Answers to Exercises in Volume II

1-1. Note that there are a great many (logically equivalent!) correct transcriptions. If your answer does not look just like the following, it does not automatically mean your answer is wrong. In a few cases I have indicated one alternative transcription.

a) Cid is taller than Eve.
b) Cid loves Eve.
c) Cid is not taller than himself.
d) Cid is blond.
e) If Cid is talller than Eve then he loves Eve.
f) Either Cid loves Eve or he loves himself.
g) Cid does not love both Eve and Adam (Also: It is not the case that both Cid Loves Eve and Cid loves Adam.)
h) Cid is blond if and only if he either loves Eve or he loves himself.

1-2. a) Ca   b) Tca   c) Cc v Tca   d) Tce -> Lce   e) Tce -> Lce
f) Lea & Lec   g) Lea v Lec   h) (Lae v Lea) & (Lac & Lec)   i) Lec -> Cc
j) Tec & -Lec

1-3. c) and g) are not sentences of predicate logic. The rest are.

1-4. a) Adam does not love himself.
b) If Adam loves himself he is not taller than himself.
c) Neither is Cid blond nor does he love Eve.
d) Adam is a cat if and only if either he is blond or he loves Eve.
e) Someone is taller than Cid.
f) Adam loves everyone and Cid loves everyone. (Also: Adam loves everyone and so does Cid.)
g) Everyone is loved by both Adam and Cid.
h) Either someone is taller than Adam or someone is taller than Cid.
i) Someone is either taller than Adam or taller than Cid.
j) All cats love Eve (Also: Of Everyone it is true that if they are a cat they love Eve.)
k) Some cat is not loved by Eve. (Also: Of someone it is true that they are both a cat and are not loved by Eve.)
l) Not all cats are loved by Eve (Also: Eve does not love all cats.)
m) All cats are loved either by Cid or by Eve.
n) Some cat is both blond and is taller that Cid. (Also: Something is a cat, is blond, and is taller than Cid.)

1-5. a) (x)Lxe   b) (x)(Lcx v Lax)   c) (x)Lax v (x)Lcx   d) (Ex)(Txa & Txc)
e) (Ex)Txa & (Ex)Txc   f) (x)(Cx -> Lex)   g) (x)(Cx -> Lxe)   h) (Ex)(Cx & Lex)
i) (x)(Cx -> -Lex) [Also: -(Ex)(Cx & Lex)]   j) (x)(Lxe -> -Cx)
k) (x)(Lxe -> -Cx)   [(j) and (k) are both equivalent to '-(Ex)(Lxe & Cx)']
l) (Ex)(Lxa & Lxc)   m) (x)-(Lxa & Lxc) [Also: -(Ex)(Lxa & Lxc)]

ANSWERS TO EXERCISES IN VOLUME II, CHAPTER 2

2-1. The following are samples of correct answers. Other correct answers can be obtained by changing the truth value of the atomic sentences listed, and by adding more names with the attendant further atomic sentences formed with the further names.

a) D = {a,b}; Laa & -Lab & Lba & -Lbb
b) D = {a,b}; Ta & -Tb & -Laa & -Lab & -Lba & Lbb
c) D = {a,b}; Laa & Lab & Lba & Lbb
d) D = {b}; -Fb & Rbb
e) D = {a,b}; Ga & -Gb & Laa & -Lab & -Lba & Lbb & -Raa & Rab & -Rba & Rbb
f) D = {a,c}; Ka & Kc & -Ma & -Mc & Raa & Rac & Rca & -Rcc

Note that the answer for a) is also a correct answer for c), and the answer for c) is a correct answer for a). The answer for b) is a correct answer for a) and for b) - an interpretation can always have more information than the minimum required. But the answers for a) and for c) are not correct answers for b) because they do not give truth values for 'Ta' and 'Tb'.

2-2. Given Sentence	Substitution Instance.
a) (Ex)Bx	Ba, false Bb, true
b) (Ex)-Lxa	-Laa, false -Lba, false
c) (x)Lxa	Laa, true Lba, true
d) (Ex)Lbx	Lba, true Lbb, false
e) (x)(Bx v Lax)	Ba v Laa, true Bb v Lab, true
f) (Ex)(Lxa & Lbx)	Laa & Lba, true Lba & Lbb, false
g) (x)(Bx -> Lbx)	Ba -> Lba, true Bb -> Lbb, false
h) (Ex)[(Lbx & Bb) v Bx]	(Lba & Bb) v Ba, true (Lbb & Bb) v Bb, true
i) (x)[Bx -> (Lxx -> Lxa)]	Ba -> (Laa -> Laa), false Bb -> (Lbb -> Lba), true
j) (x)[(Bx v Lax) -> (Lxb v -Bx)]	(Ba v Laa) -> (Lab v -Ba), true (Bb v Lab) -> (Lbb v -Bb), false
k) (Ex)[(Lax & Lxa) <-> (Bx v Lxb)]	(Laa & Laa) <-> (Ba v Lab), false (Lab & Lba) <-> (Bb v Lbb), false

2-3. a) true, b) false, c) true, d) true, e) true, f) true, g) false, h) true, i) false, j) false, k) false

2-4. a) true, b) false, c) false, d) true, e) false, f) false, g) false, h) true, i) true, j) false

2-5. a1) true: At least one US citizen is a millionaire.

a2) false: At least one US citizen is not happy.

a3) false: There is at least one happy poor person, providing a substitution instance which makes this false.

a4) true: There is at least one unhappy millionarie giving a true substituion instance of this sentence.

a5) false: one example of an unhappy millionarie provides one false substituion instances, making this false.

a6) true: One happy millinoinare gives a true substituion instance.

a7) true: One happy millinoiare makes the first conjunct true. One unhappy millionaire makes the second conjunct true.

a8) true: One happy poor person makes the antecedent of this conditional false (see a3).

b1) true. An odd integer, such as 3, gives a true substitution instance.

b2) false. An odd integer, such as 3, gives a false substituion instance.

b3) true: Every number is at least as large as itself. So any odd integer, such as 3, give a true substituion instance.

b4) false: an integer less than 17, such as 6, gives a false substituion instance.

b5) true: Every integer is either odd or not odd. So all substituion instances are true.

b6) true: An odd integer at least as large as 17, such as 21, provides a true substitution instance.

b7) false: An odd integer not equal to 17, such as 3, provides a false substitution instance.

b8) false: The second conjunct is false. The integer 17 provides a false substituion instance, being neither less than 17 nor at least as large as 18.

b9) false. 3 is odd but not at least as large as 17. Thus 3 provides a substitution instance of the first conjunct which makes the first conjunct false.

2-6. a) Suppose we have an interpretation which makes the premise true. In this interpretation all substituion instances (SIs) of (x)Lxe will be true. There must be at least one such SI in each of these interpretations because every interpretation has at least one object in it. Thus any interpretation which makes the premise true has at least one true substitution instance of (Ex)Lxe, the conclusion, making the conclusion true. So the argument is valid.

b) Invalid. An interpretation in which 'Lae' is true and '(x)Lxe' has at least one SI which is false will be an interpretation in which the premise is true and the conclusion is false, that is, a CE (counter example). For example, D = {a,e}; Laa & Lae & Lea & -Lee is such an interpretation because 'Lee' is an SI of (x)Lxe and is false in this interpretation.

c) Invalid. Let's try to make a CE. This will have to be an interpretation in which '(Ex)Lxe' is true and 'Lae' is false. We can make '(Ex)Lxe' true in our interpretation by making just one SI true, for example 'Lee'. It does not matter what truth value we assign to 'Laa' and Lea' in our CE, so a CE is D = {a,e}; Laa & -Lae & Lea & Lee.

d) Suppose we have an interpretation in which '(x)(Bx & Lxe)' is true. This will be an interpretation in which all its SI's are true, that is all sentences such as 'Ba & Lae', 'Be & Lee', and any others with the names of the interpretation subsituted for 'x' in 'Bx & Lxe'. But since all these SIs are conjunctions, all their conjuncts will be true in the interpretation. That is 'Ba', 'Be' and all other instances of '(x)Bx' will be true. So in such an interpretation, '(x)Bx' will be true. Therefore, the argument is valid.

e) Invalid. Here is a counterexample: D ={a}; -Ba & Laa.

f) Invalid. A counterexample is provided by a case in which something is B and something else bears relation L to a, but nothing does both of these. For example: D={a,b}; Ba & -Bb & -Laa & Lab & Lba & Lbb.

g) To think about this informally, let 'Bx' mean 'x is blond' and 'Lxy' mean 'x loves y'. The premise says that all blonds love e and all non-blonds love a. But if that is true, then it is true of everyone that if they are blond they love e and if not blond they love a. More exactly, consider an arbitrary interpretation in which the premise is true. Consider any one of the SIs of the conclusion in such an interpretation, formed say, with s. Each conjunct of this SI is an SI of one of the conjuncts of the premise. Since the premise is assumed true in the interpretation in question, all these SI's will be true. So, the argument is valid.

3-1. a) 'z' at 1 and 2 is free.
b) 'z' at 1 and 'y' at 2 are bound.
c) 'z' at 1 and 3 is bound. 'x' at 2 is free.
d) 'x' at 1, 3, and 6 and 'y'at 4 are bound. 'z' at 2 and 5 is free.
e) 'x' at 1, 2, and 5 is bound. 'x' at 3 and 4 is free.
(Note that we don't say of names that they are either bound or free.)
f) 'y' at 1 and 4 is free. 'x' at 2, 3, and 5 is bound.

3-2. a) (Ex)Lxa
b) (x)Bx v Ba
c) Ba <-> (x)(Lax v Bx)
d) (Ex)(Bx -> Ba) & (x)(Ba -> Bx)
e) (Ex)Bx v ((Ey)By ->Laa)
f) (Ex)[(Rxa -> Da) -> Rax]
g) (y)(z){[Say v (Hz -> Laz)] <-> (Sca & Hy)}
h) (z){(Paa -> Kz) & (Ey)[(Pay v Kc) & Paa]}
i) (y) {[(Ex)Max v (Ex)(Mxy -> Mya)] & (Ex)Max}
j) (x)(Rxa -> Rab) v [(Ex)(Rcx v Rxa) -> Raa]

3-3. X is closed, so that u does not occur free in X. Thus, the result of writing in a name for u in X is X itself. In other words, the substitution instances of (u)X and (Eu)x are X itself. Thus if X is true in an interpretation, all its substitution instances are true in the interpretation, so that (u)X and (Eu)X are both true in the interpretation. And if X is false in an interpretation, none of its substitution instances are true in the interpretation, so that (u)X and (Eu)X are both false in the interpretation.

3-4. a) Suppose we have an interpretation in which all the named objects have property B, but some unnamed object does not have property B. Then all substitution instances formed with names in the interpretation would come out true. Yet it would not be true in this interpretation that all things are Bs. Also, consider an interpretation in which no named things are B, but some unnamed thing is a B. In such an interpretation, no substitution instances formed with names in the interpretation would come out true, so that our defintion of truth of an existentially quantified sentence, would tell us that there does not exist a B in the interpretation. Yet, in such an interpretation there is a B.

b) In real life, many things do not have names. We want our system of logic to be applicable to such situations. Also, there are contexts in which there are, in some sense, MORE things than there are names. There are more numbers than there are names, for example. We cannot name all numbers at the same time.

3-5. a) We have to prove that -(u)(...u...) is true in a given interpretation if and only if (Eu)-(...u...) is true in the interpretation. So, suppose that an interpretation is given. -(u)(...u...) is true in the interpretation just in case the negation of the conjunction of the instances

-[(...a...) & (...b...) & (...c...) & ...]

is true in the interpretation, where we have included in the conjunction all the instances which can be formed using all the names which name things in the interpretation. By De Morgan's law, this is equivalent to the disjunction of the negation of the instances,

-(...a...) v -(...b...) v -(...c...) v ...

which is true in the interpretation just in case (Eu)-(...u...) is true in the interpretation.

b) To prove informally that the rule -(Eu) holds for infinite domains, we first assume that -(Eu)(...u...) is true in an interpretation (to show that (u)-(...u...) must be true in this interpretation). Now, -(Eu)(...u...) means that there is nothing in the (infinite) domain of this interpretation such that (...u...) is true of it. But to say that there is no thing such that (...u...) is true of it is just to say that everything in the domain is such that (...u...) is false for it; and this is just to say that (u)-(...u...) is true in this interpretation.

Similarly for the rule -(u): Assume that -(u)(...u...) is true in an interpretation with an infinite domain. This means that not everything in the domain is (...u...). But, to say that not everything is such-and-such is just to say that something is not such-and-such. This is just to say that (Eu)-(...u...) is true in this interpretation.

3-6. a) To see how to approach this problem, consider the open sentences

(4) -(y)Lxy

(5) (Ey)-Lxy

We want a notion of logical equivalence on which (4) and (5) will be said to be logically equivalent and which will guarantee that substituting one for the other in a closed sentence will not change the truth value of the closed sentence in any interpretation. But how can (4) and (5) affect the truth value of a closed sentence of which they are a part? They don't affect truth values directly. Rather it is sentences which result by subsituting names for the free variables in (4) and (5) which affect truth values of containing sentences. Suppose we have a longer sentence which has (4), for example, as a subsentence. The truth value of the longer sentence in an interpretation is determined, step by step, by the truth value of the longer sentence's substitution instances, and/or components and /or substitution instances of components. At some point in the process of determining the truth value of longer sentences by looking at substitution instances one will obtain sentences such as

(6) -(y)Lay

and it is (6), and the other sentences formed by writing a name for the free 'x' in (4) which do the truth determining work.

Now let us do the same with (5). We get

(7) (Ey)-Lay

If (6) and (7) are logically equivalent (as they are by the rule -(u)), in any interpretation they will have the same truth value. So their effect on the truth value of longer sentences will be the same. And clearly this will likewise be true for any pair of sentences which we form from (4) and (5) by writing in the SAME name for the SAME free variable.

In short, (4) and (5) act just like logically equivalent sentence when it comes to substitution into longer sentences if the following condition holds: (4) and (5) turn into genuine, closed, logically equivalent sentences when we replace the free variables with names, always the same name for the same variable.

Of course, if we get logically equivalent sentences from a pair like (4) and (5) when we put in one name, any name will work - which name we use won't matter. But we don't at this point have any nice way of proving that. So to get our extended defintion of logical equivalence it looks like we will have to talk about an infinitely large set of closed sentences obtained by putting in all possible names for the free variables. And this is a very messy sort of condition.

However, we can get the same effect in a very nice way. Take (4) and (5) and form their biconditional. Then form what logicians call the biconditional's Universal Closure, that is the sentence which results from the biconditional by prefixing it with universal quantifiers, one for each free variable in the biconditional. If this biconditional is a logical truth, that is true in all interpretations, then in any interpretation the substitutuion instances of both components will have the same truth value, which is just the condition we want. Of course, two closed sentences are logically equivalent just in case their biconditional is a logcial truth, and a biconditional formed from two closed sentences is its own universal closure, because such a sentence has no free variables to be bound by a prefixed universal quantifier. (See problem 12.3.) So we can give one definition which covers our old defintion of logical equivalence and which also covers the new cases we want to include:

Two open or closed sentences are Logically Equivalent if and only if the universal closure of their biconditional is a logical truth.

b) Now let us double check that this definition will make the law of substitution of logical equivalents work in the way we want. All we really need to do is to say in general what we said for the special case of (4) and (5). Suppose that X and Y are two open sentences such that the universal closure of X <-> Y is a logical truth. Suppose that we start with a sentence Z and form a new sentence Z' by subsitituting Y for X in Z. We need to show that Z and Z' are logically equivalent according to our old, more restricted definition of logical equivalence. But the truth of Z in an interpretation is completely determined by the truth value of the substituiton instances of Z and of the subsentences of Z. However, if the universal closure of X <-> Y is a logical truth, whenever a substitution instance arises which is identical to X with names substituted for X's free variables, the sentence which results by substituting the same names for the same variables in Y will have the same truth value. Thus if Y occurs in an otherwise identical sentence instead of X, the same truth values will result at all stages of evaluating truth values of components and substitution instances. In particular, Z and Z' will have the same truth value, and since this holds for all interpretations, Z and Z' will be logically equivalent.

ANSWERS TO EXERCISES IN VOLUME II, CHAPTER 4

4-1.

-(u)S(...u...)	
-(u)[Sx -> (...u...)]	Rewrite rule
(Eu)-[Su -> (...u...)]	-(u)
(Eu)[Su & -(...u...)]	C
(Eu)S-(...u...)	Rewrite Rule

-(Eu)S(...u...)	
-(Eu)[Su & (...u...)]	Rewrite rule
(u)-[Su & (...u...)]	-(Eu)
(u)[Su -> -(...u...)]	C
(u)S-(...u...)	Rewrite rule

4-2.

(Eu)S(...u...)	
--(Eu)S(...u...)	DN
-(u)S-(...u...)	-(Eu)S
-(u)[Su -> -(...u...)]	Rewrite Rule
-(u)--[Su -> -(...u...)]	DN
-(u)-[Su & (...u...)]	C, DN
--(Eu)[Su & (...u...)]	-(u)
(Eu)[Su & (...u...)]	DN

In the following answers to transcriptions, keep in mind that logically equivalent trnascriptions are always equally good transcriptions. If your answer differs from the answer given here, your answer is still right if it is logically equivalent to the answer given here. The following logical equivalences are the ones which most often convert one natural transcription into another:

X->(Y->Z) is logically equivalent to (X&Y)->Z

X->Y is logically equivalent to -Y->-X

-(Eu)(X & Y) is logically equivalent to (u)(X -> -Y)

-(u)(X -> Y) is logically equivalent to (Eu)(X & -Y)

4-3. a) (x)$_P$Lxe, (x)(Px -> Lxe)

b) (Ex)$_P$Lex, (Ex)(Px & Lex)

c) (x)$_P$Lex, (x)(Px -> Lex)

d) (Ex)$_C$(Ey)$_D$Lxy, (Ex)[Cx & (Ey)(Dy & Lxy]

e) (Ex)$_P$(-Cx & -Dx), (Ex)(Px & -Cx & -Dx)

f) (Ex)$_P$(Bx & Lxe), (Ex)(Px & Bx & Lxe)

g) (Ex)$_P$(y)$_C$Lxy, (Ex)[Px & (y)(Cy -> Lxy)]

h) (Ex)$_C$Bx, (Ex)(Cx & Bx)

i) (x)$_C$-Dx, (x)(Cx -> -Dx)

j) (Ex)$_P$(Ey)$_P$Lxy, (Ex)[Px & (Ey)(Py & Lxy]

k) (x)$_P$(y)$_P$Lxy, (x)[Px -> (y)(Py -> Lxy)]

l) (Ex)$_P$(y)$_P$Lxy, (Ex)[Px & (y)(Py -> Lxy)]

m) (Ex)$_P$(y)$_P$Lyx, (Ex)[Px & (y)(Py -> Lyx)]

n) (x)$_P$(Ey)$_P$Lxy), (x)([Px -> (Ey)(Py & Lxy)]

o) (x)$_P$(Ey)$_P$Lyx, (x)[Px -> (Ey)(Py & Lyx)]

4-4.
a) (x)(Fx -> Lxe)
b) (x)(Cx -> -Fx)
c) (Ex)(Px & Lxa) -> Lea, or equivalently, (x)[(Px & Lxa) -> Lea]
d) -(Ex)(Px & Lex), or equivalently, (x)(Px -> -Lex)
e) (x)-Fx
f) (Ex)(Px & Bx) -> Ba
g) -(x)(Cx -> Fx)
h) (Ex)(Cx & -Fx)
i) (x)(Px -> -Cx)
j) (xx)(Cx -> -Dx)
k) (x)(Qx -> Cx)
l) -(x)(Bx -> Cx)
m) (x)(Dx -> -Ax), or (Ex)(Dx & -Ax)
n) -(x)(Ax -> Dx),
o) (x)(Qx -> Cx)
p) -(x)(Fx -> Cx), or, equivalently, (Ex)(Fx & -Cx)
q) (x)(Dx -> -Cx)
r) (x)(Bx -> -Lxa)
s) (x)(Lxa -> Bx)
t) (Ex)(Dx & -Cx)
u) (x)[Fx -> (y)(Py -> -Lxy)], possibly (x)[Fx -> -(y)(Py -> Lxy)]
v) (x)[(Ey)(Dy & Lxy) -> (Ey)(Cy & Lxy)]
w) (x)[(Px & Sxa) -> Bx)]
x) (x)(Sxa -> -Sxe)
y) (x)[Px & Sxa) -> -Sxe], also (Ex)[(Px & Sxa) & -Sxe]
z) (x)[(Cx & Lxa) -> Lxe]
aa) -(x)[Px & Lxa) -> Lxe]
bb) (x)[(Px & Txe) -> Txa]
cc) (x)(Lxe -> Lxa)

4-5. a) (x)$_P$(Ey)$_P$Lxy, and (Ey)$_P$(x)$_P$Lxy

b) (Ex)$_P$(y)$_P$Lxy, and (y)$_P$(Ex)$_P$Lxy

c) (x)(Cx <-> Lax), and (Ex)(Cx <-> Lax)

d) $(x)_C$-Fx, and -$(x)_C$Fx

e) $(x)_P$-Lxa, and -$(x)_P$Lxa

4-6.
a) (x)[(Px & Fx) -> Dx], and (x)(Ax -> Dx)
b) (x)[(Ey)(Kxy & Wy & Ay) -> Bx], and (x)[Wx &(Ey)(Kxy & Ay) -> Bx]
c) (x)(Rax -> Sx), and (x)(Sx -> Lax)
d) (x)[Dx -> (Cex & Wex)], and Cee & (x)(Dx -> Wex)

4-7.
a) Someone is the son of someone.
b) Nobody is an animal.
c) Not everything that purrs is a furry cat.
d) Something that purrs is not a furry cat.
e) No one loves both Adam and Eve.
f) Everyone does not love both Adam and Eve.
g) All dogs and cats love each other.
h) All dogs and cats love each other. (g and h are logically equivalent.)
i) Someone loves some son of someone. (Or, Someone loves someone's son.)
j) Someone loves someone who has a son.
k) Any blond fuzzy tail is the tail of a cat.
l) All sons love a cat if and only if they love a dog.

4-8.
a) (x)[(Fx & Cx) -> Qx]
b) (x)[(Fx & Cx) -> Qx]
c) -(Ex)(Fx & Cx & Qx) ((c) and (d) are logically equivalent.
d) (x)[(Fx & Cx) -> -Qx] Either answer given is correct for either problem.)
e) (x)[Qx -> (Fx & Cx)]. The sentence may be ambiguous, with the following alternative transcription: (x)[Cx -> (Qx -> Fx)]
f) (Ex)(Fx & Cx & Qx)
g) (Ex)(Fx & Cx & -Qx)
h) (Ex)(Cx & Lxa) & (Ex)(Dx & Lxa)
i) (x)[(Cx & -Fx) -> Qx]
j) -(x)[(Fx & Cx) -> Qx]
k) (x)[(Cx & Fx) -> Qx]
l) (Ex)(Fx & Cx & Qx) The sentence is ambiguous. An alternative transcription is (x)[(Fx & Cx) -> Qx].
m) (x)[Qx -> (Fx & Cx)]
n) -Da & -Ca
o) (Ex)(Ey)(Px & Sxy)
p) (Ex)(Ey)(Sxy & Bx)
q) (Ex)(Bx & Cx & Lax) & (Ex)(Bx & Cx & Lex)
r) The sentence is ambiguous. One reading is the same as (q) The other is (Ex)(Bx & Cx & Lax & Lex)
s) -(x)(Px -> Lex)
t) (Ex)(Cx & Fx) & (Ex)(Cx & -Fx)
u) (x)[Cx -> (-Lxa & -Lxe)]
v) (Ex)(Fx & Lxe)

w) (x)[(Ey)(Py & Lxy) -> Px]
x) (Ex)(Px & (Ey)Syx)
y) (x)(Sxa -> Sxe)
z) (Ex)Sax & (x)(Px -> Lxa)
aa) (x)-(Ax & Fx) & (Ex)(Ax & (Ey)Tyx)
bb) (x)[(Ax & Fx) -> (Ey)Tyx]
cc) (x)(Px -> -(Ey)Syx)
dd) -(x)(Px -> (Ey)Syx)
ee) (Ex)(Bx & Lxe) & (Ex)(Bx & -Lxe)
ff) (x)[(Fx & Cx) -> Lax]
gg) (x)[(Bx & Lxx) -> Lxe]
hh) (Ex)(Lxx & Lex)
ii) (x){[(Px & (y)(Lxy -> -Cy)] -> (y)(Lxy - > -Dy)}, or equivalently, (x){[(Px & (y)(Cy -> -Lxy)] -> (y)(Dy -> -Lxy)}
jj) (x){Cx -> [(Ey)(Py & Lxy) -> Lxe]} This is ambiguous, and also transcribes as, (x){Cx -> [(y)(Py -> Lxy) -> Lxe]}
kk) (x)(Px -> (Ey)Syx) -> Lea This is ambiguous and also transcribes as, (Ex)(Px & (Ey)Syx) -> Lea.
ll) (x)[Px -> [(Ey)Syx -> Lxe)]
mm) The same as (ll)
nn) (Ex)(Px & (Ey)Syx) -> Lae (I don't think this is ambiguous.)
oo) (x)[(Px & (Ey)Syx) -> Lxa] (I don't think this is ambiguous.)
pp) (Ex)(Px & (Ey)Syx & Lxa), This is ambiguous and also transcribes as, (x)[(Px & (Ey)Syx) -> Lxa]
qq) (x)[(Cx & (Ey)Syx) -> (Lxe <-> -Fx)]
rr) (x){[Px &(Ey)(Cy & Lxy)] -> (Ey)(Ay & Lxy)}
ss) (x){[Px & (Ey)(Py & Lxy)] -> (y)(Ay -> -Lxy)}
tt) (Ex)(Sxa & -Fx)
uu) (Ex)[Sxa & (Ey)(Fy & Syx)] -> (Ex)(Fx & Sxa)
vv) (x)(Sxa -> Sxe). This is ambiguous and also transcribes as (Ex)(Sxa & Sxe)
ww) (x)[Px -> (Lxe ->Bx)] -> (x)(Px -> -Lxe)
xx) (x)[Px -> (y)(Py -> -Lxy)], This is ambiguous and also transcribes as (x)[Px -> -(y)(Py -> Lxy)]
yy) (x)[Px -> -(Ey)(Py & Lxy)]
zz) (x)[Px -> (y)(Py -> -Lxy)]
aaa) I get three ambiguous readings:
(x)[Px -> -(y)(Py -> Lxy)]
(x)[Px -> (y)(Py -> -Lxy)]
-(x)[Px -> (y)(Py -> Lxy)]
bbb) (x)[Px -> -(y)(Py -> -Lxy)], which is logically equivalent to (x)[Px -> (Ey)(Px & Lxy)]. That is, "Nobody loves nobody." comes to "Everyone loves sombody."
ccc) (x)[Ax -> (-Fx <-> Lxa)]
ddd) (x){Px -> (y)[(Ez)Lyz -> Lxy]}. An alternate ambiguous reading is (x){Px -> (Ey)[(Ez)Lyz & Lxy]}
eee) (x){[(Ey)(Cy & Oxy) & (Ey)(Dy & Oxy)] -> (Bx & Lxa)}
fff) (x)([Px & (y)(Sya ->-Lxy)] -> -(z)(Sze -> -Lxz)}
ggg) (x)[(Ey)(Oxy & Cy & -Lya) -> (Ey)(Oxy & Dy & (Ez)Tzy)]
hhh) (x)[Sxa -> -(Ey)(Oxy & Ay & Fy & -(Ez)Tzy)]
iii) (x){[Px & (y)(Lxy -> -(Ez)Tzy)] -> (z) [(w)(Aw & Lwz) -> -Oxz]}
jjj) (x){(Ey)[Sxy & (z)[(Az & -(Ew)Twz) -> -Oyz]] -> (-Lxa & -Lxe)}
kkk) (x){[Px & (y)(Ley -> Lxy)] -> (Ez)[Pz & (w)(Lwe -> Lwz) & Lxz]}

4-9.

a) (x)(Px -> -Lxs)
b) (x)(y)[(Dx & Py) -> Hxy]
c) -(x)(Rx -> Hx) & -(x)(Dx -> Hx)
d) (Ex)(Ey)(Sx & Sy & Bxy)
e) (x)[(Sx & Rxt)-> -Fxc]
f) (x)[(Px &Sx) -> (Ey)(Cy & Exy)]
g) (Ex){Px & -(Ey)(Cy & Exy) & (y)[Px & (Ey)(Bx & Exy] -> Sxy)
h) (x)[Mx -> (Lpx <-> -Sx)]
i) (Ex)[Lx & Cx & (y)(Hy -> Rxy)]
j) -(x){[(Ey)(Hy & Ry & Oxy) & (z)(w)(Hw & Bw & Ozw)] -> Txz)
k) (Ex){Tx & (Ey)(Px & Oyx) & (z)[(Dz & Ozx) -> Myz]}
l) -(Ex)[(Px & (y)(Sx -> Txy)]
m) (x)[(Px & Tx) -> -Tjx]
n) (x)(y)(Px & Py & Axs & Aym) -> - Lxy]
o) (x)[(Px & -Wx) -> (y)(Sy -> Cxy)]
p) Tsl v (Ex)(Px & Tsx & Txl)
q) (Ex)(Ey)(Lx & Sys & Txy) & (Ex)(Ey)(-Lx & Sys & Txy)
r) (x)(y)[Px & Py & Lxb & Lyd) -> Lxyc], or
s) (x){Px -> {[Mx & (Ey)(Cy & Fxy)] <-> (Ey)(Ty & Gy & Wxy)}}
t) (x)(y)[(Mx & My & Sxa & Oxy) -> Syb]
u) (x)[Lx -> (Ey)(Py & Eylx)]
v) (x)(y)[(Px & Py & Ex & Dx & Mx & -Sy & -Ly) -> Fxy]

5-1. a)

```
1  | (x)(Px & Dx)      P
   |-----------------
2  | Pk̂ & Dk̂           1,UE
3  | Pk̂                2,&E
```

5-1. b)

```
1  | (x)(Px & Dx)      P
   |-----------------
2  | Pd̂ & Dd̂           1,UE
3  | Pk̂ & Dk̂           1,UE
4  | Pd̂                2,&E
5  | Dk̂                3,&E
6  | Pd̂ & Dk̂           4,5,&I
```

5-1. c)

```
1  | (x)(Dx > Kx)      P
2  | (x)Dx             P
   |-----------------
3  | Dâ > Kâ           1,UE
4  | Dâ                2,UE
5  | Kâ                3,4,>E
```

5-1. d)

```
1  | (x)(Mx > A)          P
   |--------------------
2  |  | (x)Mx             A
   |  |-----------------
3  |  | Mâ                2,UE
4  |  | (x)(Mx > A)       1,R
5  |  | Mâ > A            4,UE
6  |  | A                 3,5,>E
7  | (x)Mx > A            2-6,>I
```

