T
F	T	T
F	F	T

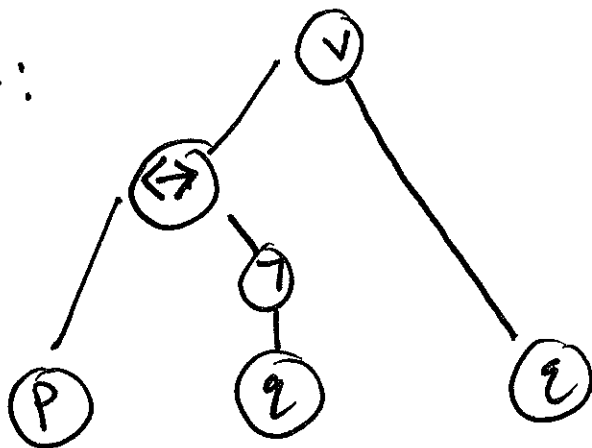
This is a tautology.

This is also satisfiable.

This is not a contradiction.

$$(b) (P \leftrightarrow \neg q) \vee q$$

Parse tree:



Truth tree:

p	q	$(p \leftrightarrow \neg q) \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

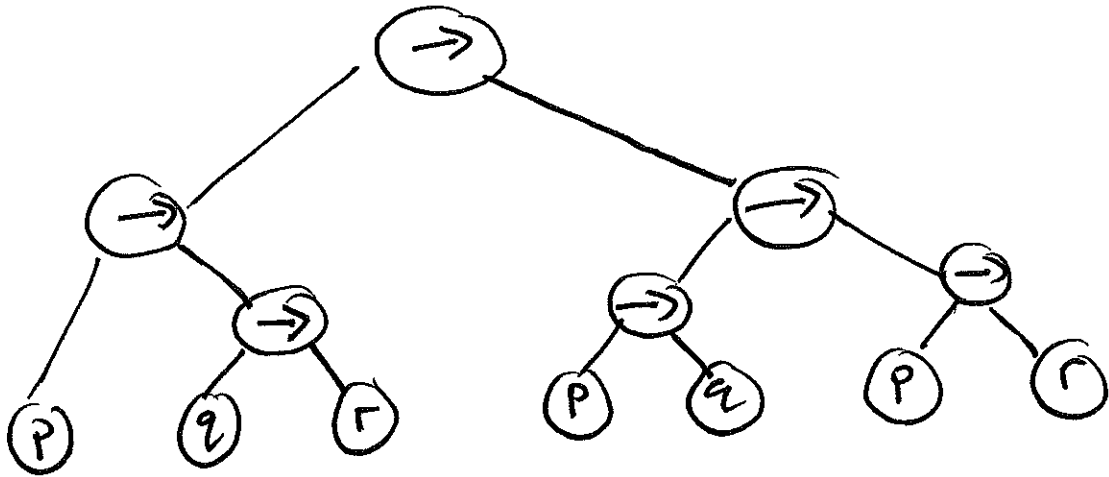
This is satisfiable.

This is not a tautology.

This is not a contradiction.

$$(c) (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$$

parse tree:



truth tree:

p	q	r	$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

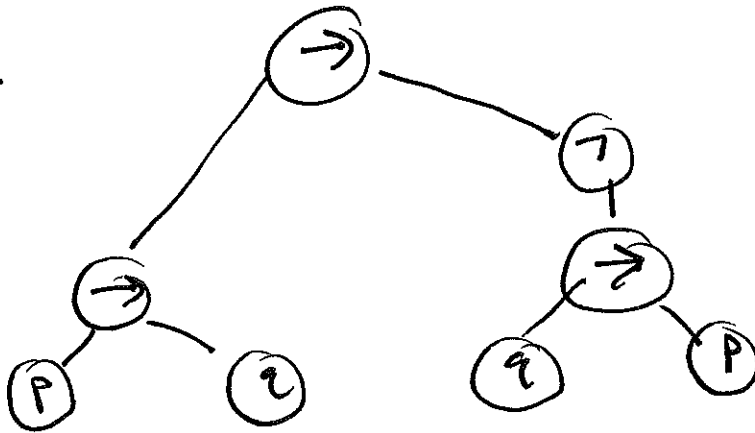
This is a tautology. (or $\models (p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$)

This is satisfiable.

