# A Modern Formal Logic Primer

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# Preface to Volumes I and II

## A Guide to the Primer

This text is a primer in the best sense of the word: A book which presents the basic elements of a subject. In other respects, I have sought to write a different kind of text, breaking with what I regard as an unfortunate tradition in teaching formal logic. From truth tables through completeness, I seek to explain, as opposed to merely presenting my subject matter. Most logic texts (indeed, most texts) put their readers to sleep with a formal, dry style. I have aimed for a livelier lecture style, which treats students as human beings and not as knowledge receptacles. In a text, as in the classroom, students need to be encouraged and to hear their difficulties acknowledged. They need variation in pace. They need shifts in focus among "I," "we," and "you," just as most of us speak in the classroom. From time to time students simply need to rest their brains.

One fault of logic textbooks especially bothers me: Some authors feel so concerned to teach rigor that they end up beating their students over the head with it. I have not sacrificed rigor. But I have sought to cultivate it rather than rubbing it in.

Now to the contents of the **Primer.** Volume I presents sentence logic. Volume II, Part I lays out predicate logic, including identity, functions, and definite descriptions; Part II introduces metatheory, including mathematical induction, soundness, and completeness. The text includes completely independent presentations of Fitch-style natural deduction and

the tree method as developed by Richard Jeffrey. I have presented the material with a great deal of modularity.

I have presented the text in two volumes to maximize flexibility of use in a variety of courses. Many introductory courses cover a mix of informal and formal logic. Too often I have heard instructors express dissatisfaction with what they find available for the formal portion of such a course. Volume I provides a new option. Using it in tandem with any of the many available inexpensive informal texts, instructors can combine the best of both subjects. Volume I will present a serious-minded introduction to formal logic, which at the same time should prove accessible and encouraging to those students who will never again take another logic course. The relatively small numbers who continue to a second course, devoted exclusively to formal logic, need only purchase Volume II to build on the foundation already laid.

The **Primer** incorporates a number of unusual features. Chapters 1, 3, and 4 emphasize the concept of a truth function. Though the idea is simple once you get it, many students need several passes. The optional section 3–4, on disjunctive normal form and the Scheffer stroke, serves the didactic function of providing yet more drill on truth functionality.

Following Richard Jeffrey, I have thoroughly presented '&', 'v', and '~' before treating '⊃' and '≡'. '&', 'v', and '~' are much less controversial correlates of their English counterparts than is '⊃'. Using '&', 'v' and '~' as a vehicle for introducing the idea of a truth function, I can deal honestly with the difficulties of giving a truth functional formulation of conditionals. In turn, this honest examination provides further drill with the concept of a truth function.

Sentences in English and logic often do not correspond very accurately. Consequently, I speak of transcription, not translation between logic and English. I treat sentence logic transcription quite briefly in chapter 1 of Volume I and further in the short, optional chapter 2. Predicate logic transcription gets a minimal introduction in chapter 1 of Volume II and then comes in for a thorough workout in chapter 4, also optional. There I deal with the subject matter of domains and the traditional square of opposition by using the much more general method of restricted quantifier subscripts and their elimination. This technique provides an all-purpose tool for untangling complicated transcription problems. Chapter 4 of Volume II also examines quantificational ambiguity in English, which most logic texts strangely ignore.

Training in metatheory begins in Volume I, chapter 1. But the training is largely implicit: I use elementary ideas, such as metavariables, and then call attention to them as use makes their point apparent. After thorough preparation throughout the text, chapter 10 of Volume II brings together the fundamental ideas of metatheory.

Standard treatments of sentence logic present sentence logic semantics, in the form of truth tables, before sentence logic derivation rules. Only in this way do students find the rules clearly intelligible, as opposed to poorly understood cookbook recipes. Often texts do not follow this heuristic for predicate logic, or they do so only half-heartedly. Presumedly, authors fear that the concept of an interpretation is too difficult. However, one can transparently define interpretations if one makes the simplifying assumption of including a name for each object in an interpretation's domain, in effect adopting a substitutional interpretation of the quantifiers. I further smooth the way by stressing the analogy of form and function between interpretations and truth value assignments in sentence logic.

This approach is ample for fixing basic ideas of semantics and for making predicate logic rules intelligible. After introducing predicate logic syntax in Volume II, chapter 1, and semantics in chapters 2 and 3, tree rules are almost trivial to teach; and derivation rules, because they can be better motivated, come more easily. I have clearly noted the limitation in my definition of an interpretation, and I have set students thinking, in an exercise, why one may well not want to settle for a substitutional interpretation. Finally, with the ground prepared by the limited but intuitive definitions of chapters 2 and 3 of Volume II, students have a relatively easy time with the full characterization of an interpretation in chapter 15.

No one has an easy time learning—or teaching—natural deduction quantifier rules. I have worked hard to motivate them in the context of informal argument. I have made some minor modifications in detail of formulation, modifications which I believe make the rules a little easier to grasp and understand. For existential elimination, I employ the superficially restrictive requirement that the instantiating name be restricted to the sub-derivation. I explain how this restriction works to impose the more complex and traditional restrictions, and I set this up in the presentation so that instructors can use the more traditional restrictions if they prefer.