5-1. e)

```
1  | (x)(Fx v Hx)      P
2  | (x)(Fx > Dx)      P
3  | (x)(Hx > Dx)      P
   |-----------------
4  | Fp̂ v Hp̂           1,UE
5  | Fp̂ > Dp̂           2,UE
6  | Hp̂ > Dp̂           3,UE
7  | Dp̂                4,5,6,AC
8  | Fb̂ v Hb̂           1,UE
9  | Fb̂ > Db̂           2,UE
10 | Hb̂ > Db̂           3,UE
11 | Db̂                8,9,10,AC
12 | Dp̂ & Db̂           7,11,&I
```

5-1. f)

```
1  | (x)(-Bx v Lcx)       P
   |--------------------
2  |  | (x)Bx             A
   |  |-----------------
3  |  | Bd̂                2,UE
4  |  | (x)(-Bx v Lcx)    1,R
5  |  | -Bd̂ v Lcd̂         4,UE
6  |  | Lcd̂               3,5,vE
7  | (x)Bx > Lcd̂          2-6,>I
```

5-1. g)

```
1  | (x)(Lxx > Lxh)       P
2  | -Lmh                 P
   |--------------------
3  |  | (x)Lxx            A
   |  |-----------------
4  |  | Lmm               3,UE
5  |  | (x)(Lxx > Lxh)    1,R
6  |  | Lmm > Lmh         5,UE
7  |  | -Lmh              2,R
8  |  | -Lmm              6,7,DC
9  | -(x)Lxx              3-8,-I
```

5-1. h)

```
1  | (x)(Rxx v Rxk)    P
2  | (y)-Ryk           P
   |-----------------
3  | Rĉĉ v Rĉk         1,UE
4  | -Rĉk              2,UE
5  | Rĉĉ               3,4,vE
6  | Rf̂f̂ v Rf̂k         1,UE
7  | -Rf̂k              2,UE
8  | Rf̂f̂               6,7,vE
9  | Rĉĉ & Rf̂f̂         5,8,&I
```

5-2. a)

```
1  | Na                P
   |-----------------
2  | Na v Ga           1,vI
3  | (Ex)(Nx v Gx)     2,EI
```

5-2. b)

```
1  |(x)(Kx & Px)       P
   |-----------------
2  |Kâ & Pâ            1,UE
3  |Kâ                 2,&E
4  |Pâ                 2,&E
5  |(Ex)Kx             3,EI
6  |(Ex)Px             4,EI
7  |(Ex)Kx & (Ex)Px    5,6,&I
```

5-2. c)

```
1  |(x)(Hx > -Dx)      P
2  |Dg                 P
   |-----------------
3  |Hg > -Dg           1,UE
4  |-Hg                2,3,DC
5  |(Ex)-Hx            4,EI
```

5-2. d)

```
1  |(x)Ax & (x)Tx      P
   |-----------------
2  |(x)Ax              1,&E
3  |(x)Tx              1,&E
4  |Aâ                 2,UE
5  |Tâ                 3,UE
6  |Aâ & Tâ            4,5,&I
7  |(Ex)(Ax & Tx)      6,EI
```

5-2. e)

```
1  |Fa v Nh              P
   |-----------------
2  | |Fa                 A
   | |---------------
3  | |(Ex)Fx             2,EI
4  | |(Ex)Fx v (Ex)Nx    3,vI
   |
5  | |Nh                 A
   | |---------------
6  | |(Ex)Nx             5,EI
7  | |(Ex)Fx v (Ex)Nx    6,vI
8  |(Ex)Fx v (Ex)Nx      1,2-4,5-7,AC
```

5-2. f)

```
1  |(x)(Sx v Jx)         P
   |-----------------
2  |Sâ v Jâ              1,UE
3  | |Sa                 A
   | |---------------
4  | |(Ex)Sx             3,EI
5  | |(Ex)Sx v (Ex)Jx    4,vI
   |
6  | |Ja                 A
   | |---------------
7  | |(Ex)Jx             6,EI
8  | |(Ex)Sx v (Ex)Jx    7,vI
9  |(Ex)Sx v (Ex)Jx      2,3-5,6-8,AC
```

5-2. g)

```
1  |(Ex)Rxa > (x)Rax     P
2  |Rea                  P
   |-----------------
3  |(Ex)Rxa              2,EI
4  |(x)Rax               1,3,>E
5  |Rab̂                  4,UE
6  |(Ex)Rax              5,EI
```

5-2. h)

```
1  |Lae v Lea            P
2  |(Ex)Lax > A          P
3  |(Ex)Lxa > A          P
   |-----------------
4  | |Lae                A
   | |---------------
5  | |(Ex)Lax            4,EI
6  | |(Ex)Lax > A        2,R
7  | |A                  5,6,>E
   |
8  | |Lea                A
   | |---------------
9  | |(Ex)Lxa            8,EI
10 | |(Ex)Lxa > A        3,R
11 | |A                  9,10,>E
12 |A                    1,4-7,8-11,AC
```

5-2. i)

```
1  |(Ex)Jx > Q           P
2  |(x)Jx                P
   |-----------------
3  |Jâ                   2,UE
4  |(Ex)Jx               3,EI
5  |Q                    1,4,>E
```


5-2. j)

```
1  |(x)(Max v Mex)       P
2  |-(Ex)Max v Bg        P
3  |-(Ex)Mex v Bg        P
   |-----------------
4  |Mab v Meb            1,UE
5  | |Mab                A
   | |---------------
6  | |(Ex)Max            5,EI
7  | |-(Ex)Max v Bg      2,R
8  | |Bg                 6,7,vE
   |
9  | |Meb                A
   | |---------------
10 | |(Ex)Mex            9,EI
11 | |-(Ex)Mex v Bg      3,R
12 | |Bg                 10,11,vE
13 |Bg                   4,5-8,9-12,AC
14 |(Ex)Bx               13,EI
```

5-2. k)

```
1  |(x)(Kxx<->Px)        P
2  |(x)(Kjx & (Px > Sx)) P
   |-----------------
3  |Kjj<->Pj             1,UE
4  |Kjj & (Pj > Sj)      2,UE
5  |Kjj                  4,&E
6  |Pj > Sj              4,&E
7  |Pj                   3,5,<>E
8  |Sj                   6,7,>E
9  |(Ex)Sx               8,EI
```

5-2. l)

```
1  |(x)(-Oxx v Ix)       P
2  |(x)(Ix > Rxm)        P
   |-----------------
3  | |(x)Oxx             A
   | |---------------
4  | |Oââ                3,UE
5  | |(x)(-Oxx v Ix)     1,R
6  | |-Oââ v Iâ          5,UE
7  | |Iâ                 4,6,vE
8  | |(x)(Ix > Rxm)      2,R
9  | |Iâ > Râm           8,UE
10 | |Râm                7,9,>E
11 | |(Ex)Rxm            10,EI
12 |(x)Oxx > (Ex)Rxm     3-11,>I
```

5-3. a) There is someone who likes both pickles and chicken (premise). Let's call this person Doe. So Doe likes pickles and Doe likes chicken. Since Doe likes pickles, someone likes pickles. Since Doe likes chicken, someone likes chicken. Therefore (conclusion) there is someone who likes pickles and there is someone who likes chicken.

(Note: The conclusion of this argument is NOT just a restatement of the premise. The conclusion could be true and the premise false!)

b) Either i) everyone likes pickles or ii) everyone likes chicken (premise). We argue by cases. Case i) Suppose everyone likes pickles. For example, Arb likes pickles. But then Arb likes either pickles or chicken. Since Arb could have been anyone, everyone likes either pickles or chicken. Case ii) Suppose everyone likes chicken. For example, Arb likes chicken. But then Arb likes either pickles or chicken. Since Arb could have been anyone, everyone likes either pickles or chicken. We see that we reach the same conclusion in case i) and case ii), but we are given in the premise that either case i) or case ii) holds. So, by argument by cases, (conclusion) everyone likes either pickles or chicken.

(Again, the conclusion is not just a restatement of the premise. The conclusion can be true and the premise false.)

5-4.

```
1  | (x)(Bx > Cx)              P
   |---
2  | Bê > Cê                    1,UE
3  | | Be                       A
   | |---
4  | | Be > Ce                  2,R
5  | | Ce                       3,4,>E
6  | | (x)Cx  MISTAKE!  e is not
                arbitrary in line 5,
                so we can't apply UI.
```

5-5. a)

```
1  | (x)(Fx & Gx)              P
   |---
2  | Fâ & Gâ                    1,UE
3  | Fâ                         2,&E
4  | (x)Fx                      3,UI
```

5-5. b)

```
1  | (x)(Mx > Nx)              P
2  | (x)Mx                     P
   |---
3  | Mâ > Nâ                    1,UE
4  | Mâ                         2,UE
5  | Nâ                         3,4,>E
6  | (x)Nx                      5,UI
```

5-5. c)

```
1  | A                         P
   |---
2  | A v Nâ                     1,vI
3  | (x)(A v Nx)                2,UI
```

5-5. d)

```
1  | (x)Hx & (x)Qx             P
   |---
2  | (x)Hx                      1,&E
3  | (x)Qx                      1,&E
4  | Hâ                         2,UE
5  | Qâ                         3,UE
6  | Hâ & Qâ                    4,5,&I
7  | (x)(Hx & Qx)               6,UI
```

5-5. e)

```
1  | (x)(Kxm & Kmx)            P
   |---
2  | Kâm & Kmâ                  1,UE
3  | Kâm                        2,&E
4  | Kmâ                        2,&E
5  | (x)Kxm                     3,UI
6  | (x)Kmx                     4,UI
7  | (x)Kxm & (x)Kmx            5,6,&I
```

5-5. f)

```
1  | (x)(Fx v Gx)              P
2  | (x)(Fx > Gx)              P
   |---
3  | Fâ v Gâ                    1,UE
4  | Fâ > Gâ                    2,UE
5  | | -Ga                      A
   | |---
6  | | Fa v Ga                  3,R
7  | | Fa                       5,6,vE
8  | | Fa > Ga                  4,R
9  | | Ga                       7,8,>E
10 | Gâ                         5-9,RD
11 | (x)Gx                      10,UI
```

Notice in 5-5. f) how there are no hats on 'a' in the sub-derivation with assumption '-Ga', but 'a' gets a hat in 'Ga' when the assumption is discharged. Because you can't put a hat on a name used in an assumption, AC won't work in this problem.

5-5. g)

```
1  | (x)-Px v C                P
   |---
2  | | (x)-Px                   A
   | |---
3  | | -Pâ                      2,UE
4  | | -Pâ v C                  3,vI
5  | | (x)(-Px v C)             4,UI
   |
6  | | C                        A
   | |---
7  | | -Pâ v C                  6,vI
8  | | (x)(-Px v C)             7,UI
9  | (x)(-Px v C)               1,2-5,6-8,AC
```


5-5. h)

```
1  | (x)(Rxb > Rax)            P
   |---
2  | | (x)Rxb                   A
   | |---
3  | | Rĉb                      2,UE
4  | | (x)(Rxb > Rax)           1,R
5  | | Rĉb > Raĉ                4,UE
6  | | Raĉ                      3,5,>E
7  | | (x)Rax                   6,UI
8  | (x)Rxb > (x)Rax            2-7,>I
```

5-5. i)

```
1  | (x)(Gxh > Gxm)            P
   |---
2  | Gâh > Gâm                  1,UE
3  | -Gâm > -Gâh                2,CP
4  | (x)(-Gxm > -Gxh)           3,UI
```

5-5. j)

```
1  | (x)(Mx > Nx)              P
2  | (x)(Nx > Ox)              P
   |---
3  | Mâ > Nâ                    1,UE
4  | Nâ > Oâ                    2,UE
5  | | Ma                       A
   | |---
6  | | Ma > Na                  3,R
7  | | Na                       5,6,>E
8  | | Na > Oa                  4,R
9  | | Oa                       7,8,>E
10 | Mâ > Oâ                    5-9,>I
11 | (x)(Mx > Ox)               10,UI
```

In lines 3 and 4, 'a' occurs arbitrarily. In line 5, 'a' occurs in an assumption, so 'a' does not occur arbitrarily while that assumption is in effect. When the assumption has been discharged in line 10, 'a' occurs arbitrarily again.

5-5. k)

```
1  | T > (x)Mdx                P
   |---
2  | | T                        A
   | |---
3  | | T > (x)Mdx               1,R
4  | | (x)Mdx                   2,3,>E
5  | | Mda                      4,UE
6  | T > Mdâ                    2-5,>I
7  | (x)(T > Mdx)               6,UI
```

5-5. l)

```
1  | (x)(Hff > Lxx)            P
   |---
2  | | Hff                      A
   | |---
3  | | (x)(Hff > Lxx)           1,R
4  | | Hff > Laa                3,UE
5  | | Laa                      2,4,>E
6  | | (x)Lxx                   5,UI
7  | Hff > (x)Lxx               2-6,>I
```

5-5. m)

```
1  | (x)Px v (x)Qx             P
   |---
2  | | (x)Px                    A
   | |---
3  | | Pâ                       2,UE
4  | | Pâ v Qâ                  3,vI
5  | | (x)(Px v Qx)             4,UI
   |
6  | | (x)Qx                    A
   | |---
7  | | Qâ                       6,UE
8  | | Pâ v Qâ                  7,vI
9  | | (x)(Px v Qx)             8,UI
10 | (x)(Px v Qx)               1,2-5,6-9,AC
```

5-5. n)

```
1  | (x)Hx                     P
2  | (Ex)Hx>(x)(Hx>Jx)         P
   |---
3  | Hâ                         1,UE
4  | (Ex)Hx                     3,EI
5  | (x)(Hx > Jx)               2,4,>E
6  | Hâ > Jâ                    5,UE
7  | Jâ                         3,6,>E
8  | (x)Jx                      7,UI
```

5-5. o)

```
1   |(x)(Sx<->Ox)          P
    |-
2   | |(x)Sx               A
    | |-
3   | |(x)(Sx<->Ox)        1,R
4   | |Sâ                  2,UE
5   | |Sâ<->Oâ             3,UE
6   | |Oâ                  4,5,<>E
7   | |(x)Ox               6,UI
    |
8   | |(x)Ox               A
    | |-
9   | |(x)(Sx<->Ox)        1,R
10  | |Oâ                  8,UE
11  | |Sâ<->Oâ             9,UE
12  | |Sâ                  10,11,<>E
13  | |(x)Sx               12,UI
14  |(x)Sx<->(x)Ox         2-7,8-13,<>I
```

5-5. p)

```
1   |(Ex)Px > A            P
    |-
2   | |Pa                  A
    | |-
3   | |(Ex)Px              2,EI
4   | |(Ex)Px > A          1,R
5   | |A                   3,4,>E
6   |Pâ > A                2-5,>I
7   |(x)(Px > A)           6,UI
```

5-5. q)

```
1   |-(Ex)Px               P
    |-
2   | |Pa                  A
    | |-
3   | |(Ex)Px              2,EI
4   | |-(Ex)Px             1,R
5   |-Pâ                   2-4,-I
6   |(x)-Px                5,UI
```

5-5. r)

```
1   |-(x)Px                P
    |-
2   | |-(Ex)-Px            A
    | |-
3   | | |-Pa               A
    | | |-
4   | | |(Ex)-Px           3,EI
5   | | |-(Ex)-Px          2,R
6   | |Pâ                  3-5,RD
7   | |(x)Px               6,UI
8   | |-(x)Px              1,R
9   |(Ex)-Px               2-8,RD
```

Note that lines 2-7 imitate 5-5. j), using -Px instead of Px.

5-5. s)

```
1   |(x)Px > A             P
    |-
2   | |-(Ex)(Px > A)       A
    | |-
3   | | |Pa > A            A
    | | |-
4   | | |(Ex)(Px > A)      3,EI
5   | | |-(Ex)(Px > A)     2,R
6   | |-(Pâ > A)           3-5,-I
7   | |Pâ & -A             6,C
8   | |Pâ                  7,&E
9   | |-A                  7,&E
10  | |(x)Px               8,UI
11  | |-(x)Px              1,9,DC
12  |(Ex)(Px > A)          2-11,RD
```

5-5. t)

```
1   |-(x)(Jx > -Kx)        P
    |-
2   | |-(Ex)(Jx & Kx)      A
    | |-
3   | | |Ja & Ka           A
    | | |-
4   | | |(Ex)(Jx & Kx)     3,EI
5   | | |-(Ex)(Jx & Kx)    2,R
6   | |-(Jâ & Kâ)          3-5,-I
7   | |-Jâ v -Kâ          6,DM
8   | |Jâ > -Kâ           7,C
9   | |(x)(Jx > -Kx)      8,UI
10  | |-(x)(Jx > -Kx)     1,R
11  |(Ex)(Jx & Kx)         2-10,RD
```


5-5. u)

```
1   |-(Ex)Qx v H           P
    |-
2   | |Qa                  A
    | |-
3   | |(Ex)Qx              2,EI
4   | |-(Ex)Qx v H         1,R
5   | |H                   3,4,vE
6   |Qâ > H                2-5,>I
7   |-Qâ v H               6,C
8   |(x)(-Qx v H)          7,UI
```

5-5. v)

```
1   |-(Ex)Dx               P
    |-
2   | |Da                  A
    | |-
3   | |(Ex)Dx              2,EI
4   | |-(Ex)Dx             1,R
5   |-Dâ                   2-4,-I
6   |-Kâ > -Dâ             5,W
7   |Dâ > Kâ               6,CP
8   |(x)(Dx > Kx)          7,UI
```

5-6.

```
1   |(y)(Ex)Lxy            P
    |-
2   |(Ex)Lxb               1,UE
3   |a|Lab                 A
    | |-
4   | |(y)Lay              MISTAKE!  b occurs in the assumption of
    | |                    the sub-derivation.  Since line 3 governs
    | |                    line 4, we can't apply UI to line 3.
    | |
5   | |(Ex)(y)Lxy          4,EI
6   |(Ex)(y)Lxy            2,3-5,EE
```

5-7. a)

```
1   |(Ex)Ix                P
2   |(x)(Ix > Jx)          P
    |-
3   |d|Id                  A
    | |-
4   | |(x)(Ix > Jx)        2,R
5   | |Id > Jd             4,UE
6   | |Jd                  3,5,>E
7   | |(Ex)Jx              6,EI
8   |(Ex)Jx                1,3-7,EE
```

5-7. b)

```
1   |(Ex)(A > Px)          P
    |-
2   |d|A > Pd              A
    | |-
3   | | |A                 A
    | | |-
4   | | |A > Pd            2,R
5   | | |Pd                3,4,>E
6   | | |(Ex)Px            5,EI
7   | |A > (Ex)Px          3-6,>I
8   |A > (Ex)Px            1,2-7,EE
```

This problem can be worked just as well by assuming 'A' first and then 'A > Pd'.

5-7. c)

```
1   | (Ex)Hmx                    P
2   | (x)(-Hmx v Gxn)            P
3   | d | Hmd                    A
4   |   | (x)(-Hmx v Gxn)        2,R
5   |   | -Hmd v Gdn             4,UE
6   |   | Gdn                    3,5,vE
7   |   | (Ex)Gxn                6,EI
8   | (Ex)Gxn                    1,3-7,EE
```

5-7. d)

```
1   | (Ex)(Cfx & Cxf)            P
2   | a | Cfa & Caf              A
3   |   | Cfa                    2,&E
4   |   | (Ex)Cfx                3,EI
5   | (Ex)Cfx                    1,2-4,EE
6   | b | Cfb & Cbf              A
7   |   | Cbf                    6,&E
8   |   | (Ex)Cxf                7,EI
9   | (Ex)Cxf                    1,6-8,EE
10  | (Ex)Cfx & (Ex)Cxf          5,9,&I
```

5-7. e)

```
1   | (Ex)(Px v Qx)              P
2   | a | Pa v Qa                A
3   |   | | Pa                   A
4   |   | | (Ex)Px               3,EI
5   |   | | (Ex)Px v (Ex)Qx      4,vI
6   |   | | Qa                   A
7   |   | | (Ex)Qx               6,EI
8   |   | | (Ex)Px v (Ex)Qx      7,vI
9   |   | (Ex)Px v (Ex)Qx        2,3-5,6-8,AC
10  | (Ex)Px v (Ex)Qx            1,2-9,EE
```

5-7. f)

```
1   | (Ex)Px v (Ex)Qx            P
2   | | (Ex)Px                   A
3   | | d | Pd                   A
4   | |   | Pd v Qd              3,vI
5   | |   | (Ex)(Px v Qx)        4,EI
6   | | (Ex)(Px v Qx)            2,3-5,EE
7   | | (Ex)Qx                   A
8   | | e | Qe                   A
9   | |   | Pe v Qe              8,vI
10  | |   | (Ex)(Px v Qx)        9,EI
11  | | (Ex)(Px v Qx)            7,8-10,EE
12  | (Ex)(Px v Qx)
                                 1,2-6,7-11,AC
```

5-7. g)

```
1   | (Ex)(Px > A)               P
2   | a | Pa > A                 A
3   |   | | (x)Px                A
4   |   | | Pa                   3,UE
5   |   | | Pa > A               2,R
6   |   | | A                    4,5,>E
7   |   | (x)Px > A              3-6,>I
8   | (x)Px > A                  1,2-7,EE
```

5-7. h)

```
1   | (x)(Px > A)                P
2   | | (Ex)Px                   A
3   | | d | Pd                   A
4   | |   | (x)(Px > A)          1,R
5   | |   | Pd > A               4,UE
6   | |   | A                    3,5,>E
7   | | A                        2,3-6,EE
8   | (Ex)Px > A                 2-7,>I
```

5-7. i)

```
1   | (Ex)(Lxa<->Lex)            P
2   | (x)Lxa                     P
3   | b | Lba<->Leb              A
4   |   | (x)Lxa                 2,R
5   |   | Lba                    4,UE
6   |   | Leb                    3,5,<>E
7   |   | (Ex)Lex                6,EI
8   | (Ex)Lex                    1,3-7,EE
```

5-7. j)

```
1   | (x)(Gsx > -Gxs)            P
2   | | (Ex)Gxs                  A
3   | | d | Gds                  A
4   | |   | (x)(Gsx > -Gxs)      1,R
5   | |   | Gsd > -Gds           4,UE
6   | |   | -Gsd                 3,5,DC
7   | |   | (Ex)-Gsx             6,EI
8   | | (Ex)-Gsx                 2,3-7,EE
9   | (Ex)Gxs > (Ex)-Gsx         2-8,>I
```

5-7. k)

```
1   | (Ex)(Px v Qx)              P
2   | (x)(Px > Kx)               P
3   | (x)(Qx > Kx)               P
4   | d | Pd v Qd                A
5   |   | | Pd                   A
6   |   | | (x)(Px > Kx)         2,R
7   |   | | Pd > Kd              6,UE
8   |   | | Kd                   5,7,>E
9   |   | | (Ex)Kx               8,EI
10  |   | | Qd                   A
11  |   | | (x)(Qx > Kx)         3,R
12  |   | | Qd > Kd              11,UE
13  |   | | Kd                   10,12,>E
14  |   | | (Ex)Kx               13,EI
15  |   | (Ex)Kx                 4,5-9,10-14,AC
16  | (Ex)Kx                     1,4-15,EE
```

5-7. l)

```
1   | (Ex)(-Mxt v Mtx)           P
2   | (x)(Mtx > Axx)             P
3   | d | -Mdt v Mtd             A
4   |   | (x)(Mtx > Axx)         2,R
5   |   | Mtd > Add              4,UE
6   |   | | Mdt                  A
7   |   | | -Mdt v Mtd           3,R
8   |   | | Mtd                  6,7,vE
9   |   | | Mtd > Add            5,R
10  |   | | Add                  8,9,>E
11  |   | Mdt > Add              6-10,>I
12  |   | -Mdt v Add             11,C
13  |   | (Ex)(-Mxt v Axx)       12,EI
14  | (Ex)(-Mxt v Axx)           1,3-13,EE
```

5-7. m)

```
1   | (Ex)Hxg v (Ex)Nxf          P
2   | (x)(Hxg > Cx)              P
3   | (x)(Nxf > Cx)              P
4   | | (Ex)Hxg                  A
5   | | d | Hdg                  A
6   | |   | (x)(Hxg > Cx)        2,R
7   | |   | Hdg > Cd             6,UE
8   | |   | Cd                   5,7,>E
9   | |   | (Ex)Cx               8,EI
10  | | (Ex)Cx                   4,5-9,EE
11  | | (Ex)Nxf                  A
12  | | e | Nef                  A
13  | |   | (x)(Nxf > Cx)        3,R
14  | |   | Nef > Ce             13,UE
15  | |   | Ce                   12,14,>E
16  | |   | (Ex)Cx               15,EI
17  | | (Ex)Cx                   11,12-16,EE
18  | (Ex)Cx                     1,4-10,11-17,AC
```

5-7. n)

```
1  | (x)((Fx v Gx) > Lxx)     P
2  | (Ex)-Lxx                 P
   |----------
3  | d| -Ldd                  A
   |  |----------
4  |  | (x)((Fx v Gx) > Lxx)   1,R
5  |  | (Fd v Gd) > Ldd        4,UE
6  |  | -(Fd v Gd)             3,5,DC
7  |  | -Fd & -Gd              6,DM
8  |  | -Fd                    7,&E
9  |  | -Gd                    7,&E
10 |  | (Ex)-Fx                8,EI
11 |  | (Ex)-Gx                9,EI
12 |  | (Ex)-Fx & (Ex)-Gx      10,11,&I
13 | (Ex)-Fx & (Ex)-Gx        2,3-12,EE
```

5-7. o)

```
1  | (x)(Fx > (Rxa v Rax))    P
2  | (Ex)-Rxa                 P
   |----------
3  | d| -Rda                  A
   |  |----------
4  |  | | (x)-Rax             A
   |  | |----------
5  |  | | -Rad                4,UE
6  |  | | (x)(Fx > (Rxa v Rax))   1,R
7  |  | | Fd > (Rda v Rad)    6,UE
8  |  | | -Rda                3,R
9  |  | | -Rda & -Rad         5,8,&I
10 |  | | -(Rda v Rad)        9,DM
11 |  | | -Fd                 7,10,DC
12 |  | | (Ex)-Fx             11,EI
13 |  | (x)-Rax > (Ex)-Fx     4-12,>I
14 | (x)-Rax > (Ex)-Fx        2,3-13,EE
```

5-7. p)

```
1  | (Ex)Qxj                      P
2  | (Ex)(Qxj v Dgx) > (x)Dgx     P
   |----------
3  | d| Qdj                       A
   |  |----------
4  |  | Qdj v Dgd                 3,vI
5  |  | (Ex)(Qxj v Dgx)           4,EI
6  |  | (Ex)(Qxj v Dgx) > (x)Dgx  2,R
7  |  | (x)Dgx                    5,6,>E
8  |  | Dga                       7,UE
9  |  | Dga v Qja                 8,vI
10 |  | (x)(Dgx v Qjx)            9,UI
11 | (x)(Dgx v Qjx)               1,3-10,EE
```

5-7. q)

```
1  | (x)-Fx                   P
   |----------
2  | | (Ex)Fx                 A
   | |----------
3  | | d| Fd                  A
   | |  |----------
4  | |  | (x)-Fx              1,R
5  | |  | -Fd                 4,UE
6  | |  | | -(A & -A)         A
   | |  | |----------
7  | |  | | Fd                3,R
8  | |  | | -Fd               5,R
9  | |  | A & -A              6-8,RD
10 | | A & -A                 2,3-9,EE
11 | | A                      10,&E
12 | | -A                     10,&E
13 | -(Ex)Fx                  2-12,-I
```

Notice that in q) we cannot take our contradiction in the form of 'Fd' and '-Fd' from the innermost to the second sub-derivation because 'd' is isolated to the innermost derivation. We solve this problem by using 'Fd' and '-Fd' to derive another contradiction which does not use 'd'. The need to do this arises in the next few problems, so we introduce two new derived rules to simplify our work.

```
X            X            -X
-X           Y&-Y         Y&-Y


Y&-Y  CD     -X    -I     X     RD
       |
for "contradiction"
```


5-7. r)

```
1  | (Ex)-Fx                  P
   |----------
2  | | (x)Fx                  A
   | |----------
3  | | (Ex)-Fx                1,R
4  | | a| -Fa                 A
   | |  |----------
5  | |  | (x)Fx               2,R
6  | |  | Fa                  5,UE
7  | |  | A & -A              4,6,CD
8  | | A & -A                 3,4-7,EE
9  | -(x)Fx                   2-8,-I
```

5-7. s)

```
1  | (x)(Jxx > -Jxf)          P
   |----------
2  | | (Ex)(Jxx & Jxf)        A
   | |----------
3  | | d| Jdd & Jdf           A
   | |  |----------
4  | |  | (x)(Jxx > -Jxf)     1,R
5  | |  | Jdd > -Jdf          4,UE
6  | |  | Jdd                 3,&E
7  | |  | Jdf                 3,&E
8  | |  | -Jdf                5,6,>E
9  | |  | A & -A              7,8,CD
10 | | A & -A                 2,3-9,EE
11 | -(Ex)(Jxx & Jxf)         2-10,-I
```

5-7. t)

```
1  | (Ex)Px v Qa              P
2  | (x)-Px                   P
   |----------
3  | | Qa                     A
   | |----------
4  | | (Ex)Qx                 3,EI
5  | | (Ex)Px                 A
   | |----------
6  | | d| Pd                  A
   | |  |----------
7  | |  | (x)-Px              2,R
8  | |  | -Pd                 7,UE
9  | |  | | -(Ex)Qx           A
   | |  | |----------
10 | |  | | Pd                6,R
11 | |  | | -Pd               8,R
12 | |  | (Ex)Qx              9-11,RD
13 | | (Ex)Qx                 5,6-12,EE
14 | (Ex)Qx                   1,3-4,5-13,AC
```

5-7. u)

```
1  | A > (Ex)Px               P
   |----------
2  | -A v (Ex)Px              1,C
3  | | -A                     A
   | |----------
4  | | -A v Pâ                3,vI
5  | | A > Pâ                 4,C
6  | | (Ex)(A > Px)           5,EI
7  | | (Ex)Px                 A
   | |----------
8  | | d| Pd                  A
   | |  |----------
9  | |  | A > Pd              8,W
10 | |  | (Ex)(A > Px)        9,EI
11 | | (Ex)(A > Px)           7,8-10,EE
12 | (Ex)(A > Px)             2,3-6,7-11,AC
```

5-8 i) Assume the rules in the text.

```
1  |(Eu)(...u...)                     Input for derived rule
2  |(u)((...u...) > X)

3  |b|(...b...)              A
   | I-----------------------
4  | |(u)((...u...) > X)    2,R
5  | |(...b...) > X         4,UE
6  | |X                     3,5,>E
7  |X                        1,3-6,EE
```

ii) Assume all rules in the text except EE. Also assume the derived rule given in 5-8 i) (derived form of EE).

```
1  |(Eu)(...u...)                     Input for derived rule

2  | |(...b...)           A        Choose b as a completely new name
   | I---------------------         so it will be arbitrary in line 4.
3  | |X

4  |(...b̂...) > X         2-3,>I
5  |(u)((...u...) > X)     4,UI
6  |X                      1,5,EE (derived form)
```

Note that in part ii) of the proof we are using the derived EE rule to get the primitive EE rule. Thus b, which appeared in the sub-derivation 2-3, can appear in line 4. But we require b to be isolated to lines 2-4 to guarantee that b is arbitrary at line 4.