This is not a contradiction.

$$(d) (P \rightarrow Q) \rightarrow \neg(Q \rightarrow P)$$

parse tree:



truth table:

p	q	$(p \rightarrow q) \rightarrow \neg(q \rightarrow p)$
T	T	F
T	F	T
F	T	T
F	F	F

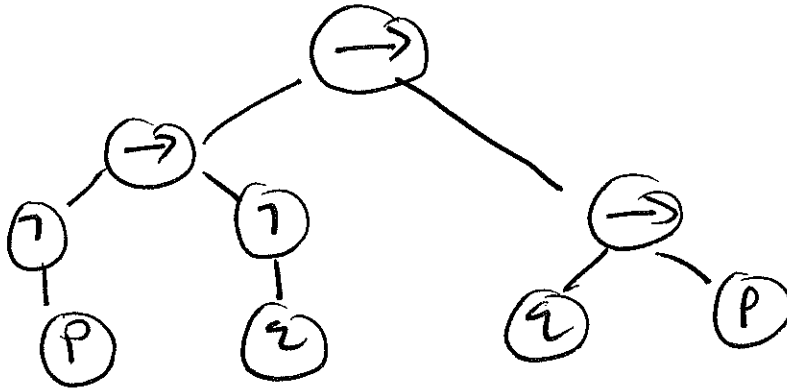
This is satisfiable.

This is not a tautology.

This is not a contradiction.

$$(e) (\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$$

parse tree:



Truth table:

p	q	$(\neg p \rightarrow \neg q) \rightarrow (q \rightarrow p)$
T	T	T
T	F	T
F	T	T
F	F	T

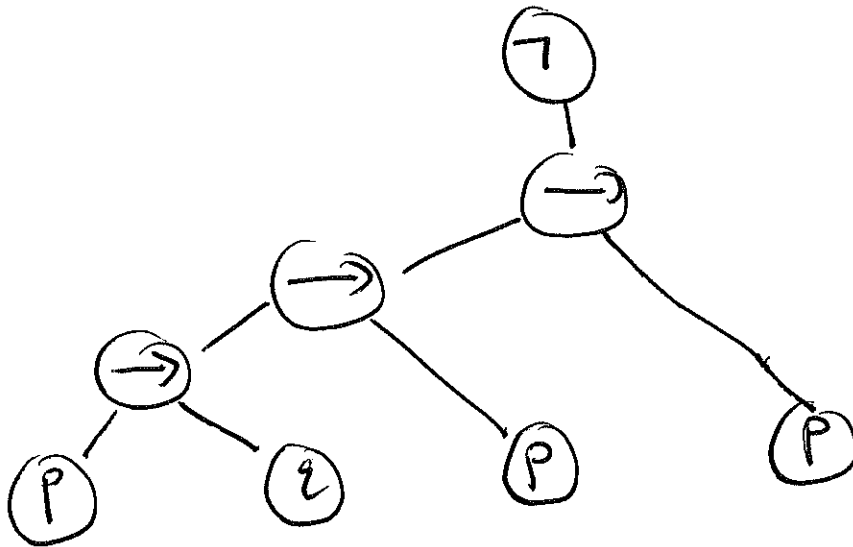
This is a tautology.

This is satisfiable.

This is not a contradiction.

$$(8) \quad \neg \left(\left((P \rightarrow Q) \rightarrow P \right) \rightarrow P \right)$$

Parse tree:



Truth tree:

p	q	$\neg(((p \rightarrow q) \rightarrow p) \rightarrow p)$
T	T	F
T	F	F
F	T	F
F	F	F

This is a contradiction.

This is not satisfiable.

This is not a tautology.