Autumn 2016
F17CC Introduction to university mathematics

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Aims. The goal of this course is to provide a bridge between school and university mathematics. In particular, the central role of proofs in mathematics will be emphasized throughout the course. Proofs tell us *why* things are true; results in mathematics are not the result of belief or experiment but of proof. There is some overlap with A-level and Advanced Highers, though I assume you have neither, and there will be elements of revision but the tone of the course is different.

Course website. This can be accessed via VISION or you can find it directly via my **homepage** (click **Teaching** then **Introduction to university mathematics**).

Lecture notes. A pdf copy of my book *Algebra & geometry: an introduction to university mathematics* will be made freely available to all students registered for the course via VISION. This contains everything in this course and more. **You are provided with a free copy of my book on the strict understanding that it is for your personal use only.**

Problem classes/tutorials. Solving mathematics problems is an essential ingredient in learning mathematics. You will be expected to prepare for the problem classes by attending the lectures and working through your notes beforehand. The exercise sheets will be made available sequentially via the course website. You can work through the problems either on your own or in groups. Both myself and PhD students will be on hand to help but it is not our role to read your notes for you or teach you stuff that you missed in the lectures. After the class, the solutions will be posted on the course website. Any issues that arise in the problems classes will be discussed in the next lecture.

Past paper policy. There are no past papers or specimen exam papers. There is more on the nature of the exam paper below but to alleviate anxiety about failing **the questions on the exam paper will be essentially based on the exercise questions (with numbers possibly changed).** This means that if you attend all the lectures and problems classes you will have seen the vast majority of questions I could possibly ask on the exam paper. The course, the exam paper and the marking of the exam paper are all my responsibility alone; they are not subcontracted out to exam boards and the like.

Assessment. The goal of assessment is to find out what you have learnt and the depth of your understanding. It is there to serve the needs of the course, the course is not run to serve the needs of the exam. This is a big difference from what you are probably used to from school and may take you some time to get used to.

- There will be two take-home assessments each of which will contribute 10% towards your final mark.
- There will be one final 2 hour exam paper worth 80% as described above.

Books for further reading and practice.

R. Hammack, *Book of proof*, VCU Mathematics Textbook Series, 2009. This book can be downloaded for free from

`http://www.people.vcu.edu/~rhammack/BookOfProof/index.html`

or click the link on the website for this course. This is an excellent reference for material dealing with sets and counting, in addition for more information about proofs in mathematics.

A. Hirst, D. Singerman, *Basic algebra and geometry*, Pearson Education, 2001. This is a textbook thats shares some of the same intentions as this course and would be useful as a source of different approaches and further examples. It covers much the same ground as my course, though oddly does not cover the solution of linear equations by Gaussian elimination.

S. Lipschutz, M. Lipson, *Discrete mathematics*, second edition and onwards, Schaum Outline Series. As with all the Schaum books, this is an excellent place to look for worked examples and further exercises. Chapters 1, 5 and 11 seem the most relevant to this course.

S. Lipschutz and M. Lipson, *Linear Algebra*, third edition and onwards, Schaum Outline Series. Chapters 1, 2, 3 and 8 are covered in this course.

J. Olive, *Maths: a student's survival guide*, second edition, CUP, 2006. This book is primarily designed for science students but is, in fact, a very useful source for a lot of A-level/(Advanced) Highers standard mathematics that you may have forgotten from school or never encountered. In addition, it contains material that I will cover from scratch in this course in Chapters 1, 2, 3, 6, 7, 10, 11.

Information on the Web. There's oodles of the stuff. Just use your favourite search-engine, such as Google, and type in key words that deal with the topic you are interested in. If you find anything you particularly like, let me know and I shall post the link on the course website. You can also use this method to find the mathematics department home-pages, my home-page or the university's homepage.

Library. This is a big building that contains books you can borrow.

Learning outcomes/syllabus

0. The conceptual aspects of mathematics

- What is mathematics? Mathematics in history and contemporary mathematics.
- Reasoning and logic. What is an argument? The notion of proof in mathematics with simple first examples such as the irrationality of $\sqrt{2}$, the triangle theorem, Pythagoras' theorem, and the proof that there are infinitely many primes.
- Solve quadratics by completing the square.
- Abstraction and rules. The meaning of key algebraic terms such as: associativity, commutativity, distributivity, identity, inverse. The rules of high-school algebra. Proof that $-1 \times -1 = 1$.
- Problem-solving.
- The need for checking.

1. Combinatorics

- Counting.
- Manipulate sets and their elements. This includes the Boolean operations and set product.
- Answer simple counting questions involving permutations and combinations. Connections with probability touched on.
- Statement and proof of the Binomial theorem (by a counting argument). Applications.

2. Complex numbers and polynomials

- How complex numbers were discovered.
- Add, subtract, multiply and divide complex numbers.
- Find square roots of complex numbers.
- Represent complex numbers in the complex plane.
- Understand the geometric interpretations of addition and multiplication of complex numbers.

- Proof that a polynomial of degree n over the complex numbers has at most n roots.
- The fundamental theorem of algebra.
- Proof of the fundamental theorem of algebra for real polynomials.
- Find nth roots.
- Use De Moivre's theorem to find expressions for $\sin^n \theta$ and $\cos^n \theta$.
- Euler's theorem and its proof.
- Find rational roots of polynomials with integer coefficients.
- Factorize real and complex polynomials appropriately.
- Understand the difference between trigonometric solutions and radical solutions.

3. Matrices

- Why matrices are important.
- Add, subtract, and multiply two matrices, and multiply a matrix by a scalar; be able to carry out sequences of such operations to obtain a single matrix as a result. The main emphasis will be on 'small' matrices often 2×2 or 3×3 throughout.
- Proof of associativity for matrix multiplication.
- Solve linear equations using Gaussian elimination.
- Proof of the fundamental theorem of linear equations.
- Compute determinants by first row expansion.
- Compute matrix inverses using the adjugate method.
- Calculate the characteristic polynomial of a matrix.
- Statement of the Cayley-Hamilton theorem and proof in the 2×2 case.

4. Vectors

- What is Euclidean geometry?
- Compute with vectors using inner products, vector products, and scalar triple products.
- Find the equation of the unique line determined by two points or a point and a vector in space.

- Find the equation of the unique plane determined by three points or by a point and a normal.
- Calculate intersections of lines or planes.
- Derivation of the volume of a parallelepiped using scalar triple products and connection with determinants.