6-1. a)

```
1  |(Ex)Lxx              P
   I----------------------
2  |a|Laa                A
   | I--------------------
3  | |(Ey)Lay            2,EI
4  | |(Ex)(Ey)Lxy        3,EI
5  |(Ex)(Ey)Lxy          1,2-4,EE
```

counterexample to $\dfrac{\text{(x)Lxx}}{\text{(x)(y)Lxy}}$:

D={a,b} ; Laa & -Lab & -Lba & Lbb.

The relevant feature of the rules is this: in applying EI you can existentially generalize on just one instance of a name with more than one occurence. But in applying UI you must universally generalize on all occurences of the name on which you are generalizing.

6-1. b)

```
1  |(x)(y)Lxy            P
   I----------------------
2  |(y)Lây               1,UE
3  |Lââ                  2,UE
4  |(x)Lxx               3,UI
```

counterexample to $\dfrac{\text{(Ex)(Ey)Lxy}}{\text{(Ex)Lxx}}$:

D={a,b} ; -Laa & Lab & Lba & -Lbb

The relevant feature of the rules is this: in applying UE you are free to instantiate a universally quantified sentence with a name that already occurs. But in applying EE you must start a sub-derivation, instantiating the existentially quantified sentence with an isolated name, and so a name which is new to the whole derivation.

6-1. c)

```
1  |(x)(y)Lxy            P
   I----------------------
2  |(y)Lây               1,UE
3  |Lâb̂                  2,UE
4  |(x)Lxb̂               3,UI
5  |(y)(x)Lxy            4,UI
```

6-1. d)

```
1 | (Ex)(Ey)Lxy              P
2 | a| (Ey)Lay               A
3 | | b| Lab                 A
4 | | | | (Ex)Lxb           3,EI
5 | | | | (Ey)(Ex)Lxy      4,EI
6 | | (Ey)(Ex)Lxy          2,3-5,EE
7 | (Ey)(Ex)Lxy              1,2-6,EE
```

6-1. e)

```
1 | (Ex)(y)Lxy               P
2 | a| (y)Lay                A
3 | | Lab̂                    2,UE
4 | | (Ex)Lxb̂               3,EI
5 | | (y)(Ex)Lxy             4,UI
6 | (y)(Ex)Lxy               1,2-5,EE
```

counterexample to $\frac{\text{(y)(Ex)Lxy}}{\text{(Ex)(y)Lxy}}$:

D={a,b} ; Laa & -Lab & Lba & -Lbb

6-1. f)

```
1 | (x)Px & (x)Qx            P
2 | (x)Px                    1,&E
3 | (x)Qx                    1,&E
4 | Pâ                       2,UE
5 | Qb̂                       3,UE
6 | Pâ & Qb̂                  4,5,&I
7 | (y)(Pâ & Qy)             6,UI
8 | (x)(y)(Px & Qy)          7,UI
```

6-1. g)

```
1 | (Ex)Px & (Ex)Qx          P
2 | (Ex)Px                   1,&E
3 | (Ex)Qx                   1,&E
4 | a| Pa                    A
5 | | (Ex)Qx                 3,R
6 | | b| Qb                  A
7 | | | Pa                   4,R
8 | | | Pa & Qb              6,7,&I
9 | | | (Ey)(Pa & Qy)           8,EI
10 | | | (Ex)(Ey)(Px & Qy)     9,EI
11 | | (Ex)(Ey)(Px & Qy)        5,6-10,EE
12 | (Ex)(Ey)(Px & Qy)
2,4-11,EE
```

6-1. h)

```
1 | (x)Px v (x)Qx            P
2 | | (x)Px                  A
3 | | Pâ                     2,UE
4 | | Pâ v Qb̂                3,vI
5 | | (y)(Pâ v Qy)           4,UI
6 | | (x)(y)(Px v Qy)        5,UI
7 | | (x)Qx                  A
8 | | Qb̂                     7,UE
9 | | Pâ v Qb̂                8,vI
10 | | (y)(Pâ v Qy)          9,UI
11 | | (x)(y)(Px v Qy)       10,UI
12 | (x)(y)(Px v Qy)
1,2-6,7-11,AC
```

6-1. i)

```
1 | (Ex)Px v (Ex)Qx          P
2 | | (Ex)Px                 A
3 | | a| Pa                  A
4 | | | Pa v Qb              3,vI
5 | | | (Ey)(Pa v Qy)        4,EI
6 | | | (Ex)(Ey)(Px v Qy)   5,EI
7 | | (Ex)(Ey)(Px v Qy)     2,3-6,EE
8 | | (Ex)Qx                 A
9 | | a| Qa                  A
10 | | | Pb v Qa             9,vI
11 | | | (Ey)(Pb v Qy)       10,EI
12 | | | (Ex)(Ey)(Px v Qy)  11,EI
13 | | (Ex)(Ey)(Px v Qy)     8,9-12,EE
14 | (Ex)(Ey)(Px v Qy)     1,2-7,8-13,AC
```

6-1. j)

```
1 | (Ex)(Ey)(PxvQy)          P
2 | a| (Ey)(Pa v Qy)         A
3 | | b| Pa v Qb             A
4 | | | | Pa                 A
5 | | | | (Ex)Px            4,EI
6 | | | | (Ex)Px v (Ex)Qx  5,vI
7 | | | | Qb                 A
8 | | | | (Ex)Qx            7,EI
9 | | | | (Ex)Px v (Ex)Qx  8,vI
10 | | | (Ex)Px v (Ex)Qx     3,4-6,7-9,AC
11 | | (Ex)Px v (Ex)Qx      2,3-10,EE
12 | (Ex)Px v (Ex)Qx        1,2-11,EE
```

6-1. k)

```
1 | (x)(y)(Lxy > -Lxy)       P
2 | (y)(Lây > -Lây)          1,UE
3 | Lââ > -Lââ               2,UE
4 | | Laa                    A
5 | | Laa > -Laa             3,R
6 | | -Laa                   4,5,>E
7 | -Lââ                     4-6,-I
8 | (x)-Lxx                  7,UI
```

6-1. l)

```
1   | (x)(y)(Px > Qy)          P
    |_____________
2   |  | (Ex)Px                A
    |  |______
3   |  | a| Pa                 A
    |  |  |___
4   |  |  | (x)(y)(Px > Qy)    1,R
5   |  |  | (y)(Pa > Qy)       4,UE
6   |  |  | Pa > Qb            5,UE
7   |  |  | Qb                 3,6,>E
8   |  |  | (x)Qx              7,UI
9   |  | (x)Qx                 2,3-8,EE
10  | (Ex)Px > (x)Qx           2-9,>I
```

6-1. m)

```
1   | (Ex)(Ey)(Px > Qy)        P
    |_____________
2   |  | (x)Px                 A
    |  |______
3   |  | (Ex)(Ey)(Px > Qy)     1,R
4   |  | a| (Ey)(Pa > Qy)      A
    |  |  |___
5   |  |  | b| Pa > Qb         A
    |  |  |  |___
6   |  |  |  | (x)Px           2,R
7   |  |  |  | Pa              6,UE
8   |  |  |  | Qb              5,7,>E
9   |  |  |  | (Ex)Qx          8,EI
10  |  |  | (Ex)Qx             4,5-9,EE
11  |  | (Ex)Qx                3,4-10,EE
12  | (x)Px > (Ex)Qx           2-11,>I
```

6-1. n)

```
1   | (Ex)(y)(Px > Qy)         P
    |_____________
2   | a| (y)(Pa > Qy)          A
    |  |___
3   |  |  | (x)Px              A
    |  |  |______
4   |  |  | (y)(Pa > Qy)       2,R
5   |  |  | Pa > Qb            4,UE
6   |  |  | Pa                 3,UE
7   |  |  | Qb                 5,6,>E
8   |  |  | (x)Qx              7,UI
9   |  | (x)Px > (x)Qx         3-8,>I
10  | (x)Px > (x)Qx            1,2-9,EE
```

6-1. o)

```
1   | (x)(Ey)(Px > Qy)         P
    |_____________
2   |  | (Ex)Px                A
    |  |______
3   |  | a| Pa                 A
    |  |  |___
4   |  |  | (x)(Ey)(Px > Qy)   1,R
5   |  |  | (Ey)(Pa > Qy)      4,UE
6   |  |  | b| Pa > Qb         A
    |  |  |  |___
7   |  |  |  | Pa              3,R
8   |  |  |  | Qb              6,7,>E
9   |  |  |  | (Ex)Qx          8,EI
10  |  |  | (Ex)Qx             5,6-9,EE
11  |  | (Ex)Qx                2,3-10,EE
12  | (Ex)Px > (Ex)Qx          2-11,>I
```

6-1. p)

```
1   | (x)Px > (x)Qx            P
    |_____________
2   |  | -(Ex)(y)(Px > Qy)     A
    |  |______
3   |  |  | -Pa                A
    |  |  |___
4   |  |  | -Qb > -Pa          3,W
5   |  |  | Pa > Qb            4,CP
6   |  |  | (y)(Pa > Qy)       5,UI
7   |  |  | (Ex)(y)(Px > Qy)   6,EI
8   |  |  | -(Ex)(y)(Px > Qy)  2,R
9   |  | Pa                    3-8,RD
10  |  | (x)Px                 9,UI
11  |  | (x)Px > (x)Qx         1,R
12  |  | (x)Qx                 10,11,>E
13  |  | Qb                    12,UE
14  |  | Pa > Qb               13,W
15  |  | (y)(Pa > Qy)          14,UI
16  |  | (Ex)(y)(Px > Qy)      15,EI
17  | (Ex)(y)(Px > Qy)         2-16,RD
```

6-1. q)

```
1   | (x)(y)(Px v Qy)          P
    |_____________
2   |  | -((x)Px v (x)Qx)      A
    |  |______
3   |  |  | (x)Px              A
    |  |  |___
4   |  |  | (x)Px v (x)Qx      3,vI
5   |  |  | -((x)Px v (x)Qx)   2,R
6   |  | -(x)Px                3-5,-I
7   |  |  | (x)Qx              A
    |  |  |___
8   |  |  | (x)Px v (x)Qx      7,vI
9   |  |  | -((x)Px v (x)Qx)   2,R
10  |  | -(x)Qx                7-9,-I
11  |  |  | -Pa                A
    |  |  |___
12  |  |  | (x)(y)(Px v Qy)    1,R
13  |  |  | (y)(Pa v Qy)       12,UE
14  |  |  | Pa v Qb            13,UE
15  |  |  | Qb                 11,14,vE
16  |  |  | (x)Qx              15,UI
17  |  |  | -(x)Qx             10,R
18  |  | Pa                    11-17,RD
19  |  | (x)Px                 18,UI
20  | (x)Px v (x)Qx            2-19,RD
```

6-1. r)

```
1   | (Ex)(y)Jxy                         P
2   | (Ey)(Ez)(Hzy & -Py)                P
3   | (z)(w)((Jzw & -Pw) > Gz)           P
    |_____________
4   | a| (y)Jay                          A
    |  |______
5   |  | (Ey)(Ez)(Hzy & -Py)             2,R
6   |  | b| (Ez)(Hzb & -Pb)              A
    |  |  |___
7   |  |  | c| Hcb & -Pb                 A
    |  |  |  |___
8   |  |  |  | (z)(w)((Jzw & -Pw) > Gz)  3,R
9   |  |  |  | (w)((Jaw & -Pw) > Ga)     8,UE
10  |  |  |  | (Jab & -Pb) > Ga          9,UE
11  |  |  |  | -Pb                       7,&E
12  |  |  |  | (y)Jay                    4,R
13  |  |  |  | Jab                       12,UE
14  |  |  |  | Jab & -Pb                 11,13,&I
15  |  |  |  | Ga                        10,14,>E
16  |  |  |  | (Ez)Gz                    15,EI
17  |  |  | (Ez)Gz                      6,7-16,EE
18  |  | (Ez)Gz                         5,6-17,EE
19  | (Ez)Gz                            1,4-18,EE
```

6-2. a)

```
1   | (x)Px                    P
2   | (x)-Qx                   P
3   | | (Ex)(Px<->Qx)          A
4   | | a| Pa<->Qa             A
5   | |  | (x)Px               1,R
6   | |  | (x)-Qx              2,R
7   | |  | Pa                  5,UE
8   | |  | Qa                  4,7,<>E
9   | |  | -Qa                 6,UE
10  | |  | A & -A              8,9,CD
11  | | A & -A                 3,4-10,EE
12  | -(Ex)(Px<->Qx)           3-11,-I
```

6-2. b)

```
1   | (x)(Fx > Gx)             P
2   | (x)(Gx > Hx)             P
3   | -(Ex)Hx                  P
4   | (x)-Hx                   3,-]
5   | -Hâ                      4,UE
6   | Gâ > Hâ                  2,UE
7   | -Gâ                      5,6,DC
8   | Fâ > Gâ                  1,UE
9   | -Fâ                      7,8,DC
10  | (x)-Fx                   9,UI
11  | -(Ex)Fx                  10,U-
```

6-2. c)

```
1   | -(x)(y)Lxy               P
2   | (Ex)-(y)Lxy              1,-U
3   | a| -(y)Lay               A
4   |  | (Ey)-Lay              3,-U
5   |  | (Ex)(Ey)-Lxy          4,EI
6   | (Ex)(Ey)-Lxy             2,3-5,EE
```

6-2. d)

```
1   | -(Ex)(Ey)Lxy             P
2   | (x)-(Ey)Lxy              1,-]
3   | -(Ey)Lây                 2,UE
4   | (y)-Lây                  3,-]
5   | (x)(y)-Lxy               4,UI
```

6-2. e)

```
1   | -(Ex)(Px v Qx)           P
2   | (x)-(Px v Qx)            1,-]
3   | -(Pâ v Qâ)               2,UE
4   | -Pâ & -Qâ                3,DM
5   | -Pâ                      4,&E
6   | -Qâ                      4,&E
7   | (x)-Px                   5,UI
8   | (x)-Qx                   6,UI
9   | (x)-Px & (x)-Qx          7,8,&I
```

6-2. f)

```
1   | -(x)(Px & Qx)                P
2   | (Ex)-(Px & Qx)               1,-U
3   | a| -(Pa & Qa)                A
4   |  | -Pa v -Qa                 3,DM
5   |  | | -Pa                     A
6   |  | | (Ex)-Px                 5,EI
7   |  | | (Ex)-Px v (Ex)-Qx       6,vI
8   |  | | -Qa                     A
9   |  | | (Ex)-Qx                 8,EI
10  |  | | (Ex)-Px v (Ex)-Qx       9,vI
11  |  | (Ex)-Px v (Ex)-Qx
                              4,5-7,8-10,AC
12  | (Ex)-Px v (Ex)-Qx            2,3-11,EE
```

6-2. g)

```
1   | (x)(-(Ey)Rxy & -(Ey)Ryx)     P
2   | -(Ey)Rây & -(Ey)Ryâ          1,UE
3   | -(Ey)Rây                     2,&E
4   | (y)-Rây                      3,-]
5   | (x)(y)-Rxy                   4,UI
```


6-2. h)

```
1   | (Ex)(Px > (y)(Py > Qy))       P
2   | -(Ex)Qx                       P
3   | | (x)Px                       A
4   | | (Ex)(Px > (y)(Py > Qy))     1,R
5   | | a| Pa > (y)(Py > Qy)        A
6   | |  | (x)Px                    3,R
7   | |  | Pa                       6,UE
8   | |  | (y)(Py > Qy)             5,7,>E
9   | |  | Pa > Qa                  8,UE
10  | |  | Qa                       7,9,>E
11  | |  | (Ex)Qx                   10,EI
12  | |  | -(Ex)Qx                  2,R
13  | |  | A & -A                   11,12,CD
14  | | A & -A                      4,5-13,EE
15  | -(x)Px                        3-14,-I
```

Notes: I could have applied -] to line 2 instead of lines 11 and 12, but it would have taken an additional step. Also I instantiated line 4 before line 5. Work on existential generalizations before universal generalizations.

6-2. i)

```
1   | (Ey)(Ez)((x)-Rxy v (x)-Rxz)          P
2   | | (y)(z)(Ex)(Rxy & Rxz)              A
3   | | (Ey)(Ez)((x)-Rxy v (x)-Rxz)        1,R
4   | | a| (Ez)((x)-Rxa v (x)-Rxz)         A
5   | |  | b| (x)-Rxa v (x)-Rxb            A
6   | |  |  | (y)(z)(Ex)(Rxy & Rxz)        2,R
7   | |  |  | (z)(Ex)(Rxa & Rxz)           6,UE
8   | |  |  | (Ex)(Rxa & Rxb)              7,UE
9   | |  |  | c| Rca & Rcb                 A
10  | |  |  |  | Rca                       9,&E
11  | |  |  |  | Rcb                       9,&E
12  | |  |  |  | (x)-Rxa v (x)-Rxb         5,R
13  | |  |  |  | | (x)-Rxa                 A
14  | |  |  |  | | -Rca                    13,UE
15  | |  |  |  | | Rca                     10,R
16  | |  |  |  | -(x)-Rxa                  13-15,-I
17  | |  |  |  | (x)-Rxb                   12,16,vE
18  | |  |  |  | -Rcb                      17,UE
19  | |  |  |  | A & -A                    11,18,CD
20  | |  |  | A & -A                       8,9-19,EE
21  | |  | A & -A                          4,5-20,EE
22  | | A & -A                             3,4-21,EE
23  | -(y)(z)(Ex)(Rxy & Rxz)               2-22,-I
```

6-3. a)

```
1 | |(x)(y)Lxy                A
  | I-------------
2 | |(y)Lây                   1,UE
3 | |Lâb̂                      2,UE
4 | |(Ey)Lây                  3,EI
5 | |(Ex)(Ey)Lxy              4,EI
6 |(x)(y)Lxy>(Ex)(Ey)Lxy   1-5,>I
```

6-3. b)

```
1 | |-(Ga v -Ga)              A
  | I-------------
2 | |-Ga & --Ga               1,DM
3 | |-Ga                      2,&E
4 | |--Ga                     2,&E
5 | |Ga                       4,-E
6 |Gâ v -Gâ          3,5,(1-5),RD
7 |(x)(Gx v -Gx)            6,UI
```

6-3. c)

```
1  | |(x)(Ey)(Ax & By)          A
   | I-------------
2  | |(Ey)(Aâ & By)             1,UE
3  | |b|Aa & Bb                 A
   | | I---------
4  | | |(x)(Ey)(Ax & By)        1,R
5  | | |(Ey)(Ab & By)           4,UE
6  | | |c|Ab & Bc               A
   | | | I---------
7  | | | |Aa & Bb               3,R
8  | | | |Bb                    7,&E
9  | | | |Ab                    6,&E
10 | | | |Ab & Bb               9,8,&I
11 | | | |(Ex)(Ax & Bx)         10,EI
12 | | |(Ex)(Ax & Bx)           5,6-11,EE
13 | |(Ex)(Ax & Bx)             2,3-12,EE
14 |(x)(Ey)(Ax&By)>(Ex)(Ax&Bx)     1-13,>I
```

6-3. d)

```
1  | |(Ey)(Ky&(x)(Dx>Rxy))            A
   | I-------------
2  | |a|Ka & (x)(Dx > Rxa)            A
   | | I---------
3  | | |Ka                            2,&E
4  | | |(x)(Dx > Rxa)                 2,&E
5  | | |Db̂ > Rb̂a                      4,UE
6  | | | |Db                          A
   | | | I-----
7  | | | |Db > Rba                          5,R
8  | | | |Rba                               6,7,>E
9  | | | |Ka                                3,R
10 | | | |Ka & Rba                          8,9,&I
11 | | | |(Ey)(Ky & Rby)                    10,EI
12 | | |Db̂>(Ey)(Ky&Rby)                     6-11,>I
13 | | |(x)(Dx>(Ey)(Ky&Rxy))                12,UI
14 | |(x)(Dx>(Ey)(Ky&Rxy))                  1,2-13,EE
15 |(Ey)(Ky&(x)(Dx>Rxy))>(x)(Dx>(Ey)(Ky&Rxy))        1-14,>I
```


6-3. e)

```
1  | |-(Ex)(y)(Fy>Fx)           A
   | I-------------
2  | | |Fa                      A
   | | I-----
3  | | |Fĉ > Fa                 2,W
4  | | |(y)(Fy > Fa)            3,UI
5  | | |(Ex)(y)(Fy > Fx)        4,EI
6  | | |-(Ex)(y)(Fy>Fx)         1,R
7  | |-Fâ                       2-6,-I
8  | |-Fb̂ > -Fâ                 7,W
9  | |Fâ > Fb̂                   8,CP
10 | |(y)(Fy > Fb̂)             9,UI
11 | |(Ex)(y)(Fy > Fx)          10,EI
12 |(Ex)(y)(Fy > Fx)           1-11,RD
```

6-3. f)

```
1  | |-(x)(Ey)(Fy>Fx)           A
   | I-------------
2  | | |-Fa                     A
   | | I-----
3  | | |-Fc > -Fa               2,W
4  | | |Fa > Fc                 3,CP
5  | | |(Ey)(Fy > Fc)           4,EI
6  | | |(x)(Ey)(Fy > Fx)        5,UI
7  | | |-(x)(Ey)(Fy>Fx)         1,R
8  | |Fâ                        2-7,RD
9  | |Fb̂ > Fâ                   8,W
10 | |(Ey)(Fy > Fâ)             9,EI
11 | |(x)(Ey)(Fy > Fx)          10,UI
12 |(x)(Ey)(Fy > Fx)           1-11,RD
```

6-3. g)

```
1  | |-(Ex)(y)(Fx>Fy)           A
   | I-------------
2  | | |-Fb                     A
   | | I-----
3  | | |-Fa > -Fb               2,W
4  | | |Fb > Fa                 3,CP
5  | | |(y)(Fb > Fy)            4,UI
6  | | |(Ex)(y)(Fx > Fy)        5,EI
7  | | |-(Ex)(y)(Fx>Fy)         1,R
8  | |Fb̂                        2-7,RD
9  | |Fâ > Fb̂                   8,W
10 | |(y)(Fâ > Fy)              9,UI
11 | |(Ex)(y)(Fx > Fy)          10,EI
12 |(Ex)(y)(Fx > Fy)           1-11,RD
```

6-4. a)

```
1  | |(x)(Ax>Bx)&(Ex)(-Bx&(y)Ay)        P
   | I-------------
2  | |(x)(Ax > Bx)             1,&E
3  | |(Ex)(-Bx&(y)Ay)          1,&E
4  | |a|-Ba & (y)Ay            A
   | | I---------
5  | | |-Ba                    4,&E
6  | | |(y)Ay                  4,&E
7  | | |(x)(Ax > Bx)           2,R
8  | | |Aa > Ba                7,UE
9  | | |Aa                     6,UE
10 | | |Ba                     8,9,>E
11 | | |A & -A                 5,10,CD
12 | |A & -A                   3,4-11,EE
```

6-4. b)

```
1 | |(x)(Rxb>-Rxb)&(Ex)Rxb        P
  | I-------------
2 | |(x)(Rxb > -Rxb)          1,&E
3 | |(Ex)Rxb                  1,&E
4 | |a|Rab                    A
  | | I-----
5 | | |(x)(Rxb > -Rxb)          2,R
6 | | |Rab > -Rab             5,UE
7 | | |-Rab                   4,6,>E
8 | | |A & -A                 4,7,CD
9 | |A & -A                   3,4-8,EE
```

6-4. c)

```
1  | |(x)((y)Lxy&(Ey)-Lyx)        P
   | I-------------
2  | |(y)Lây&(Ey)-Lyâ          1,UE
3  | |(Ey)-Lyâ                 2,&E
4  | |b|-Lba                   A
   | | I-----
5  | | |(x)((y)Lxy&(Ey)-Lyx)    1,R
6  | | |(y)Lby&(Ey)-Lyb         5,UE
7  | | |(y)Lby                  6,&E
8  | | |Lba                    7,UE
9  | | |A & -A                4,8,CD
10 | |A & -A                 3,4-9,EE
```

6-4. d)

```
1   |(x)(Ey)(Mx&-My)        P
    |___________________
2   |(Ey)(Ma & -My)         1,UE
3   |b|Ma & -Mb             A
    | |__________
4   | |-Mb                  3,&E
5   | |(x)(Ey)(Mx&-My)      1,R
6   | |(Ey)(Mb & -My)       5,UE
7   | |c|Mb & -Mc           A
    | | |__________
8   | | |Mb                 7,&E
9   | | |-Mb                4,R
10  | | |A & -A             8,9,CD
11  | |A & -A               6,7-10,EE
12  |A & -A                 2,3-11,EE
```

6-4. e)

```
1   |(x)(Ey)(w)(Ez)(Lxw&-Lyz)          P
    |__________________________
2   |(Ey)(w)(Ez)(Law&-Lyz)             1,UE
3   |b|(w)(Ez)(Law&-Lbz)               A
    | |___________________
4   | |(Ez)(Lab & -Lbz)                3,UE
5   | |c|Lab & -Lbc                    A
    | | |____________
6   | | |(x)(Ey)(w)(Ez)(Lxw&-Lyz)      1,R
7   | | |(Ey)(w)(Ez)(Lbw&-Lyz)         6,UE
8   | | |d|(w)(Ez)(Lbw&-Ldz)           A
    | | | |___________________
9   | | | |(Ez)(Lbc & -Ldz)            8,UE
10  | | | |e|Lbc & -Lde                A
    | | | | |____________
11  | | | | |Lbc                       10,&E
12  | | | | |Lab & -Lbc                5,R
13  | | | | |-Lbc                      12,&E
14  | | | | |A & -A                    11,13,CD
15  | | | |A & -A                      9,10-14,EE
16  | | |A & -A                        7,8-15,EE
17  | |A & -A                          4,5-16,EE
18  |A & -A                            2,3-17,EE
```

6-5. a)

```
1   |(x)Kx                 P
2   |(y)-(Ky v Lya)        P
    |_______________
3   |Ka                    1,UE
4   |-(Ka v Laa)           2,UE
5   |-Ka & -Laa            4,DM
6   |-Ka                   5,&E
7   |A & -A                3,6,CD
```

6-5. b)

```
1   |(x)(Ey)Rxy            P
2   |(Ex)(y)-Rxy           P
    |_____________
3   |a|(y)-Ray             A
    | |_________
4   | |(x)(Ey)Rxy          1,R
5   | |(Ey)Ray             4,UE
6   | |b|Rab               A
    | | |_____
7   | | |(y)-Ray           3,R
8   | | |-Rab              7,UE
9   | | |A & -A            6,8,CD
10  | |A & -A              5,6-9,EE
11  |A & -A                2,3-10,EE
```

6-5. c)

```
1   |(Ex)Dx                   P
2   |(x)(Dx>(y)(z)Ryz)        P
3   |(Ex)(Ey)-Rxy             P
    |_______________
4   |a|Da                     A
    | |____
5   | |(Ex)Dx                 1,R
6   | |(x)(Dx>(y)(z)Ryz)      2,R
7   | |Da > (y)(z)Ryz         6,UE
8   | |(y)(z)Ryz              4,7,>E
9   | |(Ex)(Ey)-Rxy           3,R
10  | |b|(Ey)-Rby             A
    | | |__________
11  | | |c|-Rbc               A
    | | | |______
12  | | | |(y)(z)Ryz         8,R
13  | | | |(z)Rbz            12,UE
14  | | | |Rbc               13,UE
15  | | | |A & -A            11,14,CD
16  | | |A & -A              10,11-15,EE
17  | |A & -A                9,10-16,EE
18  |A & -A                  1,4-17,EE
```


6-5. d)

```
1   |(Ex)(Ey)(Rxx&(-Ryy&Rxy))          P
2   |(x)(y)(Rxy>Ryx)                   P
3   |(x)(y)(z)((Rxy&Ryz)>Rxz)          P
    |___________________________
4   |a|(Ey)(Raa&(-Ryy&Ray))            A
    | |______________________
5   | |b|Raa & (-Rbb & Rab)            A
    | | |____________________
6   | | |Raa                           5,&E
7   | | |-Rbb & Rab                    5,&E
8   | | |-Rbb                          7,&E
9   | | |Rab                           7,&E
10  | | |(x)(y)(Rxy>Ryx)               2,R
11  | | |(y)(Ray > Rya)                10,UE
12  | | |Rab > Rba                     11,UE
13  | | |Rba                           9,12,>E
14  | | |(x)(y)(z)((Rxy&Ryz)>Rxz)     3,R
15  | | |(y)(z)((Rby&Ryz)>Rbz)        14,UE
16  | | |(z)((Rba&Raz)>Rbz)           15,UE
17  | | |(Rba & Rab) > Rbb            16,UE
18  | | |Rba & Rab                     13,9,&I
19  | | |Rbb                           17,18,>E
20  | | |A & -A                        8,19,CD
21  | |A & -A                          4,5-20,EE
22  |A & -A                            1,4-21,EE
```

6-6. a)

LOGICALLY EQUIVALENT SENTENCES:	EXERCISES WHICH SHOW L.E.:
(x)(y)(Px & Qy); (x)Px & (x)Qx	6-1. f) and 1st example in chp. 6
(Ex)(Ey)(Px & Qy); (Ex)Px & (Ex)Qx	6-1. g) and 2nd example in chp. 6
(x)(y)(Px v Qy); (x)Px v (x)Qx	6-1. h) and q)
(Ex)(Ey)(Px v Qy); (Ex)Px v (Ex)Qx	6-1. i) and j)
(x)(y)(Px -> Qy); (Ex)Px -> (x)Qx	6-1. l) and 6-8. d)
(Ex)(Ey)(Px -> Qy); (x)Px -> (Ex)Qx	6-1. m) and 3rd example in chp. 6
(Ex)(y)(Px -> Qy); (x)Px -> (x)Qx	6-1. n) and p)
(x)(Ey)(Px -> Qy); (Ex)Px -> (Ex)Qx	6-1. o) and 6-8. b)

6-6. b)

```
1   |(x)Rxa              P
    |_______________
2   |Rb̂a                 1,UE
3   |(Ex)Rxa             2,EI
```

```
1   |(x)Px               P
    |_______________
2   |Pâ                  1,UE
3   |(Ex)Px              2,EI
```

```
1   |(x)Rxx              P
    |_______________
2   |Rââ                 1,UE
3   |(Ex)Rxx             2,EI
```

6-6. c)

LOGICALLY EQUIVALENT SENTENCES:	EXERCISES WHICH SHOW L.E.:
(u)(Pu & Qu); (u)Pu & (u)Qu	5-5. d) and e)
(Eu)(Pu v Qu); (Eu)Pu v (Eu)Qu	5-7. e) and f)
A -> (u)Pu; (u)(A -> Pu)	5-5. l) and k)
A -> (Eu)Pu; (Eu)(A -> Pu)	5-5. b) and u)
(u)Pu -> A; (Eu)(Pu -> A)	5-5. s) and 5-7. g)
(Eu)Pu -> A; (u)(Pu -> A)	5-5. p) and 5-7. h)

6-6. d)

(x)(Px v Qx) is not logically equivalent to (x)Px v (x)Qx

COUNTEREXAMPLE: D={a,b} Pa & -Pb & -Qa & Qb In this interpretation, (x)(Px v Qx) is true - that is, everthing is such that it is either P or Q. [Namely, a is P and b is Q.] But, (x)Px v (x)Qx is false since both disjuncts are false - that is, it is false that everything is P [b is not], and it is false that everything is Q [a is not].