For the proof of completeness of the natural deduction system I have fashioned my own semantic tableau proof. I believe that on its own it is at least as accessible as the Henkin and other more familiar proofs. In addition, if you do tree completeness first, you can explain the natural deduction completeness proof literally in a few minutes.

I have been especially careful not to dive into unexplained proofs of soundness and completeness. Instructors will find, in separate sections, informal and intuitive explanations of the sentence logic proofs, unencumbered with formal details, giving an understanding of how the proofs work. These sections require only the first short section of the induction chapter. Instructors teaching metatheory at a more elementary level may

want to conclude with some of these sections. Those ready for the tonic of rigor will find much to satisfy them in the succeeding sections.

In some chapters I have worked as hard on the exercises as on the text. I have graded the skill problems, beginning with easy comprehension checkers, through skill builders, to some problems which will test real skill mastery. I think few will not find enough problems.

Exercises should exercise understanding as well as skills. Any decent mathematics text puts problems to this task, as well as uses them to present auxiliary material. Too few logic texts fall in this tradition. I hope that students and instructors will enjoy my efforts in some of the exercises to introduce auxiliary material, to lay foundations for succeeding material, to engage creative understanding, and to join in the activity of conceptual exploration.

For teaching plans the key word is "modularity." Those using just Volume I in an informal/formal course may teach chapters 1, 2 (optional), 3, and 4 to introduce sentence logic. Then, as taste and time permit, you may do natural deduction (chapters 5, 6, and 7) or trees (chapters 8 and 9), or both, in either order.

Volumes I and II together provide great flexibility in a first symbolic logic course. Given your introduction of sentence logic with chapters 1, 3, and 4 of Volume I and grounding of predicate logic with chapters 1, 2, and 3 of Volume II you can do almost anything you want. I have made treatment of derivations and trees completely independent. You can run through the one from sentence to predicate logic, and then go back and do the other. Or you can treat both natural deduction and trees for sentence logic before continuing to predicate logic. You can spend up to two weeks on transcription in chapter 2 of Volume I and chapter 4 of Volume II, or you can rely on the minimal discussion of transcription in the first chapters of Volumes I and II and omit chapter 2 of Volume I and chapter 4 of Volume II altogether.

If you do both trees and natural deduction, the order is up to you. Trees further familiarize students with semantics, which helps in explaining natural deduction rules. On the other hand, I have found that after teaching natural deduction I can introduce trees almost trivially and still get their benefit for doing semantics and metatheory.

Your only limitation is time. Teaching at an urban commuter university, in one quarter I cover natural deduction (Volume I, chapters 1, 2, 3, 4, 5, 6, 7; Volume II, chapters 1, 2, 3, 5, and perhaps 6), or trees and sentence logic natural deduction (Volume I, chapters 1, 2, 3, 4, 8, 9; Volume II, chapters 1, 2, 3, 7, 8; Volume I, chapters 5, 6, and 7). A semester should suffice for all of Volume I and Volume II through chapter 8, and perhaps 9. Again, you may want to follow the chapter sequencing, or you may want to do natural deduction first, all the way through predicate logic, or trees first.

If you do just natural deduction or just trees you have more time for identity, functions, definite descriptions, and metatheory. Chapter 10 of Volume II, basic metatheoretical concepts, can provide a very satisfying conclusion to a first course. A two quarter sequence may suffice for all of the metatheory chapters, especially if you do not do both natural deduction and trees thoroughly. To this end the metatheory chapters cover soundness and completeness for both natural deduction and trees independently. Or, you may choose to end with the sections presenting the informal explanations of induction and the soundness and completeness proofs. The text will provide a leisurely full year course or a faster paced full year course if you supplement it a bit at the end of the year.

I want to mention several features of my usage. I use single quotes to form names of expressions. I depart from logically correct use of quotation marks in one respect. In stating generalizations about arguments I need a formulation which makes explicit use of metavariables for premise and conclusion. But before chapter 10 of Volume II, where I make the metalanguage/object language distinction explicit, I do not want to introduce a special argument forming operator because I want to be sure that students do not mistake such an operator for a new symbol in the object language. Consequently I use the English word 'therefore'. I found, however, that the resulting expressions were not well enough set off from their context. For clarity I have used double quotes when, for example, I discuss what one means by saying that an argument, "X. Therefore Y." is valid.

Throughout I have worked to avoid sexist usage. This proves difficult with anaphoric reference to quantified variables, where English grammar calls for constructions such as 'If someone is from Chicago he likes big cities.' and ' Anyone who loves Eve loves himself.' My solution is to embrace grammatical reform and use a plural pronoun: 'If someone is from Chicago they like big cities.' and 'Anyone who loves Eve loves themself.' I know. It grates. But the offense to grammar is less than the offense to social attitudes. As this reform takes hold it will sound right to all of us.

I thank the many friends and family who have actively supported this project, and who have born with me patiently when the toil has made me hard to live with. I do not regard the project as finished. Far from it. I hope that you—instructors and students—will write me. Let me know where I am still unclear. Give me your suggestions for further clarification, for alternative ways to explain, and for a richer slate of problems. Hearing your advice on how to make this a better text will be the best sign that I have part way succeeded.

Paul Teller