Properties of sets

This is a version of Section 3.2.

Key definitions

The following three operations defined on sets are called Boolean operations, named after George Boole (1815–1864). Let $A$ and $B$ be sets. Define a set, called the intersection of $A$ and $B$, denoted by $A \cap B$, whose elements consist of all those elements that belong to $A$ and $B$.

Define a set, called the union of $A$ and $B$, denoted by $A \cup B$, whose elements consist of all those elements that belong to $A$ or $B$. The word or in mathematics does not mean quite the same as it does in everyday life. Thus $X$ or $Y$ means $X$ or $Y$ or both. It is therefore inclusive or.

Define a set, called the difference or relative complement of $A$ and $B$, denoted by $A \setminus B$ or $A - B$, whose elements consist of all those elements that belong to $A$ and not to $B$.

The diagrams used to illustrate the above definitions are called Venn diagrams where a set is represented by a region in the plane.
Properties of Boolean operations

Let \(A, B\) and \(C\) be any sets.

1. \(A \cap (B \cap C) = (A \cap B) \cap C\). Intersection is associative.
2. \(A \cap B = B \cap A\). Intersection is commutative.
3. \(A \cap \emptyset = \emptyset = \emptyset \cap A\). The empty set is the zero for intersection.
4. \(A \cup (B \cup C) = (A \cup B) \cup C\). Union is associative.
5. \(A \cup B = B \cup A\). Union is commutative.
6. \(A \cup \emptyset = A = \emptyset \cup A\). The empty set is the identity for union.
7. \(A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\). Intersection distributes over union.
8. \(A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\). Union distributes over intersection.
9. \(A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)\). De Morgan’s law part one.
10. \(A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)\). De Morgan’s law part two.
11. \(A \cap A = A\). Intersection is idempotent.
12. \(A \cup A = A\). Union is idempotent.

To illustrate these properties, we can use Venn diagrams.