(Ex)(Px & Qx) is not logically equivalent to (Ex)Px & (Ex)Qx

COUNTEREXAMPLE: D={a,b} Pa & -Pb & -Qa & Qb (Explanation is similar to that above.)

6-6. e)

To show that (u)(v)Ruv is logically equivalent to (v)(u)Ruv, we show that the arguments (u)(v)Ruv/(v)(u)Ruv and (v)(u)Ruv/(u)(v)Ruv are valid:

```
1 | (u)(v)Ruv      P
2 | (v)Râv         1,UE
3 | Râĉ            2,UE
4 | (u)Ruĉ         3,UI
5 | (v)(u)Ruv      4,UI
```

```
1 | (v)(u)Ruv      P
2 | (u)Ruâ         1,UE
3 | Rôâ            2,UE
4 | (v)Rôv         3,UI
5 | (u)(v)Ruv      4,UI
```

To show that (Eu)(Ev)Ruv is logically equivalent to (Ev)(Eu)Ruv, we show that the arguments (Eu)(Ev)Ruv/(Ev)(Eu)Ruv and (Ev)(Eu)Ruv/(Eu)(Ev)Ruv are valid:

```
1 | (Eu)(Ev)Ruv            P
2 |  a | (Ev)Rav           A
3 |    | b | Rab           A
4 |    |   | (Eu)Rub       3,EI
5 |    |   | (Ev)(Eu)Ruv   4,EI
6 |    | (Ev)(Eu)Ruv       2,3-5,EE
7 | (Ev)(Eu)Ruv            1,2-6,EE
```

```
1 | (Ev)(Eu)Ruv            P
2 |  a | (Eu)Rua           A
3 |    | b | Rba           A
4 |    |   | (Ev)Rbv       3,EI
5 |    |   | (Eu)(Ev)Ruv   4,EI
6 |    | (Eu)(Ev)Ruv       2,3-5,EE
7 | (Eu)(Ev)Ruv            1,2-6,EE
```


6-7. a)

```
1  | (x)((Ey)(LxyvLyx)>Lxx)              P
2  | (Ex)(Ey)Lxy                         P
3  | a | (Ey)Lay                         A
4  |   | b | Lab                         A
5  |   |   | (x)((Ey)(LxyvLyx)>Lxx)      1,R
6  |   |   | (Ey)(LayvLya)>Laa           5,UE
7  |   |   | Lab v Lba                   4,vI
8  |   |   | (Ey)(Lay v Lya)             7,EI
9  |   |   | Laa                         6,8,>E
10 |   |   | (Ex)Lxx                     9,EI
11 |   | (Ex)Lxx                         3,4-10,EE
12 | (Ex)Lxx                             2,3-11,EE
```

6-7. b)

```
1  | (x)(Hx > Ax)                        P
2  | | (Ey)(Hy & Tay)                    A
3  | | b | Hb & Tab                      A
4  | |   | Hb                            3,&E
5  | |   | (x)(Hx > Ax)                  1,R
6  | |   | Hb > Ab                       5,UE
7  | |   | Ab                            4,6,>E
8  | |   | Tab                           3,&E
9  | |   | Ab & Tab                      7,8,&I
10 | |   | (Ey)(Ay & Tay)                9,EI
11 | | (Ey)(Ay & Tay)                    2,3-10,EE
12 | (Ey)(Hy&Tay)>(Ey)(Ay&Tay)           2-11,>I
13 | (x)((Ey)(Hy&Txy)>(Ey)(Ay&Txy))      12,UI
```

6-7. c)

1	(x)(y)((Ez)Lyz>Lxy)	P
2	(Ex)(Ey)Lxy	P
3	a (Ey)Lay	A
4	b Lab	A
5	(x)(y)((Ez)Lyz>Lxy)	1,R
6	(y)((Ez)Lyz>Lĉy)	5,UE
7	(Ez)Lâz > Lĉâ	6,UE
8	(y)((Ez)Lyz>Lây)	5,UE
9	(Ez)Laz > Lâa	8,UE
10	(Ez)Laz	4,EI
11	Lâa	9,10,>E
12	(Ez)Lâz	11,EI
13	Lĉâ	7,12,>E
14	(y)Lĉy	13,UI
15	(x)(y)Lxy	14,UI
16	(x)(y)Lxy	3,4-15,EE
17	(x)(y)Lxy	2,3-16,EE

6-7. d)

1	(x)(y)((Ez)(Rzy&-Rxz)>Lxy)	P
2	-(Ex)Lxx	P
3	(y)((Ez)(Rzy&-Raz)>Lay)	1,UE
4	(Ez)(Rza&-Raz)>Laa	3,UE
5	-Rab	A
6	Rba	A
7	-Rab	5,R
8	Rba & -Rab	6,7,&I
9	(Ez)(Rza & -Raz)	8,EI
10	(Ez)(Rza&-Raz)>Laa	4,R
11	Laa	9,10,>E
12	(Ex)Lxx	11,EI
13	-(Ex)Lxx	2,R
14	-Rba	6-13,-I
15	-Râb̂ > -Rb̂â	5-14,>I
16	(y)(-Ryb > -Rby)	15,UI
17	(x)(y)(-Ryx>-Rxy)	16,UI

6-7. e)

1	(x)((Ey)Lxy>(Ey)((z)Lyz&Lxy))	P
2	(Ex)(Ey)Lxy	P
3	a (Ey)Lay	A
4	b Lab	A
5	(x)((Ey)Lxy>(Ey)((z)Lyz&Lxy))	1,R
6	(Ey)Lay>(Ey)((z)Lyz&Lay)	5,UE
7	(Ey)Lay	4,EI
8	(Ey)((z)Lyz&Lay)	6,7,>E
9	c (z)Lcz & Lac	A
10	(z)Lcz	9,&E
11	Lcd	10,UE
12	(y)Lcy	11,UI
13	(Ex)(y)Lxy	12,EI
14	(Ex)(y)Lxy	8,9-13,EE
15	(Ex)(y)Lxy	3,4-14,EE
16	(Ex)(y)Lxy	2,3-15,EE

6-7. f)

1	(x)(Px>(y)(Hy>Rxy))	P
2	(Ex)(Px&(Ey)-Rxy)	P
3	(x)Hx	A
4	(Ex)(Px&(Ey)-Rxy)	2,R
5	a Pa & (Ey)-Ray	A
6	Pa	5,&E
7	(Ey)-Ray	5,&E
8	b -Rab	A
9	(x)(Px>(y)(Hy>Rxy))	1,R
10	Pa > (y)(Hy > Ray)	9,UE
11	Pa	6,R
12	(y)(Hy > Ray)	10,11,>E
13	Hb > Rab	12,UE
14	(x)Hx	3,R
15	Hb	14,UE
16	Rab	13,15,>E
17	A & -A	8,16,CD
18	A & -A	7,8-17,EE
19	A & -A	4,5-18,EE
20	-(x)Hx	3-19,-I

6-7. g)

```
1  | (x)(Ex>(y)(Hy>Wxy))                P
2  | (Ex)(Hx&(y)(Dy>Wxy))               P
3  | (x)(y)(z)((Wxy&Wyz)>Wxz)           P
   |-
4  | a| Ha & (y)(Dy > Way)              A
   |  |-
5  |  | Ha                              4,&E
6  |  | (y)(Dy > Way)                   4,&E
7  |  | | Eb                            A
   |  | |-
8  |  | | | Dc                          A
   |  | | |-
9  |  | | | (x)(Ex>(y)(Hy>Wxy))        1,R
10 |  | | | Eb > (y)(Hy > Wby)         9,UE
11 |  | | | Eb                          7,R
12 |  | | | (y)(Hy > Wby)              10,11,>E
13 |  | | | (y)(Dy > Way)              6,R
14 |  | | | Dc > Wac                    13,UE
15 |  | | | Wac                         8,14,>E
16 |  | | | Ha                          5,R
17 |  | | | Ha > Wba                    12,UE
18 |  | | | Wba                         16,17,>E
19 |  | | | (x)(y)(z)((Wxy&Wyz)>Wxz)   R,3
20 |  | | | (y)(z)((Wby&Wyz)>Wbz)      19,U
21 |  | | | (z)((Wba&Waz)>Wbz)         20,UE
22 |  | | | (Wba & Wac) > Wbc          21,UE
23 |  | | | Wba & Wac                  15,18,&I
24 |  | | | Wbc                         22,23,>E
25 |  | | Dc > Wbc                      8-24,>I
26 |  | | (y)(Dy > Wby)                25,UI
27 |  | Eb > (y)(Dy > Wby)              7-26,>I
28 |  | (x)(Ex>(y)(Dy>Wxy))             27,UI
29 | (x)(Ex>(y)(Dy>Wxy))                2,4-28,EE
```

6-7. h)

```
1  | (x)(Ey)(Py > Qx)                   P
   |-
2  | | -(Ey)(x)(Py>Qx)                  A
   | |-
3  | | (y)-(x)(Py > Qx)                 2,-]
4  | | -(x)(Pa > Qx)                    3,UE
5  | | (Ex)-(Pa > Qx)                   4,-U
6  | | b| -(Pa > Qb)                    A
   | |  |-
7  | |  | Pa & -Qb                      6,C
8  | |  | Pa                            7,&E
9  | | Pa                               5,6-8,EE
10 | | (x)Px                            9,UI
11 | | c| -(Pa > Qc)                    A
   | |  |-
12 | |  | Pa & -Qc                      11,C
13 | |  | -Qc                           12,&E
14 | |  | (x)(Ey)(Py > Qx)              1,R
15 | |  | (Ey)(Py > Qc)                 14,UE
16 | |  | d| Pd > Qc                    A
   | |  |  |-
17 | |  |  | (x)Px                      10,R
18 | |  |  | Pd                         17,UE
19 | |  |  | Qc                         16,18,>E
20 | |  |  | -Qc                        13,R
21 | |  |  | A & -A                     19,20,CD
22 | |  | A & -A                        15,16-21,EE
23 | | A & -A                           5,11-22,EE
24 | (Ey)(x)(Py > Qx)                   2-23,RD
```


6-7. i)

```
1  | (Ex)Px > (Ex)Qx                    P
   |-
2  | | -(x)(Ey)(Px>Qy)                  A
   | |-
3  | | (Ex)-(Ey)(Px>Qy)                 2,-U
4  | | a| -(Ey)(Pa > Qy)                A
   | |  |-
5  | |  | (y)-(Pa > Qy)                 4,-X
6  | |  | -(Pa > Qb)                    5,UE
7  | |  | Pa & -Qb                      6,C
8  | |  | Pa                            7,&E
9  | |  | -Qb                           7,&E
10 | |  | (Ex)Px                        8,EI
11 | |  | (x)-Qx                        9,UI
12 | |  | (Ex)Px > (Ex)Qx               1,R
13 | |  | (Ex)Qx                        10,12,>E
14 | |  | c| Qc                         A
   | |  |  |-
15 | |  |  | (x)-Qx                     11,R
16 | |  |  | -Qc                        15,UE
17 | |  |  | A & -A                     14,16,CD
18 | |  | A & -A                        13,14-17,EE
19 | | A & -A                           3,4-18,EE
20 | (x)(Ey)(Px > Qy)                   2-19,RD
```

6-7. j)

```
1  | (Ex)Px > (x)Qx                     P
   |-
2  | | -(x)(y)(Px > Qy)                 A
   | |-
3  | | (Ex)-(y)(Px>Qy)                  2,-U
4  | | a| -(y)(Pa > Qy)                 A
   | |  |-
5  | |  | (Ey)-(Pa > Qy)                4,-U
6  | |  | b| -(Pa > Qb)                 A
   | |  |  |-
7  | |  |  | Pa & -Qb                   6,C
8  | |  |  | Pa                         7,&E
9  | |  |  | -Qb                        7,&E
10 | |  |  | (Ex)Px                     8,EI
11 | |  |  | (Ex)Px > (x)Qx             1,R
12 | |  |  | (x)Qx                      10,11,>E
13 | |  |  | Qb                         12,UE
14 | |  |  | A & -A                     9,13,CD
15 | |  | A & -A                        5,6-14,EE
16 | | A & -A                           3,4-15,EE
17 | (x)(y)(Px > Qy)                    16,(2-16),RD
```

6-7. k)

```
1  | (x)(Bx>((Ey)Lxy>(Ey)Lyx))        P
2  | (x)((Ey)Lyx>Lxx)                 P
3  | -(Ex)Lxx                         P
   |------------------
4  | | Ba                   A
   | |-----
5  | | (x)(Bx>((Ey)Lxy>(Ey)Lyx))      1,R
6  | | Ba>((Ey)Lay>(Ey)Lya)       5,UE
7  | | (Ey)Lay>(Ey)Lya        4,6,>E
8  | | | Lab                A
   | | |-----
9  | | | (Ey)Lay            8,EI
10 | | | (Ey)Lay>(Ey)Lya    7,R
11 | | | (Ey)Lya            9,10,>E
12 | | | (x)((Ey)Lyx>Lxx)      2,R
13 | | | (Ey)Lya > Laa      12,UE
14 | | | Laa                11,13,>E
15 | | | (Ex)Lxx            14,EI
16 | | | -(Ex)Lxx           3,R
17 | | -Lab̂                  8-16,-I
18 | | (y)-Lay              17,UI
19 | Bâ > (y)-Lây           4-18,>I
20 | (x)(Bx>(y)-Lxy)        19,UI
```

6-7. l)

```
1  | (x)(Fx>(Hx&(-Cx&-Kx)))        P
2  | (x)((Hx&-(Ey)Nxy)>Cx)         P
   |------------------
3  | | Fa                   A
   | |-----
4  | | (x)(Fx>(Hx&(-Cx&-Kx)))      1,R
5  | | Fa>(Ha&(-Ca&-Ka))       4,UE
6  | | Ha & (-Ca & -Ka)       3,5,>E
7  | | Ha                   6,&E
8  | | -Ca & -Ka            6,&E
9  | | -Ca                  8,&E
10 | | | -(Ey)Nay           A
   | | |-----
11 | | | Ha                 7,R
12 | | | Ha & -(Ey)Nay        10,11,&I
13 | | | (x)((Hx&-(Ey)Nxy)>Cx)      2,R
14 | | | (Ha&-(Ey)Nay)>Ca       13,UE
15 | | | Ca                 12,14,>E
16 | | | -Ca                9,R
17 | | (Ey)Nay              10-16,RD
18 | Fâ > (Ey)Nây           3-17,>I
19 | (x)(Fx>(Ey)Nxy)        18,UI
```


6-7. m)

```
1  | (y)(Cy > Dy)           P
2  | (x)(Ey)((Hx&Cx)&(Gy&Ryx))      P
3  | (Ex)Dx>(y)(z)(Ryz>Dy)          P
   |------------------
4  | Câ > Dâ                1,UE
5  | (Ey)((Hâ&Câ)&(Gy&Ryâ))      2,UE
6  | b | (Ha&Ca)&(Gb&Rba)       A
   |   |-----
7  |   | Ha & Ca              6,&E
8  |   | Gb & Rba             6,&E
9  |   | Ca                   7,&E
10 |   | Ca > Da              4,R
11 |   | Da                   9,10,>E
12 |   | (Ex)Dx               11,EI
13 |   | (Ex)Dx>(y)(z)(Ryz>Dy)      3,R
14 |   | (y)(z)(Ryz > Dy)      12,13,>E
15 |   | (z)(Rbz > Db)        14,UE
16 |   | Rba > Db             15,UE
17 |   | Rba                  8,&E
18 |   | Db                   16,17,>E
19 |   | Gb                   8,&E
20 |   | Gb & Db              18,19,&I
21 |   | (Ex)(Gx & Dx)        20,EI
22 | (Ex)(Gx & Dx)          5,6-21,EE
```

6-7. n)

```
1  | (x)(y)((Rdy&Rxd)>Rxy)     P
2  | (x)(Bx > Rdx)             P
3  | (Ex)(Bx & Rxd)            P
   |------------------
4  | a | Ba & Rad             A
   |   |-----
5  |   | Ba                   4,&E
6  |   | | Bb                 A
   |   | |-----
7  |   | | (x)(Bx > Rdx)      2,R
8  |   | | Bb > Rdb           7,UE
9  |   | | Rdb                6,8,>E
10 |   | | (x)(y)((Rdy&Rxd)>Rxy)      1,R
11 |   | | (y)((Rdy&Rad)>Ray)      10,UE
12 |   | | (Rdb & Rad) > Rab      11,UE
13 |   | | Ba & Rad           4,R
14 |   | | Rad                13,&E
15 |   | | Rdb & Rad          9,14,&I
16 |   | | Rab                12,15,>E
17 |   | Bb̂ > Rab̂             6-16,>I
18 |   | (y)(By > Ray)        17,UI
19 |   | Ba & (y)(By > Ray)      5,18,&I
20 |   | (Ex)(Bx&(y)(By>Rxy))      19,EI
21 | (Ex)(Bx&(y)(By>Rxy))      3,4-20,EE
```

ANSWERS TO EXERCISES IN VOLUME II, CHAPTER 7

7-1. a)

```
1          a    (Vx)(Kx & Jx)        P
2                    -Ka             -C
3                 * Ka & Ja          1 V
4                     Ka             3 &
5                     Ja             3 &
                       X
VALID
```

7-1. b)

```
1          a    (Vx)(Fx > Gx)        P
2                    -Ga             P
3                    -Fa             -C
4                 * Fa > Ga          1 V
         |-------------^-------------|
5       -Fa                         Ga    4 >
                                     X
```

INVALID. C.E.: D={a} -Fa & -Ga

7-1. c)

```
1          h    (Vx)(Cx > Ix)        P
2              * -(Ch v Ih)          -C
3                    -Ch             2 -v
4                    -Ih             2 -v
5                 * Ch > Ih          1 V
         |-------------^-------------|
6       -Ch                         Ih    5 >
                                     X
```

INVALID. C.E.: D={h} -Ch & -Ih

7-1. d)

```
1              * A > (Vx)Mx                     P
2                    A                          P
3              * -(Mg & Mi)                     -C
         |-------------^-------------|
4       -A                   g, i  (Vx)Mx       1 >
         X                    |------^------|
5                            -Mg           -Mi  3 -&
6                             Mg            Mi  4 V
                              X             X
VALID
```


7-1. e)

```
1              a, b   (Vx)(Ex > Cx)                      P
2              a, b      (Vx)Ex                          P
3                    ● -(Ca & Cb)                        -C
              |--------------^--------------|
4            -Ca                           -Cb           3 -&
5         * Ea > Ca                     * Eb > Cb        1 V
      |-------^-------|              |-------^-------|
6    -Ea             Ca             -Eb             Cb   5 >
7     Ea              X              Eb              X   2 V
      X                              X
VALID
```

7-1. f)

```
1              g, e  (Vx)(Ne<->Px)                       P
2                        -Pg                             -C
3                     * Ne<->Pg                          1 V
              |--------------^--------------|
4             Ne                           -Ne           3 <>
5             Pg                           -Pg           3 <>
6             X                         * Ne<->Pe        1 V
                                     |-------^-------|
7                                    Ne             -Ne  6 <>
8                                    Pe             -Pe  6 <>
                                     X
```

INVALID. C.E.: D={e,g} -Ne & -Pg & -Pe

7-1. g)

```
1              d, j    (Vx)(Kx v Ax)                     P
2                          -Kj                           P
3                          -Ad                           -C
4                       * Kd v Ad                        1 V
              |--------------^--------------|
5             Kd                            Ad           4 v
6         * Kj v Aj                          X           1 V
      |-------^-------|
7    Kj               Aj                                 6 v
      X
```

INVALID. C.E.: D={d,j} -Ad & Aj & Kd & -Kj

7-1. h)

```
1              a      (Vx)(Dx v Gx)              P
2              a      (Vx)(Dx > Jx)              P
3              a      (Vx)(Gx > Jx)              P
4                          -Ja                   -C
5                       * Da > Ja                2 V
                |------------^-------------|
6              -Da                         Ja    5 >
7           * Ga > Ja                      X     3 V
         |------^------|
8       -Ga            Ja                        7 >
9    * Da v Ga         X                         1 V
     |---^---|
10   Da      Ga                                  9 v
     X       X               VALID
```

7-1. i)

```
1              a, b    (Vx)(Sx<->Tx)             P
2                      * -(Sb v -Ta)             -C
3                           -Sb                  2 -v
4                         * --Ta                 2 -v
5                            Ta                  4 --
6                        * Sa<->Ta               1 V
                |------------^-------------|
7               Sa                        -Sa    6 <>
8               Ta                        -Ta    6 <>
9           * Sb<->Tb                      X     1 V
         |------^------|
10       Sb           -Sb                        9 <>
11       Tb           -Tb                        9 <>
         X
```

INVALID. C.E.: D={a,b} Ta & -Tb & Sa & -Sb

7-1. j)

```
1                    * -Tfg v (Vx)Px                     P
2                    * Ph > (Vx)Qx                       P
3                    * -(Tfg > Qh)                       -C
4                          Tfg                           3 ->
5                          -Qh                           3 ->
            |---------------^-----------------|
6          -Tfg                         h  (Vx)Px        1 v
            X                      |--------^------|
7                                 -Ph        h  (Vx)Qx   2 >
8                                  Ph              |     6 V
9                                  X               Qh    7 V
                                                   X
```

VALID. Note that I ended line 9 even though I had not yet instantiated '(Vx)Qx' with 'f' and 'g'. I noticed that I can get the branch to close by using h. If you included instances of (Vx)Qx using f and g and marked the branch as closed that would not have been a mistake (as long as you included Qh).

7-2. a)

```
1                    * A > (]x)Gx                 P
2                          A                      P
3                         -Gb                     -C
         |-----------------^-----------------|
4       -A                          * (]x)Gx      1 >
5        X                              Ga        4 ]
```

INVALID. C.E.: D={a,b} A & Ga & -Gb. If I had used 'b' in line 5, the tree would have closed.

7-2. b)

```
1                     * (]x)Dx                    P
2                     * (]x)-Dx                   P
3                          -A                     -C
4                          Da                     1 ]
5                          -Db                    2 ]
```

INVALID. C.E.: D={a,b} -A & Da & -Db. If I had used the same name in lines 4 and 5, the tree would have closed.

7-2. c)

```
1                  * (]x)(Px & Qx)                P
2                   * -(Pa v Qb)                  -C
3                        -Pa                      2 -v
4                        -Qb                      2 -v
5                     * Pc & Qc                   1 ]
6                         Pc                      5 &
7                         Qc                      5 &
```

INVALID. C.E.: D={a,b,c} -Pa & Pc & -Qb & Qc. If I had used 'a' or 'b' in line 5, the tree would have closed.

7-2. d)

```
1                     * (]x)Px                    P
2                     * (]x)Qx                    P
3                  * -(Pm<->Qm)                   -C
4                         Pa                      1 ]
5                         Qb                      2 ]
         |-----------------^-----------------|
6        Pm                                 -Pm   3 -<>
7       -Qm                                  Qm   3 -<>
```

INVALID. C.E.: D={a,b,m} (Pa & Qb & Pm & -Qm); (Pa & Qb & -Pm & Qm). If I had used 'm' in lines 4 and 5, the tree would have closed.

7-2. e)

```
1                      * A v B                         P
2                    * A > (]x)Nx                      P
3                    * B > (]x)Nx                      P
4                        -Ng                           -C
           |----------------^----------------|
5          A                                 B         1 v
   |-------^-------|                 |-------^-------|
6  -A          * (]x)Nx             -A           * (]x)Nx   2 >
7   X              Na                |               Na     6 ]
             |---^---|           |---^---|       |---^---|
8           -B   * (]x)Nx       -B   * (]x)Nx   -B   * (]x)Nx   3 >
9                    Nb          X       Nb      X       Nb     8 ]
```

INVALID. C.E.: D={a,b,g} (A & Na & Nb & -Ng); (A & -Ng & Na & -B)
(B & -A & -Ng & Nb); (B & Na & Nb & -Ng)

If I had used 'g' in line 9, the tree would have closed. Also note: the NEW NAMES only have to be new to a branch.

7-3. a)

```
1                 * (]x)(Px > Qx)                P
2               * --(Vx)(Px & -Qx)               -C
3          a       (Vx)(Px & -Qx)                2 --
4                   * Pa > Qa                    1 ]
        |---------------^---------------|
5      -Pa                              Qa       4 >
6    * Pa & -Qa                    * Pa & -Qa    3 V
7      Pa                               Pa       6 &
8      -Qa                              -Qa      6 &
        X                                X
```

VALID. Note that I worked on line 1 before line 3.

7-3. b)

```
1                   * (]x)Cx                     P
2                 * --(]x)-Cx                    -C
3                   * (]x)-Cx                    2 --
4                      Ca                        1 ]
5                      -Cb                       3 ]
```

INVALID. C.E.: D={a,b} Ca & -Cb. If I had used the same name in lines 4 and 5, the tree would have closed.

7-3. c)

```
1                * (]x)Jx v (]x)Kx               P
2          a         (Vx)-Jx                     P
3                  * --(Vx)-Kx                   -C
4          b         (Vx)-Kx                     3 --
        |---------------^---------------|
5    * (]x)Jx                        * (]x)Kx    1 v
6       Ja                              Kb       5 ]
7      -Ja                               |       2 V
8       X                               -Kb      4 V
                                         X
```

VALID

Note that in line 6the name need only be new to a branch. Also, in line 7, I only used line 2 - that was enough to get the branch to close. Similarly, I needed only line 4.

7-3. d)

```
1          a      (Vx)(Px > Qx)                  P
2                  * (]x)Px                      P
3                * --(Vx)-Qx                     -C
4          a        (Vx)-Qx                      3 --
5                     Pa                         2 ]
6                  * Pa > Qa                     1 V
        |---------------^---------------|
7      -Pa                              Qa       6 >
8       X                              -Qa       4 V
                                         X
```

VALID

7-3. e)

```
1          a, b     (Vx)(Gx v Hx)                P
2            * -(-(]x)-Gx v -(]x)-Hx)            -C
3                  * --(]x)-Gx                   2 -v
4                  * --(]x)-Hx                   2 -v
5                   * (]x)-Gx                    3 --
6                   * (]x)-Hx                    4 --
7                      -Ga                       5 ]
8                      -Hb                       6 ]
9                    * Ga v Ha                   1 V
        |---------------^---------------|
10      Ga                              Ha       9 v
11      X                           * Gb v Hb    1 V
                                 |------^------|
12                               Gb            Hb   11 v
                                                X
```

INVALID. C.E.: D={a,b} -Ga & Gb & Ha & -Hb

7-3. f)

```
1                * (]x)Px & (]x)-Px              P
2              * -(-(Vx)Px & -(Vx)-Px)           -C
3                    * (]x)Px                    1 &
4                    * (]x)-Px                   1 &
5                       Pa                       3 ]
6                       -Pb                      4 ]
        |---------------^---------------|
7    * --(Vx)Px                     * --(Vx)-Px  2 -&
8  b    (Vx)Px                   a     (Vx)-Px   7 --
9        Pb                              -Pa     8 V
         X                                X
```

VALID. On line 8, I instantiated '(Vx)Px' only with 'b' and '(Vx)-Px' only with 'a' since that was enough to get both branches to close first. Also, doing lines 3 and 4 before line 2 saves work. The practical guide: "work non-branching rules before branching rules" still applies!

7-3. g)

```
 1                 * -(Vx)Px > (]x)Qx                  P
 2               * -((]x)-Qx > -(Vx)-Px)               -C
 3                     * (]x)-Qx                       2 ->
 4                    * --(Vx)-Px                      2 ->
 5         a, b        (Vx)-Px                         4 --
 6                       -Qa                           3 ]
             |-------------^-------------------|
 7      * --(Vx)Px                      * (]x)Qx       1 >
 8    a    (Vx)Px                          Qb          7 --;]
 9          Pa                             |           8 V
10          -Pa                           -Pa          5 V
11          X                             -Pb          5 V
```

INVALID. C.E.: D={a,b} -Pa & -Pb & -Qa & Qb. I waited to instantiate line 5 with 'a' until I introduced the new name 'b' working on (]x)Qx in line 7. I could have saved a small amount of writing by doing line 6 first, writing '-Pa' as line 7. But then I would have had to return to line 6 with 'b'. In this case the one order is as good as the other.

7-3. h)

```
 1                    * Jq > (]x)Kx                    P
 2            a      (Vx)(Kx > Lx)                     P
 3               * -(Jq > -(Vx)-Lx)                    -C
 4                       Jq                            3 ->
 5                   * --(Vx)-Lx                       3 ->
 6            a         (Vx)-Lx                        5 --
          |----------------^----------------|
 7       -Jq                            * (]x)Kx       1 >
 8        X                                Ka          7 ]
 9                                     * Ka > La       2 V
                                |-----------^---------|
10                             -Ka                   La    9 >
11                              X                   -La    6 V
                                                     X
```

VALID. At line 9, I only instantiated 2 with 'a' because I looked ahead and saw that that would be enough to get all branches to close. If not all branches had closed I would have had to return to 2 and instantiate it with 'q' as well.

