Solutions 5

(1) (a)
$$\sum_{i=0}^{8} \binom{8}{i} x^i.$$
(b)
$$\sum_{i=0}^{8} \binom{8}{i} (-1)^i x^i.$$
(c) The coefficient is $\binom{10}{2}$.
(d) The coefficient is $\binom{6}{3} \cdot 3^3 \cdot 4^3$.
(e) The binomial expansion is
$$\sum_{i=0}^{9} \binom{9}{i} \left(3x^2\right)^{9-i} \left(-\frac{1}{2x}\right)^i.$$
Expanding each term carefully we get
$$\sum_{i=0}^{9} \binom{9}{i} \cdot (-1)^i \cdot 3^{9-i} \cdot 2^{-i} \cdot x^{18-2i} \cdot x^{-i}.$$
This is equal to
$$\sum_{i=0}^{9} \binom{9}{i} \cdot (-1)^i \cdot 3^{9-i} \cdot 2^{-i} \cdot x^{18-3i}.$$
We need to calculate the coefficient of x^3. This means we need to calculate the coefficient where $i = 5$. This is $-\binom{9}{5} \cdot 3^4 \cdot 2^{-5}$. We need to calculate the value of the constant term. This means we need to calculate the term where $i = 6$. This is $\binom{9}{6} \cdot 3^3 \cdot 2^{-6}$.

(2) (a) Put $x = y = 1$ in the binomial theorem.
(b) Put $x = 1$ and $y = -1$ in the binomial theorem.
(c) Put $x = 1$ and $y = \frac{1}{2}$ in the binomial theorem.