

Lecture 2

Example

$\{\}$ is the empty set.

To avoid any confusion, we give this set a special name $\emptyset = \{\}$.

Example

$$\{\} \neq \{\emptyset\}$$

empty set contains
no elements

set containing the
empty set contains 1
element

Definition Let A be a set.

The cardinality of A , written $|A|$, is the number of elements in A .

Examples

(1) $|\emptyset| = 0.$

(2) $|\{1, 2, 3\}| = 3.$

Remark

(5) Later on in this course, we shall study the Complex numbers $\mathbb{C}$.

A very common way to define a set is to describe a property that the elements of the set are to have.

Example

$\mathbb{P} = \{a : \text{such that } a \text{ is a prime}\}$
is the set of prime numbers.

Examples

$$(1) A = \{a : a \in \mathbb{R} \text{ and } a^2 = 1\} \\ = \{-1, 1\}.$$

$$(2) B = \{a : a \in \mathbb{N} \text{ and } a^2 = 1\} \\ = \{1\}.$$

$$(3) \quad C = \{a : a \in \cancel{\mathbb{Q}}_{\neq} \text{ and } a^2 = \cancel{2}\}$$

$\nearrow = \emptyset$
needs a proof.

Given a set A we can form sets B whose elements are chosen from those of A .

We say that B is a subset of A written $B \subseteq A$.

Examples

(1) $\emptyset \subseteq A$. (Choose no elements from A .)

(2) $A \subseteq A$. Choose all elements from A .

← Mathematicians use the word 'choose' in a special way.

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Definition Let A be a set.

$P(A) = \{B : B \subseteq A\}$ is called the powerset of A . It is the set of all subsets of A .

Example Let $A = \{1, 2, 3\}$. Find $P(A)$.

Cardinality of subset	subsets
0	$\emptyset$
1	$\{1\}, \{2\}, \{3\}$
2	$\{2, 3\}, \{1, 3\}, \{1, 2\}$
3	$\{1, 2, 3\}$

$$P(A) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{2, 3\}, \{1, 3\}, \{1, 2\}, \{1, 2, 3\} \}.$$

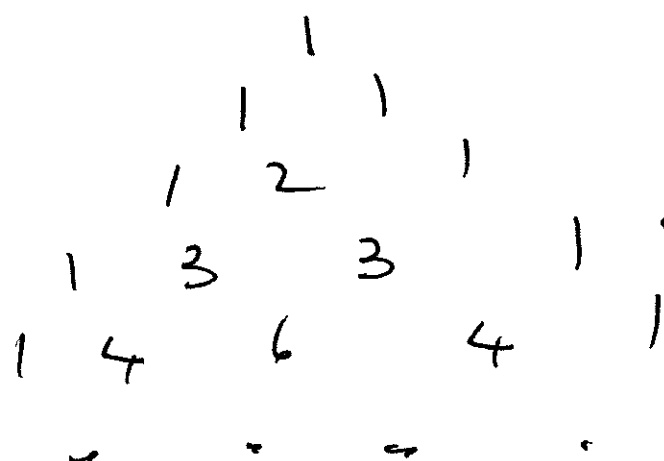
Observe that

$$|P(\{1,2,3\})| = 8.$$

There is an interesting # number pattern here.

Cardinality of subset	Number of such subsets
0	1
1	3
2	3
3	1

So, $8 = 1 + 3 + 3 + 1$



Pascal's Δ

This is not a Coincidence

And, also.

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$$\begin{aligned}(a+b)^3 &= (a+b)(a+b)^2 \\&= (a+b)(a^2 + 2ab + b^2) \\&= a^3 + \underbrace{2a^2b}_{\text{from } a \cdot 2ab} + \underbrace{ab^2}_{\text{from } b \cdot a^2} + \underbrace{2ab^2}_{\text{from } b \cdot 2ab} + b^3 \\&= \underset{=}{1}a^3 + \underset{=}{3}a^2b + \underset{=}{3}ab^2 + \underset{=}{1}b^3\end{aligned}$$

Why does the powerset
"know" about Pascal's Δ ?