7-3. i)

```
 1                 * (]x)Hx & (]x)Gx                   P
 2               * --(Vx)-(Hx & Gx)                    -C
 3                     * (]x)Hx                        1 &
 4                     * (]x)Gx                        1 &
 5          a, b     (Vx)-(Hx & Gx)                    2 --
 6                        Ha                           3 ]
 7                        Gb                           4 ]
 8                   * -(Ha & Ga)                      5 V
          |----------------^----------------|
 9       -Ha                               -Ga         8 -&
10        X                          * -(Hb & Gb)      5 V
                                |-----------^---------|
11                             -Hb                  -Gb    10 -&
```

INVALID. C.E.: D={a,b} Ha & -Hb & -Ga & -Hb X

7-3. j)

```
 1                           * (]x)-Fx                               P
 2                      * -(Vx)Fx > (Vx)-Px                          P
 3                      * -(Vx)Fx > (Vx)-Qx                          P
 4                       * --(Vx)(Px v Qx)                           -C
 5              a          (Vx)(Px v Qx)                             4 --
 6                             -Fa                                   1 ]
            |-------------------^---------------------|
 7     * --(Vx)Fx                          a      (Vx)-Px            2 >
 8     a  (Vx)Fx                                     |               7 --
    |--------^---------|              |--------------^--------|
 9 * --(Vx)Fx       (Vx)-Qx       * --(Vx)Fx       a   (Vx)-Qx       3 >
10 a  (Vx)Fx           |          a  (Vx)Fx               -Qa        9 --;V
11      Fa             Fa               |                  |         8 V
12      X              X               Fa                  |         10 V
13                                      X            * Pa v Qa       5 V
                                                     |----^----|
14                                                   Pa       Qa     13 v
15                                                  -Pa       X      7 V
                                                     X
```

VALID

7-4. a1)

```
 1                   * (Vx)Px & (Vx)Qx                 P
 2                   * -(Vx)(Px & Qx)                  -C
 3                   a     (Vx)Px                      1 &
 4                   a     (Vx)Qx                      1 &
 5                   * (]x)-(Px & Qx)                  2 -V
 6                     * -(Pa & Qa)                    5 ]
          |------------------^------------------|
 7       -Pa                                   -Qa     6 -&
 8        Pa                                    |      3 V
 9        X                                     Qa     4 V
                                                X
```

VALID

7-4. a2)

```
 1              a, b    (Vx)(Px & Qx)                  P
 2                  * -((Vx)Px & (Vx)Qx)               -C
          |------------------^------------------|
 3    * -(Vx)Px                            * -(Vx)Qx   2 -&
 4    * (]x)-Px                            * (]x)-Qx   3 -V
 5       -Pa                                  -Qb      4 ]
 6    * Pa & Qa                            * Pb & Qb   1 V
 7        Pa                                   Pb      6 &
 8        Qa                                   Qb      6 &
          X                                    X
```

VALID

7-4. b1)

```
1             * (]x)Px v (]x)Qx                 P
2             * -(]x)(Px v Qx)                  -C
3      a, b     (Vx)-(Px v Qx)                  2 -]
        |--------------^--------------|
4   * (]x)Px                       * (]x)Qx     1 v
5       Pa                            Qb        4 ]
6  * -(Pa v Qa)                  * -(Pb v Qb)   3 V
7      -Pa                           -Pb        6 -v
8      -Qa                           -Qb        6 -v
        X                             X
```

VALID

7-4. b2)

```
1              * (]x)(Px v Qx)                  P
2           * -((]x)Px v (]x)Qx)                -C
3               * -(]x)Px                       2 -v
4               * -(]x)Qx                       2 -v
5        a        (Vx)-Px                       3 -]
6        a        (Vx)-Qx                       4 -]
7               * Pa v Qa                       1 ]
        |--------------^--------------|
8      Pa                             Qa        7 v
9     -Pa                             |         5 V
10     X                             -Qa        6 V
                                      X
```

VALID

7-4. c1)

```
1             * (Vx)Px v (Vx)Qx                 P
2             * -(Vx)(Px v Qx)                  -C
3             * (]x)-(Px v Qx)                  2 -V
4               * -(Pa v Qa)                    3 ]
5                   -Pa                         4 -v
6                   -Qa                         4 -v
        |--------------^--------------|
7   a   (Vx)Px               a      (Vx)Qx      1 v
8        Pa                           Qa        7 V
         X                            X
```

VALID

7-4. c2)

```
1        a, b      (Vx)(Px v Qx)                P
2             * -((Vx)Px v (Vx)Qx)              -C
3                 * -(Vx)Px                     2 -v
4                 * -(Vx)Qx                     2 -v
5                 * (]x)-Px                     3 -V
6                 * (]x)-Qx                     4 -V
7                     -Pa                       5 ]
8                     -Qb                       6 ]
9                   * Pa v Qa                   1 V
        |--------------^--------------|
10     Pa                             Qa        9 v
11     X                          * Pb v Qb     1 V
                               |------^------|
12                             Pb            Qb 11 v
                                             X
```

INVALID. C.E.: D={a,b} -Pa & Pb & Qa & -Qb

7-4. d1)

```
1               * (]x)Px & (]x)Qx               P
2               * -(]x)(Px & Qx)                -C
3                   * (]x)Px                    1 &
4                   * (]x)Qx                    1 &
5       a, b      (Vx)-(Px & Qx)                2 -]
6                      Pa                       3 ]
7                      Qb                       4 ]
8                  * -(Pa & Qa)                 5 V
        |--------------^--------------|
9     -Pa                            -Qa        8 -&
10     X                         * -(Pb & Qb)   5 V
                               |------^------|
11                            -Pb           -Qb 10 -&
                                             X
```

INVALID. C.E.: D={a,b} Pa & -Pb & -Qa & Qb

7-4. d2)

```
1                * (]x)(Px & Qx)                P
2             * -((]x)Px & (]x)Qx)              -C
3                   * Pa & Qa                   1 ]
4                      Pa                       3 &
5                      Qa                       3 &
        |--------------^--------------|
6   * -(]x)Px                     * -(]x)Qx     2 -&
7   a (Vx)-Px                   a   (Vx)-Qx     6 -]
8      -Pa                            -Qa       7 V
        X                              X
```

VALID

7-4. e1)

```
1                 * A > (Vx)Px              P
2               * -(Vx)(A > Px)             -C
3               * (]x)-(A > Px)             2 -V
4                 * -(A > Pa)               3 ]
5                      A                    4 ->
6                     -Pa                   4 ->
         |-------------^-------------|
7        -A                  a  (Vx)Px      1 >
8         X                       Pa        7 V
VALID                             X
```

7-4. e2)

```
1          a       (Vx)(A > Px)             P
2               * -(A > (Vx)Px)             -C
3                      A                    2 ->
4                  * -(Vx)Px                2 ->
5                  * (]x)-Px                4 -V
6                     -Pa                   5 ]
7                  * A > Pa                 1 V
         |-------------^-------------|
8        -A                       Pa        7 >
VALID     X                        X
```

7-4. f1)

```
1                 * A > (]x)Px              P
2               * -(]x)(A > Px)             -C
3          a, b    (Vx)-(A > Px)            2 -]
         |-------------^-------------|
4        -A                   * (]x)Px      1 >
5   * -(A > Pa)                  |          3 V
6         A                      |          5 ->
7        -Pa                     |          5 ->
8         X                      Pb         4 ]
9                          * -(A > Pb)      3 V
10                               A          9 ->
11                              -Pb         9 ->
VALID                            X
```

7-4. f2)

```
1                * (]x)(A > Px)             P
2               * -(A > (]x)Px)             -C
3                      A                    2 ->
4                  * -(]x)Px                2 ->
5              a    (Vx)-Px                 4 -]
6                  * A > Pa                 1 ]
         |-------------^-------------|
7        -A                       Pa        6 >
8         X                      -Pa        5 V
VALID                             X
```

7-4. g1)

```
1                 * (Vx)Px > A              P
2               * -(]x)(Px > A)             -C
3          a, b    (Vx)-(Px > A)            2 -]
         |-------------^-------------|
4     * -(Vx)Px                   A         1 >
5     * (]x)-Px                   |         4 -V
6        -Pa                      |         5 ]
7    * -(Pa > A)           * -(Pb > A)      3 V
8         Pa                      Pb        7 ->
9         -A                      -A        7 ->
VALID     X                        X
```

7-4. g2)

```
1                * (]x)(Px > A)             P
2               * -((Vx)Px > A)             -C
3          a         (Vx)Px                 2 ->
4                      -A                   2 ->
5                   * Pa > A                1 ]
         |-------------^-------------|
6        -Pa                      A         5 >
7         Pa                      X         3 V
VALID     X
```

7-4. h1)

```
1                 * (]x)Px > A              P
2               * -(Vx)(Px > A)             -C
3               * (]x)-(Px > A)             2 -V
4                 * -(Pa > A)               3 ]
5                      Pa                   4 ->
6                      -A                   4 ->
         |-------------^-------------|
7     * -(]x)Px                   A         1 >
8    a  (Vx)-Px                   X         7 -]
9        -Pa                                8 V
          X
VALID
```

7-4. h2)

```
1          a       (Vx)(Px > A)             P
2               * -((]x)Px > A)             -C
3                   * (]x)Px                2 ->
4                      -A                   2 ->
5                      Pa                   3 ]
6                   * Pa > A                1 V
         |-------------^-------------|
7        -Pa                      A         6 >
VALID     X                       X
```

7-4. i1)

```
1              a    (Vx)Px                 P
2                 * -(]x)Px                -C
3              a    (Vx)-Px                2 -]
4                     -Pa                  3 V
5                     Pa                   1 V
                       X
VALID
```

7-4. i2)

```
1                 * (]x)Px                 P
2                 * -(Vx)Px                -C
3                 * (]x)-Px                2 -V
4                     Pa                   1 ]
5                     -Pb                  3 ]

INVALID. C.E.:  D={a,b}  Pa & -Pb
```

7-4. j1)

```
1                  * (Vx)Px > A                  P
2                  * -(Vx)(Px > A)               -C
3                  * (]x)-(Px > A)               2 -V
4                    * -(Pa > A)                 3 ]
5                        Pa                      4 ->
6                        -A                      4 ->
         |-----------------^-----------------|
7   * -(Vx)Px                                A   1 >
8   * (]x)-Px                                X   7 -V
9       -Pb                                      8 ]

INVALID. C.E.:  D={a,b}  -A & Pa & -Pb
```

7-4. j2)

```
1            a     (Vx)(Px > A)                 P
2                * -((Vx)Px > A)                -C
3            a       (Vx)Px                     2 ->
4                      -A                       2 ->
5                      Pa                       3 V
6                    * Pa > A                   1 V
         |----------------^-----------------|
7       -Pa                                 A   6 >
         X                                  X

VALID
```

7-4. k1)

```
1                 * (]x)(Px > A)              P
2                 * -((]x)Px > A)             -C
3                    * (]x)Px                 2 ->
4                       -A                    2 ->
5                    * Pa > A                 1 ]
6                       Pb                    3 ]
         |-----------------^-----------------|
7       -Pa                                  A    5 >
INVALID. C.E.:  D={a,b}  -A & Pb & -Pa       X
```

7-4. k2)

```
1                     * (]x)Px > A                 P
2                     * -(]x)(Px > A)              -C
3              a, b   (Vx)-(Px > A)                2 -]
         |-----------------^-----------------|
4     * -(]x)Px                              A     1 >
5   a    (Vx)-Px                             |     4 -]
6          -Pa                               |     5 V
7     * -(Pa > A)                     * -(Pb > A)  3 V
8          Pa                               Pb     7 ->
9          -A                               -A     7 ->
VALID      X                                 X
```

7-4. l)

```
1              a     (Vx)(Px > Qx)            P
2                      * (]x)Px               P
3                      * -(]x)Qx              -C
4              a        (Vx)-Qx               3 -]
5                          Pa                 2 ]
6                       * Pa > Qa             1 V
         |-----------------^-----------------|
7       -Pa                                  Qa    6 >
8        X                                   -Qa   4 V
                                              X
VALID
```

7-4. m)

```
1                  * (]x)(Lxa<->Lex)          P
2              e        (Vx)Lxa               P
3                     * -(]x)Lex              -C
4              a        (Vx)-Lex              3 -]
5                     * Lba<->Leb             1 ]
         |-----------------^-----------------|
6       Lba                                 -Lba   5 <>
7       Leb                                 -Leb   5 <>
8       Lea                                 Lea    2 V
9       -Lea                                -Lea   4 V
         X                                   X
VALID
```

ANSWERS TO EXERCISES IN VOLUME II, CHAPTER 8

8-1. a) TEST FOR CONTRADICTION

```
1                  * (Vx)Dx v (]x)-Dx                       S
              |----------------^----------------|
2     a  (Vx)Dx                       *  (]x)-Dx            1 v
3          Da                              -Db              2 V;2 ]
```

NOT A CONTRADICTION. C.E.: D={a} Da; D={b} -Db

8-1. a) TEST FOR LOGICAL TRUTH

```
1                 * -[(Vx)Dx v (]x)-Dx]                     -S
2                      * -(Vx)Dx                            1 -v
3                     * -(]x)-Dx                            1 -v
4                      * (]x)-Dx                            2 -V
5               a     (Vx)--Dx                              3 -]
6                        -Da                                4 ]
7                      * --Da                               5 V
8                         Da                                7 --
                          X
```

LOGICAL TRUTH

8-1. b) TEST FOR CONTRADICTION

```
1                 * (Vx)Kx & (]x)-Kx                        S
2                 a     (Vx)Kx                              1 &
3                    * (]x)-Kx                              1 &
4                        -Ka                                3 ]
5                         Ka                                2 V
                          X
```

CONTRADICTION

8-1. c) TEST FOR CONTRADICTION

```
1                  * (Vx)Nx v (Vx)-Nx                       S
              |----------------^----------------|
2     a  (Vx)Nx                       a  (Vx)-Nx            1 v
3          Na                              -Na              2 V
```

NOT A CONTRADICTION. C.E.: D={a} Na; D={a} -Na

8-1. c) TEST FOR LOGICAL TRUTH

```
1                 * -[(Vx)Nx v (Vx)-Nx]                     -S
2                      * -(Vx)Nx                            1 -v
3                     * -(Vx)-Nx                            1 -v
4                      * (]x)-Nx                            2 -V
5                     * (]x)--Nx                            3 -V
6                         -Na                               4 ]
7                       * --Nb                              5 ]
8                          Nb                               7 --
```

NOT A LOGICAL TRUTH. C.E.: D={a,b} -Na&Nb

8-1. d) TEST FOR CONTRADICTION

```
1                 * (Vx)Jx & (Vx)-Jx                        S
2                 a     (Vx)Jx                              1 &
3                 a     (Vx)-Jx                             1 &
4                         -Ja                               3 V
5                          Ja                               2 V
                           X
```

CONTRADICTION

8-1. e) TEST FOR CONTRADICTION

```
1                  * (]x)Bx v (]x)-Bx                       S
              |----------------^----------------|
2        * (]x)Bx                        * (]x)-Bx          1 v
3            Ba                              -Ba            2 ]
```

NOT A CONTRADICTION. C.E.: D={a} Ba; D={a} -Ba

8-1. e) TEST FOR LOGICAL TRUTH

```
1                 * -[(]x)Bx v (]x)-Bx]                     -S
2                      * -(]x)Bx                            1 -v
3                     * -(]x)-Bx                            1 -v
4               a      (Vx)-Bx                              2 -]
5               a     (Vx)--Bx                              3 -]
6                       * --Ba                              5 V
7                          Ba                               6 --
8                         -Ba                               4 V
                           X
```

LOGICAL TRUTH

8-1. f) TEST FOR CONTRADICTION

```
1                 * (]x)Px & (]x)-Px                        S
2                      * (]x)Px                             1 &
3                      * (]x)-Px                            1 &
4                          Pa                               2 ]
5                         -Pb                               3 ]
```

NOT A CONTRADICTION. C.E.: D={a,b} Pa&-Pb

8-1. f) TEST FOR LOGICAL TRUTH

```
1                 * -[(]x)Px & (]x)-Px]                     -S
              |----------------^----------------|
2       * -(]x)Px                      *  -(]x)-Px          1 -&
3     a   (Vx)-Px                      a  (Vx)--Px          2 -]
4           -Pa                              --Pa           3 V
5                                              Pa           4 --
```

NOT A LOGICAL TRUTH. C.E.: D={a} -Pa; D={a} Pa

8-1. g) TEST FOR CONTRADICTION

```
1                  * [(Vx)Gx v (Vx)Hx] & -(Vx)(Gx v Hx)          S
2                           * (Vx)Gx v (Vx)Hx                    1 &
3                           * -(Vx)(Gx v Hx)                     1 &
4                           * (]x)-(Gx v Hx)                     3 -V
5                             * -(Ga v Ha)                       4 ]
6                                 -Ga                            5 -v
7                                 -Ha                            5 -v
                 |-----------------^-----------------|
8         a     (Vx)Gx                       a     (Vx)Hx        2 v
9                 Ga                                 Ha          8 V
CONTRADICTION     X                                  X
```

8-1. h) TEST FOR CONTRADICTION

```
1            * (Vx)(Kx v Jx) > [(]x)-Kx > (]x)Jx]                S
               |-------------------^--------------------|
2     * -(Vx)(Kx v Jx)                * (]x)-Kx > (]x)Jx         1 >
               |                       |--------^--------|
3     * (]x)-(Kx v Jx)             * -(]x)-Kx      * (]x)Jx      2 -V;>
4      * -(Ka v Ja)               a  (Vx)--Kx          Ja        3 ];-]
5          -Ka                    *   --Ka                       4 -v;V
6          -Ja                         Ka                        4 -v;5 --
NOT A CONTRADICTION.  C.E.:  D={a}  -Ka&-Ja;  Ka;  Ja
```

8-1. h) TEST FOR LOGICAL TRUTH

```
1          * -{(Vx)(Kx v Jx) > [(]x)-Kx > (]x)Jx]}               -S
2                   a    (Vx)(Kx v Jx)                           1 ->
3                 * -[(]x)-Kx > (]x)Jx]                          1 ->
4                      * (]x)-Kx                                 3 ->
5                      * -(]x)Jx                                 3 ->
6                   a    (Vx)-Jx                                 5 -]
7                          -Ka                                   4 ]
8                      * Ka v Ja                                 2 V
              |--------------^-------------|
9             Ka                           Ja                    8 v
10            X                           -Ja                    6 V
LOGICAL TRUTH                              X
```

8-1. i) TEST FOR CONTRADICTION

```
1            * [(Vx)Mx > (Vx)-Nx] & (]x)(-Mx & Nx)               S
2                  * (Vx)Mx > (Vx)-Nx                            1 &
3                   * (]x)(-Mx & Nx)                             1 &
4                      * -Ma & Na                                3 ]
5                          -Ma                                   4 &
6                          Na                                    4 &
             |----------------^----------------|
7      * -(Vx)Mx                        a   (Vx)-Nx              2 >
8      * (]x)-Mx                              -Na                7 -V;V
9          -Mb                                 X                  8 ]
NOT A CONTRADICTION.  C.E.: D={a,b}   -Ma&Na&-Mb
```


8-1. i) TEST FOR LOGICAL TRUTH

```
1            * -{[(Vx)Mx > (Vx)-Nx] & (]x)(-Mx & Nx)}               -S
               |-------------------^-------------------|
2     * -[(Vx)Mx > (Vx)-Nx]                 * -(]x)(-Mx & Nx)       1 -&
3        a       (Vx)Mx                  a    (Vx)-(-Mx & Nx)       2 ->;-]
4            * -(Vx)-Nx                       * -(-Ma & Na)         2 ->;3 V
5            * (]x)--Nx                       |------^------|       4 -V
6              * --Na                    *  --Ma           -Na      5 ];4 -&
7                 Na                          Ma                    6 --
8                 Ma                                                3 V
NOT A LOGICAL TRUTH.  C.E.:  D={a}  Na&Ma;  -Na
```

8-1. j) TEST FOR CONTRADICTION

```
1       * [(]x)Hx > (Vx)(Ox > Nx)] > [(]x)(Hx & Ox) > (Vx)Nx]        S
          |-------------------^-------------------|
2 * -[(]x)Hx > (Vx)(Ox > Nx)]          * (]x)(Hx & Ox) > (Vx)Nx      1 >
3        * (]x)Hx                          |-------^-------|         2 ->
4     * -(Vx)(Ox > Nx)            * -(]x)(Hx & Ox)   a (Vx)Nx        2 ->;>
5     * (]x)-(Ox > Nx)            a (Vx)-(Hx & Ox)      Na           4 -V;-];V
6          Ha                       * -(Ha & Oa)                     3 ]
7       * -(Ob > Nb)                  |----^----|                    5 ]
8          Ob                        -Ha       -Oa                   7 ->;6 -&
9         -Nb                                                        7 ->
NOT A CONTRADICTION.  C.E.:  D={a,b}  Ha&Ob&-Nb
```

8-1. j) TEST FOR LOGICAL TRUTH

```
1     * -{[(]x)Hx > (Vx)(Ox > Nx)] > [(]x)(Hx & Ox) > (Vx)Nx]}         -S
2                 * (]x)Hx > (Vx)(Ox > Nx)                             1 ->
3                * -[(]x)(Hx & Ox) > (Vx)Nx]                           1 ->
4                      * (]x)(Hx & Ox)                                 3 ->
5                         * -(Vx)Nx                                    3 ->
6                         * (]x)-Nx                                    5 -V
7                         * Ha & Oa                                    4 ]
8                             Ha                                       7 &
9                             Oa                                       7 &
10                           -Nb                                       6 ]
          |--------------------^---------------|
11    * -(]x)Hx                       a   (Vx)(Ox > Nx)                2 >
12   a  (Vx)-Hx                           * Oa > Na                    11 -];V
13         -Ha                     |----------^----------|             12 V
14          X                     -Oa                    Na            12 >
15                                 X              * Ob > Nb            11 V
                                                  |---^---|
16                                               -Ob      Nb           15 >
                                                          X
NOT A LOGICAL TRUTH.  C.E.: D={a,b}  Ha&Oa&-Nb&Na&-Ob
```

8-1. k) TEST FOR CONTRADICTION

```
 1      * (]x)[-Sx & (Gx v Kx)] v [(Vx)Gx > (Vx)(Sx v Kx)]          S
              |-------------------^-------------------|
 2 * (]x)[-Sx & (Gx v Kx)]          *  (Vx)Gx > (Vx)(Sx v Kx)     1 v
 3   * -Sa & (Ga v Ka)                 |-----------^--------|      2 ]
 4        -Sa                       *  -(Vx)Gx   a (Vx)(Sx v Kx)  3 &;2 >
 5      * Ga v Ka                   *  (]x)-Gx   *  Sa v Ka       3 &;4 -V;V
   |--------^--------|                  -Ga        |---^---|
 6  Ga              Ka                             Sa      Ka     5 v
NOT A CONTRADICTION.  C.E.:  D={a}  -Sa&Ga; -Sa&Ka
```

8-1. k) TEST FOR LOGICAL TRUTH

```
 1   ● -{(]x)[-Sx & (Gx v Kx)] v [(Vx)Gx > (Vx)(Sx v Kx)]}        -S
 2                ● -(]x)[-Sx & (Gx v Kx)]                        1 -v
 3               ● -[(Vx)Gx > (Vx)(Sx v Kx)]                      1 -v
 4            a    (Vx)-[-Sx & (Gx v Kx)]                         2 -]
 5                       (Vx)Gx                                   3 ->
 6                   ● -(Vx)(Sx v Kx)                             3 ->
 7                   ● (]x)-(Sx v Kx)                             6 -V
 8                     ● -(Sa v Ka)                               7 ]
 9                           -Sa                                  8 -v
10                           -Ka                                  8 -v
11                 ● -[-Sa & (Ga v Ka)]                           4 V
        |--------------------^--------------------|
12   * --Sa                            * -(Ga v Ka)               11 -&
13      Sa                                 -Ga                    12 --;-v
14      X                                  -Ka                    12 -v
15              LOGICAL TRUTH               Ga                    5 V
                                            X
```

8-1. l) TEST FOR CONTRADICTION

```
 1         * [(Vx)Fx v (Vx)Gx]<->[(]x)-Fx & -(Vx)Gx]              S
                |--------------------^--------------------|
 2     * (Vx)Fx v (Vx)Gx                * -[(Vx)Fx v (Vx)Gx]      1 <>
 3    * (]x)-Fx & -(Vx)Gx              * -[(]x)-Fx & -(Vx)Gx]     1 <>
 4          * (]x)-Fx                            |                3 &
 5          * -(Vx)Gx                            |                3 &
 6          * (]x)-Gx                            |                5 -V
 7              -Fa                              |                4 ]
 8              -Gb                              |                6 ]
      |---------^---------|                      |
 9 a (Vx)Fx          b (Vx)Gx                    |                2 v
10     Fa                Gb                      |                9 V
11     X                 X                   * -(Vx)Fx            2 -v
12                                           * -(Vx)Gx            2 -v
13                                           * (]x)-Fx            11 -V
14                                           * (]x)-Gx            12 -V
15                                               -Fc              13 ]
16                                               -Gd              14 ]
                                          |------^------|
17                                 * -(]x)-Fx      * --(Vx)Gx     3 -&
18                               c (Vx)--Fx      d (Vx)Gx         17 -];--
19                                  * --Fc             Gd         18 V
20                                     Fc              X          19 --
CONTRADICTION                          X
```


8-2. a)

```
1                 * (]x)Px          S
2                 * (]x)-Px         S
3                    Pa             1 ]
4                    -Pb            2 ]
CONSISTENT.  MODEL:  D={a,b}  Pa&-Pb
```

8-2. b)

```
1                a  (Vx)Px          S
2                a  (Vx)-Px         S
3                     -Pa           2 V
4                     Pa            1 V
INCONSISTENT          X
```

8-2. c)

```
1                a  (Vx)Px          S
2                * (]x)-Px          S
3                     -Pa           2 ]
4                     Pa            1 V
INCONSISTENT          X
```

8-2. d)

```
1                 a  (Vx)-Fx                         S
2                 a  (Vx)Sx                          S
3           ● (]x)[(-Fx > Sx) > Fx]                  S
4              ● (-Fa > Sa) > Fa                     3 ]
             |------^----------------------|
5      * -(-Fa > Sa)                       Fa        4 >
6           -Fa                            |         5 ->
7           -Sa                            |         5 ->
8           Sa                             |         2 V
9           X                              -Fa       1 V
INCONSISTENT                               X
```

8-2. e)

```
1              * (]x)Gx & (]x)Qx                     S
2              * -(]x)(Gx & Qx)                      S
3                  * (]x)Gx                          1 &
4                  * (]x)Qx                          1 &
5           a    (Vx)-(Gx & Qx)                      2 -]
6                     Ga                             3 ]
7                     Qb                             4 ]
8                 * -(Ga & Qa)                       5 V
      |---------------^---------------|
9    -Ga                             -Qa             8 -&
10    X                         * -(Gb & Qb)         5 V
                             |------^------|
11                          -Gb            -Qb       10 -&
CONSISTENT.  MODEL:  D={a,b}  Ga&Qb&-Gb&-Qa          X
```

8-2. f)

```
1                    a  (Vx)(Gx v Qx)                 S
2                  * -[(Vx)Gx v (Vx)Qx]               S
3 -                     * -(Vx)Gx                     2
4                       * -(Vx)Qx                     2
5                       * (]x)-Gx                     3
6                       * (]x)-Qx                     4
7                          -Ga                        5
8                          -Qb                        6
9                       * Ga v Qa                     1
             |---------------^---------------|
10           Ga                              Qa       9
11           X                          * Gb v Qb     1
                                    |-------^-------|
12                                  Gb              Qb    11
                                                    X
```

CONSISTENT. MODEL: D={a,b} -Ga&-Qb&Gb&Qa

8-2. g)

```
1                • (]x)(Jx v Dx)                        S
2              a  (Vx)(Jx > -Hx)                        S
3               a  (Vx)(Dx > Hx)                        S
4           a (Vx)[Jx<->(Dx v Hx)]                      S
5                 • Ja v Da                             1 ]
6                 • Ja > -Ha                            2 V
7                 • Da > Ha                             3 V
8               • Ja<->(Da v Ha)                        4 V
            |-----------^-----------|
9           Ja                      Da                  5 v
        |---^---|              |---^-----|
10     -Ja     -Ha            -Ja       -Ha             6 >
        X   |---^---|     |---^---|  |---^---|
11          -Da     Ha    -Da   Ha   -Da      Ha         7 >
             |      X      X    |     X       X
      |------^------|       |---^----------|
12    Ja           -Ja      Ja            -Ja            8 <>
13  * Da > Ha  -(Da > Ha)  Da > Ha   • -(Da > Ha)        8 <>
  |----^----|       X         X            |
14 -Da      Ha                             Da            13 >;-
15          X                             -Ha            13 ->
                                           X
```

CONSISTENT. MODEL: D={a} -Da & Ja & -Ha

8-3. A contradiction is true in NO circumstances. A consistent sentence is true in SOME (one or more) interpretations. A logical truth is true in ALL interpretations. (A set of sentences is consistent if there is an interpretation in which they are all true. So one sentence is consistnet if there is an interpretation in which that one sentence is true.) Thus a logical truth is consistent, and a contradiction is inconsistent (i.e., not consistent).

8-4. a) The sentences are logically equivalent. If all things are P and all things are Q, then all things are both P and Q. And, conversely, if all

things are both P and Q, then all things are P and also all things are Q.

b) The sentences are logically equivalent. If either something is P or something is Q, then there is something which is either P or is Q. And conversely, if there is something which is either P or is Q, then there is something which is P or there is something which is Q.

c) The sentences are not logically equivalent. It can be true that everything is either P or is Q, but not either everything is P or everything is Q. The counterexample shows you how this can happen. If there are two things and one is P and the other is Q, everything is either P or Q. But if the first is not Q and the second is not P, then it is false that either everything is P or everything is Q.

d) Again, the sentences are not logically equivalent. There can be something which is P and ANOTHER which is Q. This makes it true that something is P and something is Q. But if the first is not Q and the second is not P, there won't be anything which is both P and Q.

Now go back and compare (a)-(d). Note how the universal quantifier distributes over conjunctions but not disjunctions, while the existential quantifier distributes over disjunctions but not conjunctions. If you think about it a bit you will see that this symmetry can be explained by appeal to DeMorgan's laws and the rules of logical equivalence -(u) and -(Eu).

e), f), g), and h): In these cases the pairs of sentences are logically equivalent.

i), j), and k): The sentences are not logically equivalent.

Note for e), f), g), h), j) and k): You can move either quantifier which governs a conditional to the consequent of the conditional, if the quantified variable is not feree in the antecedent. But you con't move the quantifier to the antecedent even if the quantified variable is not free in the consequent. Instead a universal quantifier governing a conditional converts to an existential quantifier and an existential quantifier governing a conditional converts to a universal quantifier when the quantifiers are moved to the antecedent (with the quantified variable not occurring free in the consequent). It is not hard to see that this is because a conditional of the form X -> Y is logically equivalent to -X v Y. When the quantifier moves in from the whole conditional to the antecedent it has to move through the negation sign, and this changes a universal quantifier to an existential quantifier and and existential quantifier to a universal quantifier.

