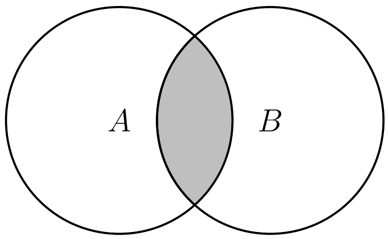
Properties of sets

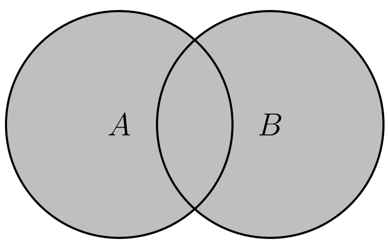
This is a version of Section 3.2.

Key definitions

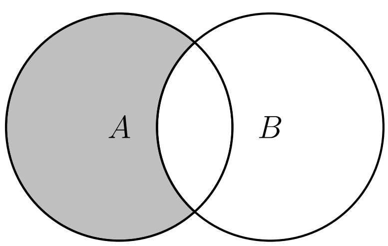
The following three operations defined on sets are called *Boolean operations*, named after George Boole (1815–1864). Let A and B be sets. Define a set, called the *intersection* of A and B, denoted by $A \cap B$, whose elements consist of all those elements that belong to A **and** B.



Define a set, called the *union* of A and B, denoted by $A \cup B$, whose elements consist of all those elements that belong to A **or** B. The word *or* in mathematics does not mean quite the same as it does in everyday life. Thus X *or* Y means X *or* Y *or both*. It is therefore *inclusive or*.



Define a set, called the *difference* or *relative complement* of A and B, denoted by $A \setminus B$ or $A - B$, whose elements consist of all those elements that belong to A **and not** to B.



The diagrams used to illustrate the above definitions are called *Venn diagrams* where a set is represented by a region in the plane.

Properties of Boolean operations

Let A, B and C be any sets.

(1) $A \cap (B \cap C) = (A \cap B) \cap C$. Intersection is associative.
(2) $A \cap B = B \cap A$. Intersection is commutative.
(3) $A \cap \emptyset = \emptyset = \emptyset \cap A$. The empty set is the zero for intersection.
(4) $A \cup (B \cup C) = (A \cup B) \cup C$. Union is associative.
(5) $A \cup B = B \cup A$. Union is commutative.
(6) $A \cup \emptyset = A = \emptyset \cup A$. The empty set is the identity for union.
(7) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Intersection distributes over union.
(8) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Union distributes over intersection.
(9) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$. De Morgan's law part one.
(10) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$. De Morgan's law part two.
(11) $A \cap A = A$. Intersection is idempotent.
(12) $A \cup A = A$. Union is idempotent.

To *illustrate* these properties, we can use Venn diagrams.