8-5. a) TEST FOR LOGICAL TRUTH

```
1                 * -(Vx)((Vy)Py > Px)          -S
2                 * (]x)-((Vy)Py > Px)          1 -V
3                   * -[(Vy)Py > Pa]            2 ]
4                       a  (Vy)Py               3 ->
5                             -Pa               3 ->
6                              Pa               4 V
LOGICAL TRUTH                  X
```

8-5. b) TEST FOR CONTRADICTION

```
1                 a   (Vx)[(]y)By > Bx]          S
2                      * (]y)By > Ba             1 V
               |------------^------------|
3        * -(]y)By                       Ba      2 >
4     a    (Vy)-By                               3 -]
5           -Ba                                  4 V
```

NOT A CONTRADICTION. C.E.: D={a} -Ba; D={a} Ba

8-5. b) TEST FOR LOGICAL TRUTH

```
1             * -(Vx)[(]y)By > Bx]      -S
2             * (]x)-[(]y)By > Bx]      1 -V
3               * -[(]y)By > Ba]        2 ]
4                   * (]y)By            3 ->
5                      -Ba              3 ->
6                      Bb               4 ]
```

NOT A LOGICAL TRUTH. C.E.: D={a,b} -Ba&Bb

8-5. c) TEST FOR CONTRADICTION

```
1             b   (Vx)[(]y)Cy & -Cx]     S
2                 * (]y)Cy & -Ca         1 V
3                   * (]y)Cy             2 &
4                      -Ca               2 &
5                      Cb                3 ]
6                 * (]y)Cy & -Cb         1 V
7                     (]y)Cy             6 &
8                      -Cb               6 &
CONTRADICTION           X
```

8-5. d) TEST FOR CONTRADICTION

```
1             * (]x)(]y)(Lxy & -Lyx)     S
2               * (]y)(Lay & -Lya)       1 ]
3                 * Lab & -Lba           2 ]
4                     Lab                3 &
5                     -Lba               3 &
```

NOT A CONTRADICTION. C.E.: D={a,b} Lab&-Lba. Note: you can skip line 2 if you don't neglect to use different names when instantiating (]x) and (]y).

8-5. d) TEST FOR LOGICAL TRUTH

```
1             * -(]x)(]y)(Lxy & -Lyx)       -S
2          a    (Vx)-(]y)(Lxy & -Lyx)       1 -]
3               * -(]y)(Lay & -Lya)         2 V
4          a      (Vy)-(Lay & -Lya)         3 -]
5                 * -(Laa & -Laa)           4 V
          |-------------^-------------|
6       -Laa                        --Laa   5 -&
7                                    Laa    6 --
```

NOT A LOGICAL TRUTH. C.E.: D={a} Laa; D={a} -Laa

8-5. e) TEST FOR CONTRADICTION

```
1              * (]y)[(Vx)Rxy > (Vx)Ryx]        S'
2                * (Vx)Rxa > (Vx)Rax            1 ]
          |-------------^-------------|
3       * -(Vx)Rxa               a   (Vx)Rax    2 >
4       * (]x)-Rxa                    Raa       3 -V;V
5          -Rba                                 4 ]
```

NOT A CONTRADICTION. C.E.: D={a,b} -Rba; D={a} Raa

8-5. e) TEST FOR LOGICAL TRUTH

```
1              * -(]y)[(Vx)Rxy > (Vx)Ryx]      -S
2          b     (Vy)-[(Vx)Rxy > (Vx)Ryx]      1 -]
3                * -[(Vx)Rxa > (Vx)Rax]        2 V
4              a      (Vx)Rxa                  3 ->
5                   * -(Vx)Rax                 3 ->
6                   * (]x)-Rax                 5 -V
7                      -Rab                    6 ]
8                       Raa                    4 V
9                * -[(Vx)Rxb > (Vx)Rbx]        2 V
10             a      (Vx)Rxb                  9 ->
11                  * -(Vx)Rbx                 9 ->
12                  * (]x)-Rbx                 11 -V
13                     -Rbc                    12 ]
14                      Rab                    10 V
                         X
```

LOGICAL TRUTH. Note that in this problem you must be very choosy about what you work on first, if the problem is to be completed as quickly as shown here. Unfortunate choice of applying the (u) rule would make the tree much longer.

8-5. f) TEST FOR CONTRADICTION

```
1             a,b  (Vx)[(Vy)Txy & (]y)-Tyx]     S
2               *    (Vy)Tay & (]y)-Tya         1 V
3                       (Vy)Tay                 2 &
4                    *   (]y)-Tya               2 &
5                         -Tba                  4 ]
6                 *  (Vy)Tby & (]y)-Tyb         1 V
7                  a    (Vy)Tby                 6 &
8                    *  (]y)-Tyb                6 &
9                         Tba                   7 V
CONTRADICTION              X
```

8-5. g) TEST FOR CONTRADICTION

```
1                 * (]x)(Vy)(Fx > Fy)           S
2             a      (Vy)(Fa > Fy)              1 ]
3                    * Fa > Fa                  2 V
           |-------------^-------------|
4         -Fa                          Fa       3 >
```

NOT A CONTRADICTION. C.E.: D={a} -Fa; D={a} Fa

8-5. g) TEST FOR LOGICAL TRUTH

```
1                 * -(]x)(Vy)(Fx > Fy)          -S
2             a     (Vx)-(Vy)(Fx > Fy)          1 -]
3                   * -(Vy)(Fa > Fy)            2 V
4                   * (]y)-(Fa > Fy)            3 -V
5                     * -(Fa > Fb)              4 ]
6                         Fa                    5 ->
7                         -Fb                   5 ->
8                   * -(Vy)(Fb > Fy)            2 V
9                   * (]y)-(Fb > Fy)            8 -V
10                    * -(Fb > Fc)              9 ]
11                        Fb                    10 ->
12                        -Fc                   10 ->
LOGICAL TRUTH             X
```

8-5. h) TEST FOR CONTRADICTION

```
1          a,b,c   (Vx)(]y)(Rxy & -Ryx)         S
2                * (]y)(Ray & -Rya)             1 V
3                *    Rab & -Rba                2 ]
4                     Rab                       3 &
5                     -Rba                      3 &
6                * (]y)(Rby & -Ryb)             1 V
7                *    Rbc & -Rcb                6 ]
8                     Rbc                       7 &
9                     -Rcb                      7 &
10                 (]y)(Rcy & -Ryc)             1 V
                        .
                        .
```

INFINITE TREE. NOT A CONTRADICTION.
C.E.: D={a,b,c....}; Rab & -Rba & Rbc & -Rcb & ......

8-5. h) TEST FOR LOGICAL TRUTH

```
1              * -(Vx)(]y)(Rxy & -Ryx)          -S
2              * (]x)-(]y)(Rxy & -Ryx)          1 -V
3                * -(]y)(Ray & -Rya)            2 ]
4             a    (Vy)-(Ray & -Rya)            3 -]
5                  * -(Raa & -Raa)              4 V
         |-------------^-------------|
6       -Raa                        --Raa       5 -&
7                                    Raa        6 --
```

NOT A LOGICAL TRUTH. C.E.: D={a} -Raa; Raa

8-6. a)

```
1                 * (]x)(]y)Lxy                 S
2                * (]x)(]y)-Lxy                 S
3                   * (]y)Lay                   1 ]
4                  * (]y)-Lby                   2 ]
5                      Lac                      3 ]
6                      -Lbd                     4 ]
```

CONSISTENT. MODEL: D={a,b,c,d} Lac&-Lbd. Note: You can skip lines 3 and 5 if you are sure not to neglect using new names!

8-6. b)

```
1             a    (Vx)(Vy)Lxy                  S
2             a    (Vx)(Vy)-Lyx                 S
3              a     (Vy)-Lya                   2 V
4                     -Laa                      3 V
5               a    (Vy)Lay                    1 V
6                     Laa                       5 V
INCONSISTENT           X
```

8-6. c)

```
1             a    (Vx)(]y)Kxy                  S
2               * (]x)(Vy)-Kxy                  S
3             b     (Vy)-Kay                    2 ]
4                  * (]y)Kay                    1 V
5                     Kab                       4 ]
6                     -Kab                      3 V
INCONSISTENT           X
```

8-6. d)

```
1                          a   (Vx)Ax                                   S
2                            * -(]x)Bx                                  S
3           a   (Vx)((]y)(Ax & -By) > [(]y)Ay > (Vy)-By]}               S
4                          a   (Vx)-Bx                                  2 -]
5                                -Ba                                    4 V
6                 * (]y)(Aa & -By) > [(]y)Ay > (Vy)-By]                 3 V
                  |-----------------^-----------------|
7      * -(]y)(Aa & -By)                 * (]y)Ay > (Vy)-By             6 >
8    a   (Vy)-(Aa & -By)                 |-------^--------|             7 -]
9        * -(Aa & -Ba)              * -(]y)Ay        a   (Vy)-By        7 V;>
       |------^------|                  |                    |
10    -Aa         * --Ba          a   (Vy)-Ay               -Ba         9 -&;-];V
11    Aa              |                 |                   Aa          1 V
12     X             Ba                -Aa                              10 --;V
13                    X                 Aa                              1 V
                                        X
```

CONSISTENT. MODEL: D={a} -Ba&Aa

8-6. e)

```
1              a,b   (Vx)(]y)Mxy                S
2                  * (]y)(Vx)-Mxy               S
3            a,b,c   (Vx)-Mxa                   2 ]
4                    * (]y)May                  1 V
5                       Mab                     4 ]
6                       -Maa                    3 V
7                       -Mba                    3 V
8                     * (]y)Mby                 1 V
9                       Mbc                     8 ]
10                      -Mca                    3 V
                         .
                         .
                         .
```

INFINITE TREE. CONSISTENT. MODEL: D={a,b,c,...} Mab&-Maa&-Mba&Mbc&-Mca...
[The model has objects a,b,c,.... Each bears relation M to the next; each fails to bear the relation M to a.]

8-6. f)

```
1              * (]x)(]y)[(Rxx & -Ryy) & Rxy]              S
2                  a    (Vx)(Vy)(Rxy > Ryx)                S
3           b  (Vx)(Vy)(Vz)[(Rxy & Ryz) > Rxz]             S
4               *  (]y)[(Raa & -Ryy) & Ray]                1 ]
5                   *  (Raa & -Rbb) & Rab                  4 ]
6                        *  Raa & -Rbb                     5 &
7                           Rab                            5 &
8                           Raa                            6 &
9                           -Rbb                           6 &
10                 b  (Vy)(Ray > Rya)                      2 V
11                    * (Rab > Rba)                        10 V
             |-------------^-------------|
12         -Rab                         Rba                11 >
             X                           |
13                  a  (Vy)(Vz)[(Rby & Ryz) > Rbz]         3 V
14                      b  (Vz)[(Rba & Raz) > Rbz]         13 V
15                          b  (Rba & Rab) > Rbb           14 V
                        |---------^---------|
16              * -(Rba & Rab)             Rbb             15 >
               |-------^-------|            X
17           -Rba            -Rab                          16 -&
               X               X
```

INCONSISTENT. Line 2 was instantiated just with x=a and y=b. On line 3, I instantiated just with x=b, y=a, and z=b. Knowing about relations, I was able to see that these instances would result in all branches closing. This was a hard problem.

8-7. a)

```
1             * -{(Vx)(Vy)(Px & Qy)<->[(Vx)Px & (Vx)Qx]}             -S
                 |-----------------^-----------------|
2    a,b  (Vx)(Vy)(Px & Qy)               * -(Vx)(Vy)(Px & Qy)       1 -<>
3        * -[(Vx)Px & (Vx)Qx]             * (Vx)Px & (Vx)Qx          1 -<>
          |--------^--------|                    |
4    * -(Vx)Px        * -(Vx)Qx                  |                   3 -&
5    * (]x)-Px        * (]x)-Qx                  |                   4 -V
6       -Pa              -Qb                     |                   5 ]
7 a (Vy)(Pa & Qy)  b (Vy)(Pb & Qy)   * (]x)-(Vy)(Px & Qy)            2 V;-V
8    * Pa & Qa        * Pb & Qb                  |                   7 V
9       Pa               Pb                      |                   8 &
10      Qa               Qb                      |                   8 &
11      X                X               c   (Vx)Px                  3 &
12                                       d   (Vx)Qx                  3 &
13                                    * -(Vy)(Pc & Qy)               7 ]
14                                    * (]y)-(Pc & Qy)               13 -V
15                                      * -(Pc & Qd)                 14 ]
                                      |------^------|
16                                   -Pc           -Qd               15 -&
17                                    Pc             |               11 V
18                                    X             Qd               12 V
                                                     X
```

LOGICALLY EQUIVALENT

8-7. b)

```
1            * -{(]x)(]y)(Px & Qy)<->[(]x)Px & (]x)Qx]}             -S
                |-----------------^-----------------|
2       * (]x)(]y)(Px & Qy)              * -(]x)(]y)(Px & Qy)       1 -<>
3       * -((]x)Px & (]x)Qx)             * (]x)Px & (]x)Qx          1 -<>
4         * (]y)(Pa & Qy)            c   (Vx)-(]y)(Px & Qy)         2 ];-]
5           * Pa & Qb                           |                   4 ]
6              Pa                               |                   5 &
7              Qb                               |                   5 &
         |------^------|                        |
8     * -(]x)Px    * -(]x)Qx                    |                   3 -&
9   a  (Vx)-Px   b  (Vx)-Qx                     |                   8 -]
10       -Pa          -Qb                       |                   9 V
11        X            X                   * (]x)Px                 3 &
12                                         * (]x)Qx                 3 &
13                                            Pc                    11 ]
14                                            Qd                    12 ]
15                                     * -(]y)(Pc & Qy)             4 V
16                                d      (Vy)-(Pc & Qy)             15 -]
17                                       * -(Pc & Qd)               16 V
                                       |------^------|
18                                    -Pc           -Qd             17 -&
                                       X             X
```

LOGICALLY EQUIVALENT

8-7. c)

```
1           * -{(Vx)(Vy)(Px v Qy)<->[(Vx)Px v (Vx)Qx]}              -S
                |-----------------^-----------------|
2    a   (Vx)(Vy)(Px v Qy)               * -(Vx)(Vy)(Px v Qy)       1 -<>
3     * -[(Vx)Px v (Vx)Qx]                 * (Vx)Px v (Vx)Qx        1 -<>
4          * -(Vx)Px                               |                3 -v
5          * -(Vx)Qx                               |                3 -v
6          * (]x)-Px                               |                4 -V
7          * (]x)-Qx                               |                5 -V
8             -Pa                                  |                6 ]
9             -Qb                                  |                7 ]
10   b    (Vy)(Pa v Qy)                  * (]x)-(Vy)(Px v Qy)       2 V;-V
11         * Pa v Qb                       * -(Vy)(Pc v Qy)         10 V;]
       |-------^-------|                           |
12    Pa               Qb                  * (]y)-(Pc v Qy)         11 v;-V
13    X                X                     * -(Pc v Qd)           12 ]
14                                              -Pc                 13 -v
15                                              -Qd                 13 -v
                                          |------^------|
16                                 c  (Vx)Px       d  (Vx)Qx        3 v
17                                       Pc              Qd         16 V
                                         X               X
```

LOGICALLY EQUIVALENT

8-7. d)

```
1            * -{(]x)(]y)(Px v Qy)<->[(]x)Px v (]x)Qx]}              -S
                  |------------------^------------------|
2    * (]x)(]y)(Px v Qy)                 * -(]x)(]y)(Px v Qy)         1 -<>
3    * -[(]x)Px v (]x)Qx]                  * (]x)Px v (]x)Qx          1 -<>
4           * -(]x)Px                             |                   3 -v
5           * -(]x)Qx                             |                   3 -v
6         a   (Vx)-Px                             |                   4 -]
7         b   (Vx)-Qx                             |                   5 -]
8       * (]y)(Pa v Qy)             c,d    (Vx)-(]y)(Px v Qy)         2 ];-]
9           * Pa v Qb                             |                   8 ]
      |--------^--------|                         |
10    Pa                Qb                        |                   9 v
11    -Pa               |                         |                   6 V
12    X                -Qb             |----------^----------|        7 V
13                      X           * (]x)Px             * (]x)Qx     3 v
14                                     Pc                   Qd        13 ]
15                            * -(]y)(Pc v Qy)    * -(]y)(Pd v Qy)    8 V
16                           c  (Vy)-(Pc v Qy)   d (Vy)-(Pd v Qy)     15 -]
17                               * -(Pc v Qc)       * -(Pd v Qd)      16 V
18                                   -Pc                -Pd           17 -v
19                                   -Qc                -Qd           17 -v
                                      X                  X
```

LOGICALLY EQUIVALENT

8-7. e)

```
1            * -{(Vx)(Vy)(Px > Qy)<->[(]x)Px > (Vx)Qx]}              -S
                  |------------------^------------------|
2     a  (Vx)(Vy)(Px > Qy)               * -(Vx)(Vy)(Px > Qy)         1 -<>
3      * -[(]x)Px > (Vx)Qx]                * (]x)Px > (Vx)Qx          1 -<>
4            * (]x)Px                             |                   3 ->
5           * -(Vx)Qx                             |                   3 ->
6           * (]x)-Qx                             |                   5 -V
7               Pa                                |                   4 ]
8               -Qb                               |                   6 ]
9      b   (Vy)(Pa > Qy)                 * (]x)-(Vy)(Px > Qy)         2 V;-V
10          * Pa > Qb                      * -(Vy)(Pc > Qy)           9 V;]
       |--------^--------|                        |
11     -Pa              Qb                 * (]y)-(Pc > Qy)           10 >;-V
12     X                X                    * -(Pc > Qd)             11 ]
13                                                Pc                  12 ->
14                                               -Qd                  12 ->
                                        |---------^---------|
15                                   * -(]x)Px        d   (Vx)Qx      3 >
16                               c   (Vx)-Px                 Qd       15 -];V
17                                     -Pc                   X        16 V
                                        X
```

LOGICALLY EQUIVALENT

8-7. f)

```
1            * -{(]x)(]y)(Px > Qy)<->[(Vx)Px > (]x)Qx]}              -S
                  |------------------^------------------|
2    * (]x)(]y)(Px > Qy)                 * -(]x)(]y)(Px > Qy)         1 -<>
3    * -[(Vx)Px > (]x)Qx]                  * (Vx)Px > (]x)Qx          1 -<>
4       a      (Vx)Px                             |                   3 ->
5           * -(]x)Qx                             |                   3 ->
6       b     (Vx)-Qx                             |                   5 -]
7        * (]y)(Pa > Qy)            c,d    (Vx)-(]y)(Px > Qy)         2 ];-]
8           * Pa > Qb                             |                   7 ]
      |--------^--------|                         |
9     -Pa               Qb                        |                   8 >
10    Pa                |                         |                   4 V
11    X                -Qb             |----------^----------|        6 V
12                      X          * -(Vx)Px             * (]x)Qx     3 >
13                                 * (]x)-Px                Qd        12 -V;]
14                                    -Pc                   |         13 ]
15                            * -(]y)(Pc > Qy)    * -(]y)(Pd > Qy)    7 V
16                           c  (Vy)-(Pc > Qy)   d (Vy)-(Pd > Qy)     15 -]
17                               * -(Pc > Qc)       * -(Pd > Qd)      16 V
18                                    Pc                 Pd           17 ->
19                                   -Qc                -Qd           17 ->
                                      X                  X
```

LOGICALLY EQUIVALENT

8-7. g)

```
1            * -{(]x)(Vy)(Px > Qy)<->[(Vx)Px > (Vx)Qx]}              -S
                  |------------------^------------------|
2    * (]x)(Vy)(Px > Qy)                 * -(]x)(Vy)(Px > Qy)         1 -<>
3    * -[(Vx)Px > (Vx)Qx]                  * (Vx)Px > (Vx)Qx          1 -<>
4       a      (Vx)Px                             |                   3 ->
5           * -(Vx)Qx                             |                   3 ->
6           * (]x)-Qx                             |                   5 -V
7     b   (Vy)(Pa > Qy)             c,e    (Vx)-(Vy)(Px > Qy)         2 ];-]
8               -Qb                               |                   6 ]
9           * Pa > Qb                             |                   7 V
      |--------^--------|                         |
10    -Pa               Qb                        |                   9 >
11    Pa                X              |----------^----------|        4 V
12    X                            * -(Vx)Px          e  (Vx)Qx       3 >
13                                 * (]x)-Px                Qe        12 -V;V
14                                    -Pc                   |         13 ]
15                            * -(Vy)(Pc > Qy)    * -(Vy)(Pe > Qy)    7 V
16                            * (]y)-(Pc > Qy)    * (]y)-(Pe > Qy)    15 -V
17                               * -(Pc > Qd)       * -(Pe > Qf)      16 ]
18                                    Pc                 Pe           17 ->
19                                   -Qd                -Qf           17 ->
20                                    X                  Qf           12 V
                                                         X
```

LOGICALLY EQUIVALENT

8-7. h)

```
 1          * -{(Vx)(]y)(Px > Qy)<->[(]x)Px > (]x)Qx]}           -S
                 |------------------^------------------|
 2   a   (Vx)(]y)(Px > Qy)               * -(Vx)(]y)(Px > Qy)     1 -<>
 3    * -((]x)Px > (]x)Qx)               * (]x)Px > (]x)Qx        1 -<>
 4          * (]x)Px                              |               3 ->
 5        * -(]x)Qx                               |               3 ->
 6      b    (Vx)-Qx                              |               5 -]
 7              Pa                                |               4 ]
 8    * (]y)(Pa > Qy)                    * (]x)-(]y)(Px > Qy)     2 V;-V
 9        * Pa > Qb                        * -(]y)(Pc > Qy)       8 ]
      |------^------|                             |
10   -Pa            Qb               c,d   (Vy)-(Pc > Qy)         9 >;-]
11    X            -Qb                    |-------^-------|       6 V
12                  X               * -(]x)Px        * (]x)Qx     3 >
13                                c   (Vx)-Px           Qd        12 -];]
14                                      -Pc             |         13 V
15                                * -(Pc > Qc)    * -(Pc > Qd)    10 V
16                                      Pc              Pc        15 ->
17                                     -Qc             -Qd        15 ->
LOGICALLY EQUIVALENT                    X               X
```

8-7. i) TEST ARGUMENT (]y)(Vx)Lxy/(Vx)(]y)Lxy

```
 1                  * (]y)(Vx)Lxy              P
 2                 * -(Vx)(]y)Lxy              -C
 3                 * (]x)-(]y)Lxy              2 -V
 4                 * (]x)(Vy)-Lxy              3 -]
 5                b   (Vx)Lxa                  1 ]
 6                a   (Vy)-Lby                 4 ]
 7                      Lba                    5 V
 8                     -Lba                    6 V
The argument is valid.  X
```

8-7. i) TEST ARGUMENT (Vx)(]y)Lxy/(]y)(Vx)Lxy

```
 1       a,b,c     (Vx)(]y)Lxy              P
 2              * -(]y)(Vx)Lxy              -C
 3              *  (Vy)-(Vx)Lxy             2 -]
 4       a,b,c     (Vy)(]x)-Lxy             3 -V
 5                *  (]y)Lay                1 V
 6                     Lab                  5 ]
 7              *  (]x)-Lxa                 4 V
 8                    -Lca                  7 ]
 9                    (]y)Lby               1 V
10                    (]y)Lcy               1 V
11                   (]x)-Lxb               4 V
12                   (]x)-Lxc               4 V
                         .
                         .
```

Lines 9,10,11, and 12 will produce further names which must go back into lines 1 and 4. Clearly the tree is infinite, so the argument is invalid. The counter-example is quite complicated. Since the argument from the first to the second sentence is invalid the sentences are NOT LOGICALLY EQUIVALENT.

8-7. j)

```
 1          * -{(Vy)[(]x)Bx & Hy]<->[(]x)Bx & (Vy)Hy]}              -S
                 |-------------------^-------------------|
 2 a,c (Vy)[(]x)Bx & Hy]                 * -(Vy)[(]x)Bx & Hy]       1 -<>
 3    * -[(]x)Bx & (Vy)Hy]                * (]x)Bx & (Vy)Hy         1 -<>
       |--------^--------|                        |
 4 * -(]x)Bx        * -(Vy)Hy                     |                 3 -&
 5 a (Vx)-Bx        * (]y)-Hy                     |                 4 -];-V
 6     -Ba              -Hc                       |                 5 V;]
 7 (]x)Bx & Ha    * (]x)Bx & Hc          * (]y)-[(]x)Bx & Hy]       2 V;-V
 8 * (]x)Bx           (]x)Bx                      |                 7 &
 9     Ha               Hc                        |                 7 &
10     Bb               X                         |                 8 ]
11    -Bb                                         |                 5 V
12     X                                     * (]x)Bx               3 &
13                                        d    (Vy)Hy               3 &
14                                       * -[(]x)Bx & Hd]           7 ]
15                                              Be                  12 ]
                                         |-------^-------|
16                                   * -(]x)Bx          -Hd         14 -&
17                                   e  (Vx)-Bx          |          16 -]
18                                       -Be             Hd         17 V;13 V
                                          X              X
```

LOGICALLY EQUIVALENT

8-8. a)

```
 1              * (]x)(]y)[(Vz)Lzx > Lxy]          P
 2                 * -(]x)(]y)Lxy                  -C
 3                 * (Vx)-(]y)Lxy                  2 -]
 4         a,b,c     (Vx)(Vy)-Lxy                  3 -]
 5              * (]y)[(Vx)Lza > Lay]              1 ]
 6                 * (Vx)Lza > Lab                 5 ]
                   |------^------|
 7          * -(Vz)Lza          Lab                6 >
 8                |         b  (Vy)-Lay            4 V
 9          * (]z)-Lza          -Lab               7 -V;8 V
10                -Lca            X
11                -Laa                             Lines 10-19
12                -Lab                             are from
13                -Lac                             line 4, V
14                -Lba                             all combinations
15                -Lbb                             of a,b,c into
16                -Lbc                             (Vx)(Vy)-Lxy
17                -Lca
18                -Lcb
19                -Lcc
```

INVALID. C.E.: D={a,b,c} -Laa&-Lab&-Lac&-Lba&-Lbb&-Lbc&-Lca&-Lcb&-Lcc

8-8. b)

```
1                  a     (Vx)(]y)Lxy                        P
2                a   (Vx)[(]y)Lxy > Lxx]                    P
3                       * -(Vx)Lxx                          -C
4                       * (]x)-Lxx                          3 -V
5                          -Laa                             4 ]
6                     * (]y)Lay > Laa                       2 V
                  |-----------^-----------|
7        * -(]y)Lay                      Laa                6 >
8      b   (Vy)-Lay                       X                 7 -]
9        * (]y)Lay                                          1 V
10           Lab                                            9 ]
11           -Lab                                           8 V
VALID         X
```

8-8. c)

```
1                  a     (Vx)(]y)Lyx                        P
2                a   (Vx)(Vy)(Lxy > Txy)                    P
3                     * -(Vx)(]y)Tyx                        -C
4                     * (]x)-(]y)Tyx                        3 -V
5                       * -(]y)Tya                          4 ]
6                    a    (Vy)-Tya                          5 -]
7                           -Taa                            6 V
8                 a    (Vy)(Lay > Tay)                      2 V
9                       * Laa > Taa                         8 V
                  |-----------^-----------|
10               -Laa                    Taa                9 >
11           * (]y)Lya                    X                 1 V
12               Iba                                        11 ]
13     a    (Vy)(Iby > Tby)                                 2 V
14           * Iba > Tba                                    13 V
          |------^------|
15      -Iba           Tba                                  14 >
16        X            -Tba                                 6 V
                        X        VALID
```

8-8. d)

```
1            a    (Vx)(Vy)(Vz)[(Jxy & Jyz) > Jxz]           P
2                a   (Vx)(Vy)(Jxy > Jyx)                    P
3                       * -(Vx)Jxx                          -C
4                       * (]x)-Jxx                          3 -V
5                          -Jaa                             4 ]
6                  a    (Vy)(Jay > Jya)                     2 V
7                       * Jaa > Jaa                         6 V
                  |-----------^-----------|
8                -Jaa                    Jaa                7 >
9  a   (Vy)(Vz)[(Jay & Jyz) > Jaz]        X                 1 V
10   a    (Vz)[(Jaa & Jaz) > Jaz]                           9 V
11       * (Jaa & Jaa) > Jaa                                10 V
              |------^------|
12     *-(Jaa & Jaa)       Jaa                              11 >
         |---^---|          X
13     -Jaa    -Jaa                                         12 -&
INVALID.  C.E.:  D={a}  -Jaa
```


8-8. e)

```
1           a,b,c,d  (Vx)(]y)Hxy                            P
2                  * -(]y)(Vx)Hxy                           -C
3                  * (Vy)-(Vx)Hxy                           3 -]
4               a,b   (Vy)(]x)-Hxy                          4 -V
5                    *    (]y)Hay                           1 V
6                           Hab                             5 ]
7                    *  (]x)-Hxa                            4 V
8                    *  (]x)-Hxb                            4 V
9                          -Hca                             7 ]
10                         -Hdb                             8 ]
11                        (]y)Hby                           1 V
12                        (]y)Hcy                           1 V
13                        (]y)Hdy                           1 V
                             .
                             .
                             .
```

INFINITE TREE. INVALID. The C.E. is very complicated!

8-8. f)

```
1                 b     (Vx)(Cx > Ax)                            P
2        * -(Vx)[(]y)(Cy & Txy) > (]y)(Ay & Txy)]                -C
3        * (]x)-[(]y)(Cy & Txy) > (]y)(Ay & Txy)]                2 -V
4          * -[(]y)(Cy & Tay) > (]y)(Ay & Tay)]                  3 ]
5                  * (]y)(Cy & Tay)                              4 ->
6                  * -(]y)(Ay & Tay)                             4 ->
7               b   (Vy)-(Ay & Tay)                              6 -]
8                     * Cb & Tab                                 5 ]
9                         Cb                                     8 &
10                        Tab                                    8 &
11                   * -(Ab & Tab)                               7 V
          |-------------------^-------------------|
12       -Ab                                     -Tab            11 -&
13    * Cb > Ab                                   X              1 V
   |------^------|
14 -Cb          Ab                                               13 >
    X            X
```

VALID. Note: I never had to instantiate line 1 with 'a'.

ANSWERS TO EXERCISES IN VOLUME II, CHAPTER 9

9-1.
a) (Ex)Cx
b) (x)(y)[(Cx & Cy) -> x=y]
c) (Ex)(Ey)(Ez)(Cx & Cy & Cz & x≠y & y≠z & y≠z)
d) (Ex)(Ey)(Ez){Cx & Cy & Cz & x≠y & x≠z & y≠z & (w)[Cw ->(w=x v w=y v w=z)]
e) (x)(y)(z)(w)[(Cs & Cy & Cz & Cw) -> (x=y v x=z v x=w v y=z v y=w v z=w)]

9-2.
a) (Ex)(Ey)(Ez)[Px & Py & Pz & Lxa & Lya & Lza & x≠y & x≠z &y≠z]
b) (Ex)(Ey)(Ez){Px & Py & Pz & Lxa & Lya & Lza & x≠y & x≠z & y≠z &
(w)[(Pw & Lwa) -> (x=w v y=w v z=w)]}
c) Pe & Re & (x)[(Px & Rx) -> x=e]
d) (x)[(Px & x≠a) -> Rx] & -Ra & Pa
e) Sea & (x)(Sxa -> x=e)
f) (x)[(Px & Rx & x≠a) -> Sxa]
g) Pe & Re & (x)[(Px & Rx & x≠e) -> Sex]
h) Pa & -Sae & (x)[(Px & x≠a) -> Sxe]
i) (Ex){Cx & Oax & (y)[(Cy & Oay) -> x=y] & Fx}
j) (x){Px -> (Ey)(Ex)[Myx & Qzy & (w)(Qwy -> w=z)]}
k) (x){Px -> [(Ey)(Ez)(Ew)(Qyx & Qzx & Qwx) -> (y=z v y=w v z=w)]}, OR
(x)[Px -> -(Ey)(Ez)(Ew)(Qyx & Qzx & Qwx & y≠z & y≠w &z≠w)]

9-3. Suppose we are given the rules:

```
|s=s   =I                         |s=t
                                  |P(s)
                                  |P(t)   =E
```

Suppose we have a derivation on which s=t and P(t) already appear. We need to show that P(s) can be derived from the first two rules:

```
1 |s=t
2 |P(t)                      [The argument from the second
3 |s=s       =I              to the first form of =E works
4 |t=s       1,3,=E          the same way.]
5 |P(s) 2,4,=E
```

9-4. To see that this holds, consider a case in which we have a branch on which a≠a appears. Now, since the =I rule licenses us to write n=n (for any name, n) at any place on a branch, we could simply write a=a on the branch on which a≠a appears. So, on one branch, we would have both a≠a [or, -(a=a)] and a=a, which would close the branch. In general, if we wrote self-identities for each name occuring on a branch, any branch having n≠n for some name would close. This is just what the ≠ rule does for us.

NOTE: Answers for exercises 9-5 through 9-8 are given in the form of both derivations and truth trees. All of the derivations come first, followed by all of the truth trees.

9-5. a)

```
1 |â=â            =I
2 |(Ex)(x=a)      1,EI
```


9-5. b)

```
1 |   |-(Fa > Fb)              A
2 |   |   |a=b                 A
3 |   |   |Fa                  1,->
4 |   |   |-Fb                 1,->
5 |   |   |-Fa                 2,4,=E
6 |   |a≠b                     2-5,-I
7 |-(Fâ > Fb̂) > â≠b̂            1-6,>I
8 |(y)[-(Fâ > Fy) > â≠y]          7,(y)I
9 |(x)(y)[-(Fx > Fy) > x≠y]      8,(x)I
```

9-5. c)

```
1  |  |Pa                        A
2  |  |a=a                       =I
3  |  |a=a & Pa                  1,2 &I
4  |  | (Ey)(a=y & Py)           3,EI
5  |Pâ > (Ey)(â=y & Py)         1-4,>I
6  |  |(Ey)(a=y & Py)            A
7  |  |b|a=b & Pb                A
8  |  | |a=b                     7,&E
9  |  | |Pb                      7,&E
10 |  | |Pa                      8,9,=E
11 |  |Pa                       6,7-10,EE
12 |(Ey)(â=y & Py) > Pâ        6-11,>I
13 |Pâ<->(Ey)(â=y & Py)        5,12,<>I
14 |(x)[Px<->(Ey)(x=y & Py)]   13,UI
```

9-5. d)

```
1  |  |Pa                         A
2  |  |  |b=a                     A
3  |  |  |Pa                      1,R
4  |  |  |Pb                      2,3,=E
5  |  |b=a > Pb                   2-4,>I
6  |  | (x)(x=a > Px)             5,UI
7  |Pâ -> (x)(x=â > Px)          1-6,>I
8  |  |(x)(x=a > Px)              A
9  |  |a=a > Pa                   8,UE
10 |  |a=a                        =I
11 |  |Pa                         9,10,>E
12 |(x)(x=a > Px) > Pa           8-12,>I
13 |Pâ<->(x)(x=â > Px)           7,12,<>I
```

9-5. e)

```
1  |  |(Ex)(Ey)(Fx & -Fy)              A
2  |  |a|(Ey)(Fa & -Fy)                A
3  |  | |b|Fa & -Fb                    A
4  |  | | |  |a=b                      A
5  |  | | |  |Fa                       3,&E
6  |  | | |  |-Fb                      3,&E
7  |  | | |  |-Fa                      4,6,=E
8  |  | | |a≠b                         4-7,-I
9  |  | | | (Ey)(a≠y)                  8,EI
10 |  | | |(Ex)(Ey)(x≠y)              9,EI
11 |  | |(Ex)(Ey)(x≠y)             2,3-10,EE
12 |  |(Ex)(Ey)(x≠y)               1,2-11,EE
13 |(Ex)(Ey)(Fx&-Fy) > (Ex)(Ey)(x≠y)
                                      1-12,>I
```

9-6.

```
1  |(Ex)(y)(Fy<->y=x)               P
2  |a|(y)(Fy<->y=a)                 A
3  | |Fa<->a=a                      2,UE
4  | |a=a                           =I
5  | |Fa                            3,4,<>E
6  | |Fb̂<->b̂=a                      2,UE
7  | |Fb̂ > b̂=a                      5,<>E
8  | | (y)(Fy > y=a)                6,UI
9  | |Fa & (y)(Fy > y=a)            5,10,&I
10 | | (Ex)[Fx&(y)(Fy > y=x)]  8,EI
11 |(Ex)[Fx&(y)(Fy > y=x)] 1,2-10,EE
```

```
1  |(Ex)[Fx&(y)(Fy > y=x)]          P
2  |a|Fa&(y)(Fy > y=a)              A
3  | |Fa                            2,&E
4  | | (y)(Fy > y=a)                2,&E
5  | |Fb̂ > b̂=a                      4,UE
6  | |  |b=a                        A
7  | |  |Fa                         3,R
8  | |  |Fb                         6,7,-E
9  | |b̂=a > Fb̂                      6-8,>I
10 | |Fb̂<->b̂=a                      5,9,<>I
11 | | (y)(Fy<->y=a)                10,UI
12 | | (Ex)(y)(Fy<->y=x)            11,EI
13 |(Ex)(y)(Fy<->y=x)           1,2-12,EE
```

9-7.
REFLEXIVITY:

```
1 |â=â           =I
2 |(x)(x=x)      1,UI
```

SYMMETRY:

```
1 |   |a=b                       A
2 |   |a=a                       =I
3 |   |b=a                       1,2,=E
4 |â=b̂ > b̂=â                      1-3,>I
5 |(y)(â=y > y=â)                4,UI
6 |(x)(y)(x=y > y=x)             5,UI
```

TRANSITIVITY:

```
1 |   |a=b & b=c                 A
2 |   |a=b                       1,&E
3 |   |b=c                       1,&E
4 |   |a=c                       2,3,=E
5 |(â=b̂&b̂=ĉ) > â=ĉ               1-4,>I
6 |(z)[(â=b̂&b̂=z) > â=z]          5,UI
7 |(y)(z)[(â=y & y=z) > â=z]     6,UI
8 |(x)(y)(z)[(x=y&y=z) > x=z]    7,UI
```

9-8. a)

```
1 |(x)(x=a > Fx)                 P
2 |a=a > Fa                      1,UE
3 |a=a                           =I
4 |Fa                            2,3,>E
```

9-8. b)

```
1 |Fa                            P
2 |   |b=a                       A
3 |   |Fa                        1,R
4 |   |Fb                        2,3,=E
5 |b̂=a > Fb̂                      2-4,>I
6 |(x)(x=a > Fx)                 5,UI
```

9-8. c)

```
1 |(Ex)(Fx & x=a)                P
2 |b|Fb & b=a                    A
3 | |Fb                          2,&E
4 | |b=a                         2,&E
5 | |Fa                          3,4,=E
6 |Fa                            1,2-5,EE
```

9-8. d)

```
1 |(x)(x=a > Fx)                 P
2 |(x)(Fx > x=b)                 P
3 |a=a > Fa                      1,UE
4 |a=a                           =I
5 |Fa                            3,4,>E
6 |Fa > a=b                      2 UE
7 |a=b                           5,6,>E
8 |Fb                            5,7,=E
```

9-8. e)

```
1 |Pa                            P
2 |(Ey)(y=a & y=b)               P
3 |c|c=a & c=b                   A
4 | |c=a                         3,&E
5 | |c=b                         3,&E
6 | |a=b                         4,5,=E
7 | |Pa                          1,R
8 | |Pb                          6,7,=E
9 |Pb                            2,3-8,EE
```

9-8. f)

```
1  |a=b                          P
2  |   |Fa                       A
3  |   |a=b                      1,R
4  |   |Fb                       2,3,=E
5  |Fa > Fb                      2-4,>I
6  |   |Fb                       A
7  |   |a=b                      1,R
8  |   |Fa                       6,7,=E
9  |Fb > Fa                      6-8,>I
10 |Fa<->Fb                      5,9,<>I
```

9-8. g)

```
1  |a=b                          P
2  |   |a=c                      A
3  |   |a=b                      1,R
4  |   |b=c                      2,3,=E
5  |a=ĉ > b=ĉ                    2-4,>I
6  |   |b=c                      A
7  |   |a=b                      1,R
8  |   |a=c                      6,7,=E
9  |b=ĉ > a=ĉ                    6-8,>I
10 |a=ĉ<->b=ĉ                    5,9,<>I
11 |(x)(a=x<->b=x)               10,UI
```


9-8. h)

```
1 |(x)(a=x<->b=x)                P
2 |a=b<->b=b                     1,UE
3 |b=b                           =I
4 |a=b                           2,3,=E
```

9-8. i)

```
1  |(Ex)(y)(Py<->y=x)            P
2  |Pa                           P
3  |Pb                           P
4  |c|(y)(Py<->y=c)              A
5  | |Pa                         2,R
6  | |Pb                         3,R
7  | |Pa<->a=c                   4,UE
8  | |Pb<->b=c                   4,UE
9  | |a=c                        5,7,<>E
10 | |b=c                        6,8,<>E
11 | |a=b                        9,10,=E
12 |a=b                          1,4-11,EE
```

9-8. j)

```
1  |(Ex)Px                       P
2  |(x)(x=a v x=b)               P
3  |c|Pc                         A
4  | |(x)(x=a v x=b)             2,R
5  | |c=a v c=b                  4,UE
6  | |   |c=a                    A
7  | |   |Pc                     3,R
8  | |   |Pa                     6,7,=E
9  | |   |Pa v Pb                8,vI

10 | |   |c=b                    A
11 | |   |Pc                     3,R
12 | |   |Pb                     10,11,=E
13 | |   |Pa v Pb                12,vI
14 | |Pa v Pb                    5,6-9,10-13,AC
15 |Pa v Pb                      1,3-14,EE
```

9-8. k)

```
1  |(Ex)(y)(x=y)                 P
2  |  |(Ex)Px                    A
3  |  |a|Pa                      A
4  |  | |(Ex)(y)(x=y)            1,R
5  |  | |b|(y)(b=y)              A
6  |  | | |Pa                    3,R
7  |  | | |b=a                   5,UE
8  |  | | |b=c                   5,UE
9  |  | | |a=c                   7,8,=E
10 |  | | |Pc                    6,9,=E
11 |  | | |(x)Px                 10,UI
12 |  | |(x)Px                   4,5-11,EE
13 |  |(x)Px                     2,3-13,EE
14 |(Ex)Px -> (x)Px              2-13,>I
```

9-8. l)

```
1  |(x)(Ey)Rxy                   P
2  |(x)-Rxx                      P
3  |(Ey)Rây                      1,UE
4  |b|Rab                        A
5  | |   |a=b                    A
6  | |   |Rab                    4,R
7  | |   |Raa                    5,6,=E
8  | |   |(x)-Rxx                2,R
9  | |   |-Raa                   8,UE
10 | |   a≠b                     5-9,-I
11 | |(Ey)(a≠y)                  10,EI
12 | |(Ex)(Ey)(x≠y)              11,EI
13 |(Ex)(Ey)(x≠y)                3,4-12,EE
```

9-8. m)

```
1  |(x)(Ey)Rxy                   P
2  |(x)-Rxx                      P
3  |(Ey)Rây                      1,UE
4  |b|Rab                        A
5  | |   |a=b                    A
6  | |   |Rab                    4,R
7  | |   |Raa                    5,6,=E
8  | |   |(x)-Rxx                2,R
9  | |   |-Raa                   8,UE
10 | |   a≠b                     5-9,-I
11 | |Rab & a≠b                  4,10,&I
12 | |(Ey)(Ray & a≠y)            11,EI
13 |(Ey)(Rây & â≠y)              3,4-12,EE
14 |(x)(Ey)(Rxy & x≠y)           13,UI
```

9-8. n)

```
1  |(Ex)(Kx & Jx)                P
2  |(Ex)(Kx &-Jx)                P
3  |a|Ka & Ja                    A
4  | |(Ex)(Kx & -Jx)             2,R
5  | |b|Kb & -Kb                 A
6  | | |Ka & Ja                  3,R
7  | | |Ja                       6,&E
8  | | |-Jb                      5,&E
9  | | |   |a=b                  A
10 | | |   |Ja                   7,R
11 | | |   |-Jb                  8,R
12 | | |   |-Ja                  9,11,=E
13 | | |   a≠b                   9-12,-I
14 | | |Kb                       5,&E
15 | | |Ka                       6,&E
16 | | |Ka&Kb&a≠b                13,14,15,&I
17 | | |(Ey)(Ka&Ky&a≠y)         16,EI
18 | | |(Ex)(Ey)(Kx&Ky&x≠y)     17,EI
19 | |(Ex)(Ey)(Kx & Ky & x≠y)
                                 4,5-18,EE
20 |(Ex)(Ey)(Kx & Ky & x≠y)
                                 1,3-19,EE
```

9-8. o)

```
 1 | (Ex)(Px & (y)(Py > y=x))          P
 2 | (Ex)(Px & Qx)                     P
   |------------------
 3 | a | Pa & Qa                       A
   |   |----------
 4 |   | | Pb                          A
   |   | |----
 5 |   | | (Ex)(Px & (y)(Py > y=x))    1,R
 6 |   | | c | Pc & (y)(Py > y=c)      A
   |   | |   |----------------
 7 |   | |   | Pc                      6,&E
 8 |   | |   | (y)(Py > y=c)           6,&E
 9 |   | |   | Pa > a=c                8,UE
10 |   | |   | Pa & Qa                 3,R
11 |   | |   | Pa                      10,&E
12 |   | |   | a=c                     9,11,>E
13 |   | |   | Pb > b=c                8,UE
14 |   | |   | Pb                      4,R
15 |   | |   | b=c                     13,14,>E
16 |   | |   | b=a                     12,15,=E
17 |   | |   | Qa                      10,&E
18 |   | |   | Qb                      16,17,=E
19 |   | | Qb                          5,6-18,EE
20 |   | Pb > Qb                       4-19,>I
21 |   | (x)(Px > Qx)                  20,UI
22 | (x)(Px > Qx)                      2,3-21,EE
```

9-8. p)

```
 1 | -(Ex)(Fx & (y)(Fy > y=x))         P
 2 | (Ex)Fx                            P
   |------------------
 3 | a | Fa                            A
   |   |----
 4 |   | | (y)(Fy > y=a)               A
   |   | |-------------
 5 |   | | Fa                          3,R
 6 |   | | Fa & (y)(Fy > y=a)          4,5,&I
 7 |   | | (Ex)(Fx & (y)(Fy > y=x))    6,EI
 8 |   | | -(Ex)(Fx & (y)(Fy > y=x))   1,R
 9 |   | -(y)(Fy > y=a)                4-8,-I
10 |   | (Ey)-(Fy > y=a)               9,-U
11 |   | b | -(Fb > b=a)               A
   |   |   |-------------
12 |   |   | Fb                        11,->
13 |   |   | b≠a                       11,->
14 |   |   | Fa                        3,R
15 |   |   | Fb & Fa & b≠a             12,13,14,&I
16 |   |   | (Ey)(Fb & Fy & b≠y)       15,EI
17 |   |   | (Ex)(Ey)(Fx & Fy & x≠y)   16,EI
18 |   | (Ex)(Ey)(Fx & Fy & x≠y)       10,11-17,EE
19 | (Ex)(Ey)(Fx & Fy & x≠y)           2,3-18,EE
```

9-5. a)

```
1        -(]x)(x=a)        -S
2    a   (x)-(x=a)         1 -V
3          a≠a             2 V
            X
```


9-5. b)

```
1   *  -(x)(y)[-(Fx > Fy) > x≠y]       -S
2   *  (]x)(]y)-[-(Fx > Fy) > x≠y]     1 -V
3      *  -[-(Fa > Fb) > a≠b]          2 ]
4         *  -(Fa > Fb)                3 ->
5            * -(a≠b)                  3 ->
6               a=b                    5 --
7               Fa                     4 ->
8               -Fb                    4 ->
9               -Fa                    6,8 =
                 X
```

9-5. c)

```
 1            *  -(x)[Px<->(]y)(x=y & Py)]          -S
 2            *(]x)-[Px<->(]y)(x=y & Py)]           1 -V
 3             * -[Pa<->(]y)(a=y & Py)]             2 ]
          |-----------------^-----------------|
 4        Pa                                         3 -<>
 5   * -(]y)(a=y & Py)                 -Pa           3 -<>
 6 a   (y)-(a=y & Py)          *  (]y)(a=y & Py)     5 -]
 7   *   -(a=a & Pa)                    |            6 V
        |----^----|                     |
 8      a≠a      -Pa                    |            7 -&
 9       X        X                     |            5 V
10                              *  a=b & Pb          9 &
11                                   a=b             9 &
12                                   Pb              10,11 =
                                     Pa
                                      X
```

9-5. d)

```
1              *  -[Pa<->(x)(x=a > Pa)]                  -S
              |-------------^-------------|
2             Pa                                          1 -<>
3     *  -(x)(x=a > Pa)                  -Pa              1 -<>
4     *   (]x)-(x=a > Pa)       a   (x)(x=a > Pa)         3 -V;V
5     *   -(b=a > Pa)               *  a=a > Pa           4 ]
                                   |-----^-----|
6             b=a                 a≠a         Pa          5 ->;4 >
7             -Pa                  X           X          5 ->
               X
```

9-5. e)

```
 1   *  -[(]x)(]y)(Fx & -Fy) > (]x)(]y)(x≠y)]     -S
 2          *  (]x)(]y)(Fx & -Fy)                 1 ->
 3           *   -(]x)(]y)(x≠y)                   1 ->
 4              *  Fa & -Fb                       2 ]
 5         a,b  (x)(y)-(x≠y)                      3 -]
 6               *   -(a≠b)                       5 V
 7                    a=b                         6 --
 8                    Fa                          4 &
 9                    -Fb                         4 &
10                    -Fa                         7,9 =
                       X
```

9-6. Test argument from first to second sentence.

```
 1               * (]x)(y)(Fy<->y=x)                    P
 2          * -(]x)[Fx & (y)(Fy > y=x)]                 -C
 3          a  (x)-[Fx & (y)(Fy > y=x)]                 2 -V
 4            b,a      (y)(Fy<->y=a)                    1 ]
 5            * -[Fa & (y)(Fy > y=a)]                   3 V
 6                 *   Fa<->a=a                         4 V
          |---------------^----------------|
 7       Fa                                -Fa          6 <>
 8      a=a                                a≠a          6 <>
    |------^-------|                         X
 9 -Fa        * -(y)(Fy > y=a)                          5 -&
10  X         * (]y)-(Fy > y=a)                         9 -V
11            * -(Fb > b=a)                             10 ]
12                   Fb                                 11 ->
13                   b≠a                                11 ->
14             * Fb<->b=a                               4 V
             |------^------|
15           Fb           -Fb                           14 <>
16          b=a           b≠a                           14 <>
             X             X
```

Test argument from the second to the first sentence.

```
 1          * (]x)[Fx & (y)(Fy > y=x)]                  P
 2              * -(]x)(y)(Fy<->y=x)                    -C
 3            a   (x)(]y)-(Fy<->y=x)                    2 -];-V
 4              * Fa & (y)(Fy > y=a)                    1 ]
 5                     Fa                               4 &
 6            b   (y)(Fy > y=a)                         4 &
 7              * (]y)-(Fy<->y=a)                       3 V
 8               *   -(Fb<->b=a)                        7 ]
 9                 *   Fb > b=a                         6 V
           |------------^------------|
10         Fb                       -Fb                 9 <>
11        b≠a                       b=a                 9 <>
     |------^-----|                 -Fa                 10,11 =
12
13  -Fb          b=a                 X                  9 >
     X            X
```

9-7. We have to show that the statements that = is reflexive, symmetric and transitive are all logical truths.

REFLEXIVITY

```
1            * -(x)(x=x)                                -S
2           * (]x)-(x=x)                                1 -V
3                a≠a                                    2 ]
                  X
```


SYMMETRY

```
1          * -(x)(y)(x=y > y=x)                         -S
2          * (]x)(]y)-(x=y > y=x)                       1 -V
3             * -(a=b > b=a)                            2 ]
4                   a=b                                 3 ->
5                   a≠b                                 3 ->
                     X
```

TRANSITIVITY

```
1     * -(x)(y)(z)[(x=y & y=z) > x=z]                   -S
2     * (]x)(]y)(]z)-[(x=y & y=z) > x=z]                1 -V
3         * -[(a=b & b=c) > a=c]                        2 ]
4             * a=b & b=c                               4 ->
5                  a≠c                                  4 ->
6                  a=b                                  4 &
7                  b=c                                  4 &
8                  a=c                                  6,7 =
                    X
```

9-8. a)

```
1          a   (x)(x=a > Fx)                            P
2                 -Fa                                   -C
3              *  a=a > Fa                              1 V
     |--------------^---------------|
4   a≠a                             Fa                  3 V
     X                               X
```

9-8. b)

```
1                  Fa                                   P
2          * -(x)(x=a > Fx)                             -C
3         * (]x)-(x=a > Fx)                             2 -V
4            * -(b=a > Fb)                              4 ]
5                 b=a                                   5 ->
6                 -Fb                                   5 ->
7                 -Fa                                   6,7 =
                   X
```

9-8. c)

```
1          * (]x)(Fx & x=a)                             P
2                 -Fa                                   -C
3             * Fb & b=a                                1 ]
4                 Fb                                    3 &
5                 b=a                                   3 &
6                 Fa                                    4,5 =
                   X
```

9-8. d)

```
1        a   (x)(x=a > Fx)            P
2        a   (x)(Fx > Fb)             P
3                -Fb                  -C
4              * a=a > Fa             1 V
5              * Fa > Fb              2 V
          |-----------^------------|
6        a≠a                      Fa            4 >
                           |-------^-------|
7                         -Fa             -Fb   5 >
                           X               X
```

9-8. e)

```
1                  Pa                 P
2         *  (]y)(y=a & y=b)          P
3                  -Pb                -C
4            *  c=a & c=b             2 ]
5                  c=a                4 &
6                  c=b                4 &
7                  Pc                 1,5 =
8                  -Pc                3,6 =
                    X
```

9-8. f)

```
1                  a=b                P
2          *  -(Fa<->Fb)              -C
   |------------------^------------------|
3  Fa                                   -Fa      2 -<>
4  -Fb                                  Fb       2 -<>
5  Fb                                   -Fb      1,3 =
   X                                     X
```

9-8. g)

```
1                  a=b                P
2         *  -(x)(a=x<->b=x)          -C
3         *  (]x)-(a=x<->b=x)         2 -V
4           *  -(a=c<->b=c)           3 ]
   |------------------^------------------|
5  a=c                                  a≠c      4 -<>
6  b≠c                                  b=c      4 -<>
7  b=c                                  b≠c      1,5 =
   X                                     X
```

9-8. h)

```
1        a   (x)(a=x<->b=x)           P
2                  a≠b                -C
3           *   a=a<->b=a             1 V
   |------------------^------------------|
4  a=a                                  a≠a      3 <>
5  b=a                                  b≠a      3 <>
6  b=a                                   X       4,5 =
   X
```

9-8. i)

```
1              *  (]x)(y)(Py<->y=x)              P
2                       Pa                       P
3                       Pb                       P
4                       a≠b                      -C
5        a,b        (y)(Py<->y=c)                1 ]
6                *    Pa<->a=c                   5 V
7                *    Pb<->b=c                   5 V
          |------------------^-------------------|
8         Pa                                    -Pa       6 <>
9         a=c                                   a≠c       6 <>
   |-------^-------|                             X
10 Pb             -Pb                                     7 <>
11 b=c            b≠c                                     7 <>
12 a=b             X                                      9,11 =
   X
```

9-8. j)

```
1              •     (]x)Px                      P
2        c       (x)(x=a v x=b)                  P
3             *     -(Pa v Pb)                   -C
4                      Pc                        1 ]
5                      -Pa                       3 -V
6                      -Pb                       3 -V
7              •     c=a v c=b                   2 V
     |------------------^-------------------|
8    c=a                                   c=b       7 V
9    Pa                                    Pb        4,8 =
     X                                      X
```

9-8. k)

```
1          *   (]x)(y)(x=y)               P
2         *  -[(]x)Px > (x)Px]            -C
3        a,b     (y)(a=y)                 1 ]
4             *    (]x)Px                 2 ->
5             *   -(x)Px                  2 ->
6             *   (]x)-Px                 5 -V
7                   Pb                    4 ]
8                   -Pc                   6 ]
9                   a=b                   3 V
10                  a=c                   3 V
11                  Pa                    7,9 =
12                  -Pa                   8,10 =
                     X
```

9-8. l)

```
 1        a     (x)(]y)Rxy                 P
 2        a       (x)-Rxx                  P
 3        * -(]x)(]y)(x≠y)                 -C
 4       *    (x)(y)-(x≠y)                 3 -]
 5   a,b      (x)(y)(x=y)                  4 --
 6          *    (]y)Ray                   1 V
 7                 Rab                     6 ]
 8                -Raa                     2 V
 9                 a=b                     5 V
10                 Raa                     7,9 =
                    X
```

9-8. m)

```
 1        a     (x)(]y)Rxy                 P
 2           a     (x)-Rxx                 P
 3      *  -(x)(]y)(Rxy & x≠y)             -C
 4       * (]x)(y)-(Rxy & x≠y)             3 -V;-]
 5     b     (y)-(Ray & a≠y)               4 ]
 6          *    (]y)Ray                   1 V
 7                 Rab                     6 ]
 8        *  -(Rab & a≠b)                  5 V
      |-----------^-----------|
 9  -Rab                 * -(a≠b)          8 -&
10    X                     a=b            9 --
11                         -Raa            2 V
12                          Raa            7,10 =
                             X
```

9-8. n)

```
 1        *     (]x)(Kx & Jx)                  P
 2        *    (]x)(Kx & -Jx)                  P
 3    *    -(]x)(]y)(Kx & Ky & x≠y)            -C
 4  a,b     (x)(y)-(Kx & Ky & x≠y)             3 -]
 5           *    Ka & Ja                      1 ]
 6           *   Kb & -Kb                      2 ]
 7                  Ka                         5 &
 8                  Ja                         5 &
 9                  Kb                         6 &
10                 -Jb                         6 &
11        *  -(Ka & Kb & a≠b)                  4 V
    |-------------|---------------|
12 -Ka           -Kb         * -(a≠b)          11 -&
13  X             X             a=b            12 --
14                              -Ja            10,13 =
                                 X
```


9-8. o)

```
 1      *  (]x)[Px & (y)(Py > y=x)]            P
 2          *  (]x)(Px & Qx)                   P
 3          *  -(x)(Px > Qx)                   -C
 4          *  (Ex)-(Px > Qx)                  3 -V
 5          *    -(Pa > Qa)                    4 ]
 6                  Pa                         5 ->
 7                 -Qa                         5 ->
 8            *   Pb & Qb                      2 ]
 9       *  Pc & (y)(Py > y=c)                 1 ]
10                  Pb                         8 &
11                  Qb                         8 &
12                  Pc                         9 &
13   a,b       (y)(Py > y=c)                   9 &
14            *   Pa > a=c                     13 V
15            *   Pb > b=c                     13 V
      |---------------^---------------|
16   -Pa                             a=c       14 >
      X                     |----------^----|
17                         -Pb             b=c 15 >
18                          X              -Qc 16,17 =
19                                          Qc 11,17 =
                                             X
```

9-8. p)

```
 1     *   -(]x)[Fx & (y)(Fy > y=x)]           P
 2              *     (]x)Fx                   P
 3     *    -(]x)(]y)(Fx & Fy & x≠y)           -C
 4  a,b       (x)(y)-(Fx & Fy & x≠y)           3 -]
 5    a      (x)-[Fx & (y)(Fy > y=x)]          1 -]
 6                     Fa                      2 ]
 7     *      -[Fa & (y)(Fy > y=a)]            5 V
       |---------------^---------------|
 8    -Fa                   *  -(y)(Fy > y=a)  7 -&
 9     X                    *  (]y)-(Fy > y=a) 8 -V
10                          *    -(Fb > b=a)   9 ]
11                                  Fb         10 ->
12                                  b≠a        10 ->
13                          *  -(Fa & Fb & a≠b) 4 V
                       |-----------|-----------|
14                    -Fa         -Fb    * -(a≠b)  13 -&
15                     X           X        a=b    14 --
                                             X
```

9-9. Suppose player A belongs to two soccer teams, Team I and Team II. Suppose further that A is teammates with player B on Team I, and with player C on Team II. In this case, A holds the relation being-teammates-on-a-soccer-team to B, and A holds this relation to C, but B does not hold the relation to C. Thus, the relation being-teammates-on-a-soccer-team fails to be an equivalence relation in this case, as it is non-transitive.

There may be circumstances under which this relation is an equivalence relation even though one or more people belong to more than one team -

namely, those circumstances in which whenever a player, P, is on more than one team, all of his teammates are also on all of the teams that P plays on. This would correspond, in the above example, to the circumstances in which player C was also a member of Team I, and Player B a member of Team II. In such circumstances, the relation being-teammates-on-a-soccer-team would be an equivalence one, since it would be transitive as well as reflexive and symmetric.

NOTE: Answers for exercises 9-10 and 9-11 are given in the form of both derivations and truth trees. All of the derivations come first, followed by all of the truth trees.

9-10. a)

```
1 | |f(a)=c & f(a)=b                 A
2 | |f(a)=c                          1,&E
3 | |f(a)=b                          1,&E
4 | |c=b                             2,3,=E
5 |(f(â)=ĉ & f(â)=b̂) -> ĉ=b̂          1-4,>I
6 |(z)[(f(z)=ĉ&f(z)=b̂)->ĉ=b̂]         5,UI
7 |(y)(z)[f(z)=ĉ&f(z)=y)->ĉ=y]       6,UI
8 |(x)(y)(z)[f(z)=x&f(z)=y)->x=y]
                                     7,UI
```

9-10. b)

```
1 | |-(Ex)(Ff(x)v-Fx)                A
2 | |(x)-(Ff(x)v-Fx)                 1,-]
3 | |-(Ff(a)v-Fa                     2,UE
4 | |-Ff(a)&--Fa                     3,DM
5 | |-Ff(a)                          4,&E
6 | |--Fa                            5,&E
7 | |Fa                              6,--
8 | |(x)Fx                           7,UI
9 | |Ff(a)                           8,UE
10|(Ex)(Ff(x)v-Fx)                   1-9,RD
```

9-11. a)

```
1 |(x)Fx             P
2 |Fg(â)             1,UE
3 |(x)Fg(x)          2,UI
```

9-11. b)

```
1 |(x)(y)(x=y)           P
2 |(y)(f(â)=y)           1,UE
3 |f(â)=â                2,UE
4 |(x)(f(x)=x)           3,UI
```

9-11. c)

```
1 |(x)(f(x)≠x)           P
2 |f(â)≠â                1,UE
3 |(Ey)(f(â)≠y)          2,EI
4 |(Ex)(Ey)(x≠y)         3,EI
```

9-11. d)

```
1 |(Ex)(f(x)≠x)               P
2 |a|f(a)≠a                   A
3 |  |(Ey)(f(a)≠y)            2,EI
4 |  |(Ex)(Ey)(f(x)≠y)        3,EI
5 |(Ex)(Ey)(f(x)≠y)           1,2-4,EE
```

9-11. e)

```
1 |(Ex)(y)(f(y)=x)                P
2 |a|(y)(f(y)=a)                  A
3 |  |f(b)=a                      2,UE
4 |  |f(c)=a                      2,UE
5 |  |f(b)=f(c)                   3,4,=E
6 |  |(y)(f(b)=f(y))              5,UI
7 |  |(x)(y)(f(x)=f(y))           6,UI
8 |(x)(y)(f(x)=f(y))              1,2-7,EE
```

9-11. f)

```
1 |(x)(y)(g(x,y)=g(y,x))           P
2 |  |Fg(a,b)                      A
3 |  |(x)(y)(g(x,y)=g(y,x))        1,R
4 |  |(y)(g(a,y=g(y,a))            3,UE
5 |  |g(a,b)=g(b,a)                4,UE
6 |  |Fg(b,a)                      2,5,=E
7 |Fg(â,b̂)->Fg(b̂,â)                2-6,>I
8 |(y)[Fg(â,y)->Fg(y,â)]           7,UI
9 |(x)(y)[Fg(x,y)->Fg(y,x)]        8,UI
```


9-11. g)

```
1 |(x)(ff(x)=x)                 P
2 |  |f(a)=f(b)                 A
3 |  |ff(a)=ff(a)               =I
4 |  |ff(a)=ff(b)               2,3,=E
5 |  |(x)(ff(x)=x)              1,R
6 |  |ff(a)=a                   5,UE
7 |  |ff(b)=b                   5,UE
8 |  |a=ff(b)                   4,6,=E
9 |  |a=b                       7,8,=E
10|f(â)=f(b̂) > â=b̂              2-9,>I
11|(y)[f(â)=f(y) > â=y]         10,UI
12|(x)(y)[f(x)=f((y) > x=y]     11,UI
```

9-11. h)

```
1 |(Ex)(y)(z)(g(y,z)=x)              P
2 |a|(y)(z)(g(y,z)=a)                A
3 |  |(z)(g(y,ĉ)=a)                  2,UE
4 |  |g(b̂,ĉ)=a                       3,UE
5 |  |(z)(g(d̂,z)=a)                  2,UE
6 |  |g(d̂,ê)=a                       5,UE
7 |  |g(b̂,ĉ)=g(d̂,ê)                  4,6,=E
8 |  |(w)(g(b̂,ĉ)=g(d̂,w))             7,UI
9 |  |(z)(w)(g(b̂,ĉ)=g(z,w))          8,UI
10|  |(y)(z)(w)(g(b̂,y)=g(z,w))       9,UI
11|  |(x)(y)(z)(w)(g(x,y)=g(z,w))
                                     10,UI
12|(x)(y)(z)(w)(g(x,y)=g(z,w))
                                     1,2-11,EE
```

9-11. i)

```
1 |(z)(Ex)(Ey)(z=g(x,y))             P
2 |  |(x)(y)(Fg(x,y)                 A
3 |  |(z)(Ex)(Ey)(z=g(x,y))          1,R
4 |  |(Ex)(Ey)(â=g(x,y))             3,UE
5 |  |b|(Ey)(a=g(b,y)                A
6 |  |  |c|a=g(b,c)                  A
7 |  |  |  |(x)(y)Fg(x,y)            2,R
8 |  |  |  |(y)Fg(b,y)               7,UE
9 |  |  |  |Fg(b,c)                  8,UE
10|  |  |  |Fa                       6,9,=E
11|  |  |Fa                          5,6-10,EE
12|  |Fâ                             4,5-11,EE
13|  |(x)Fx                          12,UI
14|(x)(y)Fg(x,y)->(x)Fx              2-13,>I
```

9-11. j)

```
1 |(Ex)(Ey)(Ff(x)&-Ff(y))               P
2 |a|(Ey)(Ff(a)&-Ff(y))                 A
3 |  |b|Ff(a)&-Ff(b)                    A
4 |  |  |Ff(a)                          3,&E
5 |  |  |-Ff(b)                         3,&E
6 |  |  |  |f(a)=f(b)                   A
7 |  |  |  |Ff(a)                       4,R
8 |  |  |  |-Ff(b)                      5,R
9 |  |  |  |Ff(b)                       6,7,=E
10|  |  |f(a)≠f(b)                      6-10,-I
11|  |  |(Ey)(f(a)≠f(y))                10,EI
12|  |  |(Ex)(Ey)(f(x)≠f(y))            11,EI
13|  |(Ex)(Ey)(f(x)≠f(y))               2,3-13,EE
14|(Ex)(Ey)(f(x)≠f(y))                  1,2-14,EE
```

9-11. k)

```
 1 | (x)(y)(x≠y > g(x,y)≠g(y,x))                          P
 2 |  | a≠b                                                A
 3 |  | (x)(y)[x≠y > g(x,y)≠g(y,x)]                        1,R
 4 |  | (y)[a≠y > g(a,y)≠g(y,a)]                           3,UE
 5 |  | a≠b > g(a,b)≠g(b,a)                                4,UE
 6 |  | g(a,b)≠g(b,a)                                      2,5,>E
 7 |  | (y)[g(a,b)≠y > g(g(a,b),y)≠g(y,g(a,b))]            3,UE
 8 |  | g(a,b)≠g(b,a) > g(g(a,b),g(b,a))≠g(g(b,a),g(a,b))  7,UE
 9 |  | g(g(a,b),g(b,a))≠g(g(b,a),g(a,b))                  6,8,>E
10 | â≠b̂ > g(g(â,b̂),g(b̂,â))≠g(g(b̂,â),g(â,b̂))              2-9,>I
11 | (y){â≠y > [g(g(â,y),g(y,â))≠g(g(y,â),g(â,y))]}       10,UI
12 | (x)(y){x≠y > [g(g(x,y),g(y,x))≠g(g(y,x),g(x,y))]}    11,UI
```

9-11. l)

```
 1 | (x)(y)[x≠y > (Fg(x,y)<->-Fg(y,x))]         P
 2 | (y)[â≠y > (Fg(â,y)<->-Fg(y,â))             1,UE
 3 | â≠b̂ > (Fg(â,b̂)<->-Fg(b̂,â))                  2,UE
 4 |  | a≠b                                     A
 5 |  | a≠b > (Fg(a,b)<->-Fg(b,a))              3,R
 6 |  | Fg(a,b)<->-Fg(b,a)                      4,5,>E
 7 |  |  | g(a,b)=g(b,a)                        A
 8 |  |  |  | Fg(a,b)                           A
 9 |  |  |  | Fg(a,b)<->-Fg(b,a)                6,R
10 |  |  |  | -Fg(b,a)                          8,9,<>E
11 |  |  |  | g(a,b)=g(b,a)                     7,R
12 |  |  |  | -Fg(a,b)                          10,11,=E
13 |  |  | -Fg(a,b)                             8-12,-I
14 |  |  | -Fg(b,a)                             7,13,=E
15 |  |  | Fg(a,b)<->-Fg(b,a)                   6,R
16 |  |  | Fg(a,b)                              14,15,<>E
17 |  | g(a,b)≠g(b,a)                           7-16,-I
18 | â≠b̂ > (g(â,b̂)≠g(b̂,â))                      4-17,>I
19 | (y)[â≠y > (g(â,y)≠g(y,â))]                 18,UI
20 | (x)(y)[x≠y > (g(x,y)≠g(y,x))]              19,UI
```

9-10. a)

```
1       * -(Vx)(Vy)(Vz)[(f(z)=x & f(z)=y) > x=y]      -S
2       *  (]x)(]y)(]z)-[(f(z)=x & f(z)=y) > x=y]     1 -V
3               * -[(f(c)=a & f(c)=b) > a=b]          2 ]
4                   *  f(c)=a & f(c)=b                3 ->
5                          a≠b                        3 ->
6                          f(c)=a                     4 &
7                          f(c)=b                     4 &
8                          a=b                        6,7 =
                            X
```


9-10. b)

```
1              *  -(]x)(Ff(x) v -Fx)           -S
2       a,f(a)   (Vx)-(Ff(x) v -Fx)            1 -]
3                *  -(Ff(a) v -Fa)             2 V
4                     -Ff(a)                   3 -v
5                     --Fa                     3 -v
6              *  -(Fff(a) v Ff(a))            2 V
7                     -Fff(a)                  6 -v
8                  *  --Ff(a)                  6 -v
9                     Ff(a)                    8 --
                        X
```

9-11. a)

```
1       g(a)    (Vx)Fx                 P
2             * -(Vx)Fg(x)             -C
3             * (]x)-Fg(x)             2 -V
4               -Fg(a)                 3 ]
5               Fg(a)                  1 V
                  X
```

9-11. b)

```
1       f(a)   (Vx)(Vy)(x=y)           P
2            * -(Vx)(f(x)=x)           -C
3            * (]x)-(f(x)=x)           2 -V
4                f(a)≠a                3 ]
5          a   (Vy)(f(a)=y)            1 V
6                f(a)=a                5 V
                  X
```

9-11. c)

```
1          a   (Vx)(f(x)≠x)            P
2            * -(]x)(]y)(x≠y)          -C
3            * (Vx)-(]y)(x≠y)          2 -]
4       f(a)   (Vx)(Vy)-(x≠y)          3 -]
5                f(a)≠a                1 V
6          a   (Vy)-(f(a)≠y)           4 V
7               -(f(a)≠a)              6 V
                  X
```

9-11. d)

```
1            * (]x)(f(x)≠x)            P
2         *  -(]x)(]y)(f(x)≠y)         -C
3          * (Vx)-(]y)(f(x)≠y)         2 -]
4      f(a)  (Vx)(Vy)-(f(x)≠y)         3 -]
5                f(a)≠a                1 ]
6          a   (Vy)-(f(a)≠y)           4 V
7               -(f(a)≠a)              6 V
                  X
```

9-11. e)

1		* (]x)(Vy)(f(y)=x)	P
2		* -(Vx)(Vy)[f(x)=f(y)]	-C
3		* (]x)-(Vy)[f(x)=f(y)]	2 -V
4		* (]x)(]y)[f(x)≠f(y)]	3 -V
5	b,c	(Vy)(f(y)=a)	1]
6		* (]y)[f(b)≠f(y)]	4]
7		f(b)≠f(c)	6]
8		f(b)=a	5 V
9		f(c)=a	5 V
10		f(b)=f(c)	6,7 =
		X	

9-11. f)

1	a	(Vx)(Vy)[g(x,y)=g(y,x)]	P
2		* -(Vx)(Vy)[Fg(x,y) > Fg(y,x)]	-C
3		* (]x)-(Vy)[Fg(x,y) > Fg(y,x)]	2 -V
4		* (]x)(]y)-[Fg(x,y) > Fg(y,x)]	3 -V
5		* (]y)-[Fg(a,y) > Fg(y,a)]	4]
6		* -[Fg(a,b) > Fg(b,a)]	5]
7		Fg(a,b)	6 ->
8		-Fg(b,a)	6 ->
9	b	(Vy)[g(a,y)=g(y,a)]	1 V
10		g(a,b)=g(b,a)	9 V
11		-Fg(a,b)	8,10 =
		X	

9-11. g)

1	a,b	(Vx)(ff(x)=x)	P
2		* -(Vx)(Vy)[f(x)=f(y) > x=y]	-C
3		* (]x)-(Vy)[f(x)=f(y) > x=y]	2 -V
4		* (]x)(]y)-[f(x)=f(y) > x=y]	3 -V
5		* (]y)-[f(a)=f(y) > a=y]	4]
6		* -[f(a)=f(b) > a=b]	5]
7		f(a)=f(b)	6 ->
8		a≠b	6 ->
9		ff(a)=a	1 V
10		ff(b)=b	1 V
11		ff(a)=b	7,10 =
12		a=b	9,11 =
		X	

9-11. h)

1		* (]x)(Vy)(Vz)(g(y,z)=x)	P
2		* -(Vx)(Vy)(Vz)(Vw)[g(x,y)=g(z,w)]	-C
3	b,d	(Vy)(Vz)(g(y,z)=a)	1]
4		* (]x)-(Vy)(Vz)(Vw)[g(x,y)=g(z,w)]	2 -V
5		* (]x)(]y)-(Vz)(Vw)[g(x,y)=g(z,w)]	4 -V
6		* (]x)(]y)(]z)-(Vw)[g(x,y)=g(z,w)]	5 -V
7		* (]x)(]y)(]z)(]w)-[g(x,y)=g(z,w)]	6 -V
8		* (]y)(]z)(]w)[g(b,y)≠g(z,w)]	7]
9		* (]z)(]w)[g(b,c)≠g(z,w)]	8]
10		* (]w)[g(b,c)≠g(d,w)]	9]
11		g(b,c)≠g(d,e)	10]
12	c	(Vz)(g(b,z)=a)	3 V
13		g(b,c)=a	12 V
14	e	(Vz)(g(d,z)=a)	3 V
15		g(d,e)=a	14 V
16		g(b,c)=g(d,e)	13,15 =
		X	

9-11. i)

1	a	(Vz)(]x)(]y)[z=g(x,y)]	P
2		* -[(Vx)(Vy)Fg(x,y) > (Vx)Fx]	-C
3	b	(Vx)(Vy)Fg(x,y)	2 ->
4		* -(Vx)Fx	2 ->
5		* (]x)-Fx	4 -V
6		-Fa	5]
7		* (]x)(]y)[a=g(x,y)]	1 V
8		* (]y)[a=g(b,y)]	7]
9		a=g(b,c)	8]
10	c	(Vy)Fg(b,y)	3 V
11		Fg(b,c)	10 V
12		-Fg(b,c)	6,9 =
		X	

9-11. j)

1		* (]x)(]y)[Ff(x) & -Ff(y)]	P
2		* -(]x)(]y)[f(x)≠f(y)]	-C
3		* (Vx)-(]y)[f(x)≠f(y)]	2 -]
4	a	(Vx)(Vy)-[f(x)≠f(y)]	3 -]
5		* (]y)[Ff(a) & -Ff(y)]	1]
6		* Ff(a) & -Ff(b)	5]
7		Ff(a)	6 &
8		-Ff(b)	6 &
9	b	(Vy)-[f(a)≠f(y)]	4 V
10		* -[f(a)≠f(b)]	9 V
11		f(a)=f(b)	10 --
12		-Ff(a)	8,11 =
		X	

9-11. k)

```
1          a, g(a,b)     (Vx)(Vy)[x≠y > g(x,y)≠g(y,x)]                          P
2           * -(Vx)(Vy)[x≠y > g(g(x,y),g(y,x))≠g(g(y,x),g(x,y))]                 -C
3           * (]x)-(Vy)[x≠y > g(g(x,y),g(y,x))≠g(g(y,x),g(x,y))]                 2 -V
4           * (]x)(]y)-[x≠y > g(g(x,y),g(y,x))≠g(g(y,x),g(x,y))]                 3 -V
5            *  (]y)-[a≠y > g(g(a,y),g(y,a))≠g(g(y,a),g(a,y))]                   4 ]
6               * -[a≠b > g(g(a,b),g(b,a))≠g(g(b,a),g(a,b))]                     5 ]
7                             a≠b                                                 6 ->
8                  * -[g(g(a,b),g(b,a))≠g(g(b,a),g(a,b))]                         6 ->
9                     g(g(a,b),g(b,a))=g(g(b,a),g(a,b))                           8 --
10               b      (Vy)[a≠y > g(a,y)≠g(y,a)]                                 1 V
11                       *  a≠b > g(a,b)≠g(b,a)
                         |------------^-------------|                            10 V
12                     -a≠b                   g(a,b)≠g(b,a)                       11 >
                        X                           |
13             g(b,a)        (Vy)[g(a,b)≠y > g(g(a,b),y)≠g(y,g(a,b))]            1 V
14           *  g(a,b)≠g(b,a)  > g(g(a,b),g(b,a))≠g(g(b,a),g(a,b))]              13 V
                                                    |
             |--------------------------------------^-------|
15   -[g(a,b)≠g(b,a)]              g(g(a,b),g(b,a))≠g(g(b,a),g(a,b))             14 >
             X                                              X
```

9-11. l)

```
1                a    (Vx)(Vy){x≠y > [Fg(x,y)<->-Fg(y,x)]}                       P
2                   *  -(Vx)(Vy)[x≠y > g(x,y)≠g(y,x)]                            -C
3                    * (]x)-(Vy)[x≠y > g(x,y)≠g(y,x)]                            2 -V
4                   *  (]x)(]y)-[x≠y > g(x,y)≠g(y,x)]                            3 -V
5                    *   (]y)-[a≠y > g(a,y)≠g(y,a)]                              4 ]
6                      *  -[a≠b > g(a,b)≠g(b,a)]                                 5 ]
7                              a≠b                                               6 ->
8                        * -[g(a,b)≠g(b,a)]                                      6 ->
9                           g(a,b)=g(b,a)                                        8 --
10           b   (Vy){a≠y > [Fg(a,y)<->-Fg(y,a)]}                                1 V
11                   * a≠b > [Fg(a,b)<->-Fg(b,a)]                                10 V
            |-----------------------^-------------------------|
12        -a≠b                              * Fg(a,b)<->-Fg(b,a)                 11 >
           X                     |---------------^---------|
13                            Fg(a,b)                   -Fg(a,b)                 12 <>
14                           -Fg(b,a)                  --Fg(b,a)                 12 <>
15                           -Fg(a,b)                * --Fg(a,b)                 9,14 =
16                              X                       Fg(a,b)                  15 --
                                                           X
```

9-12. [Note: Many other placements of the existential quantifiers provide logically equivalent, and so equally correct answers. Also, in every case uniqueness can be expressed with a biconditional, as in problem 9-6.]

a) (Ex)[Sxe & (y)(Sye -> y=x) & Bx]
b) (Ex)[Sxe & (y)(Sye -> y=x) & Cxa]
c) (Ex)[Fxc & (y)(Fyc & (y)(Fyc -> y=x) & a=x]
d) (Ex)(Sxe & (y)(Sye -> y=x) & Lax]

e) (Ex)(Sxa & (y)(Sya -> y=x) & Lax]
f) (Ex)[Bx & (y)(By -> y=x) & Lcx]
g) (Ex){(Ey)(Fxy & Fya) & (z)[(Ey)(Fzy & Fya) -> z=x] & Dx)
h) (Ex)(Ey)[Sxe & (z)(Sze -> z=x) & Sya & (z)(Sya -> z=y) & x=y]
i) (Ex)(Ey)[Bx & (z)(Bx -> z=x) & Dy & (z)(Dz -> z=y) & Cxy]
j) (Ex)(Ey){Sxa & (z)[(Sza & z≠x) -> Cxz] & Fye & (z)(Fze -> z=y) & x=y)
k) (Ex)(Ey)(Eu)(Ev)[Fye & (z)Fze -> z=y) & Sxy & (z)(Szy -> z=x) & Sva & (z)(Sza -> z=v) & Fuv & (z)(Fzv -> z=u) & Cxu]

9-13. When an expression has two definite descriptions we have four options for negation. We can let neither, or just the first, or just the second, or both definite description have primary scope. Thus the negation of i) could be:

-(Ex)(Ey)[Bx & (z)(Bx -> z=x) & Dy & (z)(Dz -> z=y) & Cxy]

or

(Ex){[Bx & (z)(Bx -> z=x) & -(Ey)[Dy & (z)(Dz -> z=y) & Cxy]}

or

(Ey)(Dy & (z)(Dz -> z=y) & -(Ex)[Bx & (z)(Bz -> z=x) & Cxy]}

or

(Ex)(Ey)[Bx & (z)(Bx -> z=x) & Dy & (z)(Dz -> z=y) & -Cxy]

More than two definite descriptions can be treated analogously. In most cases the only interesting options are when all definite descriptions or none of the definite descriptions have primary scope, and so these are the only ones I provide here in the answers.

Answers in which all the definite descriptions have primary scope:

a) (Ex)[Sxe & (y)(Sye -> y=x) & -Bx]
b) (Ex)[Sxe & (y)(Sye -> y=x) & -Cxa]
c) (Ex)[Fxc & (y)(Fyc & (y)(Fyc -> y=x) & a≠x]
d) (Ex)(Sxe & (y)(Sye -> y=x) & -Lax]
e) (Ex)(Sxa & (y)(Sya -> y=x) & -Lax]
f) (Ex)[Bx & (y)(By -> y=x) & -Lcx]
g) (Ex){(Ey)(Fxy & Fya) & (z)[(Ey)(Fzy &Fya) -> z=x] & -Dx)
h) (Ex)(Ey)[Sxe & (z)(Sze -> z=x) & Sya & (z)(Sya -> z=y) & x≠y]
i) (Ex)(Ey)[Bx & (z)(Bx -> z=x) & Dy & (z)(Dz -> z=y) & -Cxy]
j) (Ex)(Ey){Sxa & (z)[(Sza & z≠x) -> Cxz] & Fye & (z)(Fze -> z=y)& x≠y)
k) (Ex)(Ey)(Eu)(Ev)[Fye & (z)Fze -> z=y) & Sxy & (z)(Szy -> z=x)& Sva & (z)(Sza -> z=v) & Fuv & (z)(Fzv -> z=u) & -Cxu]

Answers in which all the definite descriptions have secondary scope are obtained by simply negating all of the answers in exercise 9-12.

10-1. a) Used as part of the metalanguage. b) Mentioned as part of the object language. c) Used as part of the metalanguage. d) Mentioned as part of the metalanguage.

10-2. a) Semantic fact b) Syntactic fact c) Syntactic fact d) Semantic fact e) Syntactic fact f) Semantic fact g) Semantic fact h) Syntactic fact

10-3. Given the assumption that Z⊨ X and Z⊨ Y we have to show that Z⊨ X&Y, that is that if we have an interpretation, I, in which all the sentences in Z are true, I is also an interpretation in which X&Y is true. So let us suppose that we have an interpretation, I, in which all the sentences in Z are true. Since we are given that Z⊨ X, we know that X is true in I. Likewise, since we are given that Z⊨ Y we know that Y is true in I. But since X and Y are both true in I, so is X&Y, by the truth table definition of '&'.

10-4. &E: If Z⊢ X&Y then Z⊢ X and Z⊢ Y. To show that this is sound we must show that if Z⊨ X&Y, then Z⊨ X and Z⊨ Y. Suppose that Z⊨ X&Y and suppose that we have an interpretation, I in which all the sentences in Z are true. Then X&Y is true in I. But then X and Y are themselves each true in I, by the truth table definition of '&'.

For the remaining rules I will streamline exposition. The rule is always stated using single turnstiles. To show soundness we must demonstrate the corresponding statement with double turnstiles. In each caseI will do this by assuming that we are given an interpretation, I, in which all the sentences in Z are true and then show that the sentence or sentences following the relevant double turnstile are also true in I.

vI: If Z⊢ X, then Z⊢ XvY, and if Z⊢ Y, then Z⊢ XvY.
Since all sentences in Z are true in I and Z⊨ X is assumed, X is true in I. It follows that XvY is true in I by the truth table definition of 'v'. If Z⊨ Y is assumed, XvY is likewise true in I.

vE: If Z⊢ XvY and Z⊢ -X, then Z⊢ Y. If Z⊢ XvY and Z⊢ -Y, then Z⊢ X.
Since all sentences in Z are true in I and Z⊨ XvY and Z⊨ -X are assumed, both XvY and -X are true in I. Then Y is true in I, by the truth table definition of 'v'. Soundness of the second half of the rule is the same.

<->I: If Z⊢ X->Y and Z⊢ Y->X, then Z⊢ X<->Y.
We are given that all sentences in Z are true in I and both Z⊨ X->Y and Z⊨ Y->X. Suppose that X is true in I, Then, since X->Y is true in I, Y is also true in I. On the other hand, suppose that Y is true in I. Then, since Y->X is true in I, X is also true in I. Then the truth table definition of '<->' tells us that X<->Y is true in I.

<->E: If Z⊢ X<->Y, then both Z⊢ X->Y and Z⊢ Y->X.
Since all sentences in Z are true in I and Z⊨ X<->Y is assumed, X<->Y is true in I. Then inspection of the truth table definition of '<->' immediately tells us that X->Y and Y->X are both true in I.

->E: If Z⊢ X->Y and Z⊢ X, then Z⊢ Y.
Since all sentences in Z are true in I and Z⊨ X->Y and Z⊨ X, X->Y and Y are both true in I. Then the truth table definition of '->' tells us that Y is true in I also.

-E: If Z⊢ --X, then Z⊢ X.
Since all sentences in Z are true in I and Z⊨ --X, --X is true in I. The truth table definition of '-' then tells us that X is true in I.

Treatment of ->I and -I require a somewhat different strategy, since these rules call for appeal to sub-derivations. This matter is discussed more fully in Volume II, Section 13-2. But briefly:

->I: If Z,X⊢ Y, then Z⊢ X->Y.
We have to show that if Z,X⊨ Y, then Z⊨ X->Y. Let I, with all sentences in Z true in I, be given. We have to consider two cases. Case 1: X is false in I. Then X->Y is true in I, since a conditional with a false antecedent is always true. Case 2: X is true in I. Since we are assuming that all sentences in Z are true in I, we have that all sentences in Z,X are true in I. Since we are assuming that Z,X⊨ Y, we then have that Y is true in I. Since X and Y are both true in I, X->Y is true in I.

-I: If Z,X⊢ Y&-Y, then Z⊢ X.
Let's suppose that all sentences in Z are true in I and that Z,X⊨ Y&-Y. But Y&-Y can't be true in I, as it would have to be if all sentences in Z are true in I, X were true in I and Z,X⊨ Y&-Y. Since we are assuming that Z,X⊨ Y&-Y and all sentences in Z are true in I, X must not be true in I, and so is false in I. That is, -X is true in I.

10-5. a) Not sound. Let Z be the set {-A,B}. Then we have-A,B⊨ A->B and -A,B⊨ B but NOT -A,B⊨ A.

b) Sound. Suppose that both Z⊢ X<->Y and Z⊢ -X. Now suppose that all sentences in Z are true in I. Then X<->Y and -X are true in I, and the truth table definition of '<->' tells us that -Y is true in I also. So Z⊨ -Y

c) Sound. Suppose we have Z⊢ [(u)P(u) v (u)Q(u)] and an interpretation, I, in which all the sentences in Z are true. Then(u)P(u) v (u)Q(u) is true in I, so that one of the disjuncts is true in I. Let us suppose that it is (u)P(u) which is true in I. Then all the substitution instances, P(s), of (u)P(u) are true in I. But then so are all substitution instances, P(s) v Q(s), of (u)[P(u) v Q(u)] true in I, so that(u)[P(u) v Q(u)] is true in I. The argument is the same if the other disjunct, (u)Q(u), is true in I.

d) Not sound. Let Z be the set {Pa,-Pb}. Then Pa,-Pb⊨ (Ex)Px but NOT Pa,-Pb⊨ Pb

e) Not sound. Let Z be the set {Pa,-Pb,-Qa,Qb}. Then Pa,-Pb,-Qa,Qb⊨ [(Ex)Px & (Ex)Qx] but NOT Pa,-Pb,-Qa,Qb⊨ (Ex)[Px& Qx].

10-6. The truth table method for determining the validity of sentence logic arguments works by writing down a truth table for all the sentences involved and then simply inspecting the lines of the truth table to see whether in all lines in which the premises are all true the conclusion is also true. Given a finite number of sentences which use a finite number of sentence letters, such a truth table can always be constructed. Since there are only finitely many lines to inspect, the procedure will give a definite answer after some finite number of steps.

10-7. We need no subscripts on the double turnstile. The double turnstile means that any interpretation which makes all sentences to the left of the double turnstile true also makes the sentence to the right true. But there is only one definition of truth in an interpretation. Consequently there is only one way to understand the double turnstile.

10-8. Assume that Y$\vdash$ Z and Y*sX. Y$\vdash$ Z means that there is a proof of Z using premises in Y. But Y*sX just means that any sentence in Y is a sentence in X. Hence a proof of Z using premises in Y is also a proof of Z using premises in X. So there is a proof of Z using sentences in X, that is X$\vdash$ Z.

10-9. Assume that Y$\models$ Z and Y*sX. Assume that I is an interpretation in which all the sentences of X are true. We have to show that Z is true in I. Y*sX just means that any sentence in Y is a sentence in X, so all the sentences in Y are true in I. Since we are assuming Y$\models$ Z, it follows that Z is true in I.

10-10. $\vdash$ X means that there is a proof of X using no premises. $\models$ X means that X is true in all interpretations.

10-11. We need to show that (EI)Mod(I,X) iff NOT X$\models$ A&-A. I will do this by showing that -(EI)Mod(I,X) iff X$\models$ A&-A. The key is that no I makes A&-A true. How then could every I which makes all the sentences in X true make A&-A true? Only by there being no I which makes all the sentences in X true, that is, -(EI)Mod(I,X). Conversely, if -(EI)Mod(I,X), then since there are no I's which make all the sentences in X true, it is vacuously true that any such I makes A&-A true. (The last step can be confusing. The point is that if every substitution instance of 'Px' is false, then '(x)(Px-> Qx)' is automatically true, since all substitution instances of '(x)(Px->Qx)' are conditionals true because of having a false antecedent.)

10-12. The demonstration of the equivalence of D6 and D6' suffices to show that D7 and D7' are likewise equivalent if we note that the rule-E tells us that -(EI)Mod(I,X) and (I)-Mod(I,X) are equivalent.

10-13. As defined in D6, D6', D7 and D7' consistency and inconsistency all concern which sentences are true or fail to be true in some or all interpretations. Truth in interpretations is a semantic fact. So these definitions all characterize semantic notions of consistency and inconsistency.

To say that a set of sentences, X, is semantically inconsistent is to say that there is no I in which all the sentences in X are true. A parallel notion of syntactic consistency and inconsistency will concern what is or is not provable. We can prove that X is semantically inconsistent by providing a derivation using premises taken from X and deriving a contradiction. So the sensible syntactic counterpart of semantic inconsistency is provability of a contradiction: