Texture Segmentation based on the Wavelet Packet Frame and Principle Component Analysis

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Abstract—The wavelet packet frame is applied to decompose the texture image into subimages, which have the same size with the original image and contain the middle frequency information. Among all of the subimages, only the one having the maximum variance in a subchannel is selected out to be the initial feature subimage. Then the mean variance, energy and entropy features are extracted from every initial feature subimage. In order to improve these features, first the normalization is done; then a quadrant-mean filter is proposed to smooth noise while keeping the boundary exact; at last a set of new features are gotten through the PCA method, which decrease greatly in dimension while keeping the main information of all the features only in few new features. The feature segmentation is obtained by fuzzy c-means clustering. The performance of this new method is demonstrated on the segmentation of Brodatz texture set.

Keywords—Texture segmentation, wavelet packet frame, quadrant-mean filter, principle component analysis (PCA)

I. INTRODUCTION

Texture segmentation is an important task in many computer vision applications. The multichannel/multiresolution methods have drawn much attention recently. With the development of the wavelet theory, it provides a new space-frequency analysis method. Mallat [1] applied the pyramid structure wavelet transform to texture analysis in the multiresolution way. Unser [2] proposed wavelet frame transform to yield a translation invariant description of texture. The two transforms are focus on the successive decomposition in the low frequency channel, but the textures’ main spatial frequencies are usually in the middle frequency regions. The tree structure wavelet packet transform has been applied to extend the decomposition into various frequency regions containing important information for the texture analysis tasks [3].

In order to make the segmentation more precise, the raw features need to be improved, which are extracted from the subimages of the wavelet packet frame decomposition. It proved a quadrant-ratio noise filter to preserve the borders of texture regions in [4]. Although the filter could maintain the boundary, it also maintained some noise. We propose a new method to smooth the noise while preserve the boundary in the feature images. Not all of the features, however, contain the useful information for segmentation. There are many correlations among the features. Principle component analysis (PCA) is the optimal transform in minimum mean square error and maximum energy packing sense. The transformed data are totally uncorrelated and contain most of the segmentation information in the first few coordinates [5]. In section 2, we apply Unser’s wavelet frame decomposition method to the wavelet packet decomposition. In section 3 we give out the standard to choose the initial feature images from the decomposed subimages and extract three different features from each subimage selected out. To these raw feature images, a new method quadrant mean filter is proposed to improve the feature. Additionally the PCA method is used to optimize these features. Section 4 and section 5 provide the experimental results and conclusion respectively.

II. WAVELET PACKET FRAME

A. Briefly review of the wavelet frame

Unser [2] described an overcomplete wavelet representation called a discrete wavelet frame which is similar to the traditional discrete wavelet transform, except that no downsampling occurs between levels of the hierarchy. The wavelet frame decomposition for an image can be obtained by successive 1-D processing along the rows and columns of the image. In the implementation, Unser made good use of the two-scale relation (1) obtained a fast iterative decomposition algorithm

\[
\begin{align*}
  h_{i+1}(k) &= [h]_{2i} * h_i(k) \\
  g_{i+1}(k) &= [g]_{2i} * h_i(k)
\end{align*}
\]

where notation \([.]_m\) denotes the upsampling by a factor of \(m\). The factor of one iteration is more or less to dilate the filters \(h_i\) and \(g_i\).

\[
\begin{align*}
  s_{i+1}(k) &= [h]_{2i} * s_i(k) \\
  d_{i+1}(k) &= [g]_{2i} * s_i(k)
\end{align*}
\]

where \(i = 0, \ldots, J\). Each step involves a convolution with the basic filters \(h\) and \(g\) which are expanded by inserting of an appropriate number of zeros between taps. The complexity of this algorithm is same to all iterations. It is simply proportional to the number of samples.

B. Wavelet Packet Frame Decomposition

The pyramid-structured wavelet transform decomposes the signal into a set of frequency channels that have narrower bandwidths in the lower frequency region. The transform is suitable for the signal that its main information is concentrated in the low frequency regions. However, it may not be suitable for the texture whose primary frequency channels are focused on the middle frequency region. To analyze such signals, the concept of wavelet bases has been generalized to include a library of waveform orthogonal bases, called wavelet packet...
bases or simply wavelet packet. The library of wavelet packet basis functions $\{W_n\}_{n=0}^{\infty}$ can be generated from a given function $W_0$ as follows:

$$W_{2n}(x) = \sqrt{2} \sum_k h(k) W_n(2x - k)$$
$$W_{2n+1}(x) = \sqrt{2} \sum_k g(k) W_n(2x - k)$$

(3)

where the function $W_0$ is the scaling function $\psi$ and $W_1$ is the wavelet function $\phi$. The library of wavelet packet bases can be defined as the collection of orthogonal bases in the form $W_n(2^l(x - k))$, where $l, k \in \mathbb{Z}, n \in \mathbb{N}$. Each element of the library is determined by a subset of the indices: a scaling parameter $l$, a localization parameter $k$, and an oscillation parameter $n$.

In the implementation, the traditional method is to downsample the signal and then convolute with the filters. The width or length of the subimage is the $1/2^n$ (ndenotes the level of decomposition) of the original which is not very suitable for the segmentation. For every pixel in the subimages stands for $2^n$ pixels in original image, accordingly the segmentation is the block-based not the pixel-based to the original image. Therefore we apply the wavelet frame decomposition method to the wavelet packet decomposition, the so-called wavelet packet frame decomposition.

We use (1) to upsample the wavelet packet filters and then apply these filters to decompose signal. The subimages got from the wavelet packet frame keeping the size same with the original image and containing the middle frequency information, which combine the merit of wavelet frame and wavelet packet decomposition.

III. EXTRACTING AND IMPROVING FEATURES

In the feature based texture segmentation, some common questions need to be overcome. One is the misclassification caused by the noise of the feature, which takes the form of holes and other fragments [6]. Another is the so-called boundary effect question. It usually appears as inaccurate segmentation of boundaries or superfluous narrow regions at the boundary between two textures. To make matter worse, the misclassification may be interpreted as a third texture.

The use of multiple types of features in texture segmentation may be exploited to take full advantage of the merit of each feature and alleviate these problems. And some of the post-processing works can improve the features furthermore.

A. Selecting feature subimages

The wavelet packet decomposition can provide a fully description of the texture images whose information is focused in the middle frequency domain. However, with the increasing of the decomposition level, the number of subimages is also increased exponentially, which bring much difficulty for the clustering and computation. In order to decrease the number of the wavelet packet decomposition nodes and keep the dimension of the feature stable, we choose 2-level wavelet packet decomposition [7]. The second-level decomposition subimage with the maximum variance in its subchannel is selected out as the initial feature. In this method, we can get 8 initial feature subimages.

Let $S(x, y), x = 1, \ldots, M, y = 1, \ldots, N$ is a sub-image, its variance is

$$\sigma^2 = \frac{1}{M \times N} \sum_{i=1}^{M} \sum_{j=1}^{N} (s(x, y) - \overline{s})$$

(4)

where $\overline{s}$ denote the average value of the subimage.

The relation of these initial features is shown in Fig. 1, the A, H, V, D denotes the first-level approximate, horizontal detail, vertical detail and diagonal detail subimages respectively and the A2, H2, V2, D2 are the features selected out from second-level subimages.

B. Extracting Feature

Mallat [1] suggested that by using wavelet decomposition, statistics based on first-order distribution of gray levels might be sufficient for the perception of textural difference. In this paper, we extract three features from every subimage. These features are energy feature (5), entropy feature (6) [8] and mean variance feature (7) respectively.

$$F_1(i, j) = \frac{1}{255^2} \sum_{x=i-n}^{i+n} \sum_{y=j-n}^{j+n} s(x, y)^2 p(s(x, y))$$

(5)

$$F_2(i, j) = -\frac{1}{\log 255} \sum_{x=i-n}^{i+n} \sum_{y=j-n}^{j+n} p(s(x, y)) \log p(s(x, y))$$

(6)

$$F_3(i, j) = \frac{1}{(2n+1)^2} \sum_{x=i-n}^{i+n} \sum_{y=j-n}^{j+n} |s(x, y) - \overline{s}|$$

(7)

where $p(s(x, y))$ is the probability of the intensity of point $s(x, y)$ in the sub-image, $\overline{s}$ denotes the mean value of all the points in the local window.

Then computing these three features for every selected subimages point by point, we can get a feature matrix $F(x, y) = \{F_k(x, y)\}, i = 1, 2, \ldots, 8, k = 1, 2, 3$ ,where a feature $F_k(x, y)$ represents the $k$th feature of the $i$th subimage $x = 1, 2, \ldots, M, y = 1, 2, \ldots, N$.

C. Smoothing Features

The raw features extracted from the method described in previous section are not suitable for the clustering. It needs some improvement without losing the features’ information. Different subimages have different magnitude after wavelet packet decomposition. Usually the low frequency subbands
We propose to use the mean value as the judging standard in the quadrant method. Compared with the window mean filter, the quadrant mean filter assigns every point to a certain region according to the local mean value, however the window mean filter just averages the possibility of the point assigned to the regions around it, i.e. the quadrant mean filter can preserve the boundary better than the window mean filter. Compared with the quadrant ratio filter, quadrant mean filter smoothes the noise but the quadrant ratio filter only smoothes the slowly changed region but sharpens the acutely changed region, i.e. the quadrant mean filter can smooth the noise better than the quadrant ratio filter.

Fig. 3 shows the results of these three filters. The feature is the mean variance feature of test mosaic T1 made up of Brodatz textures. From these three smoothing results, we can find that in the region with single feature, the quadrant-mean operator can smooth the noise better than the quadrant ratio; in the boundary region, it can preserve the boundary better than the window mean operator.

Fig. 2. the relation of the four local windows

According to the method subscripted previous we get a feature matrix with 24 features that are not totally uncorrelated. That’s is to say, there are some redundancy between these features. On the basis of keeping the main information of original feature matrix, we should make a new feature matrix with fewer features that are uncorrelated to each other.

PCA involves rotating and transforming the original \( p \) axes that represent the original variable into a new set of \( q \) orthonomal axes which lie along the direction of maximum variance. The first axis represents the maximum variance and variance value represented by subsequence axes decreases rapidly. The number of new \( q \) axes that can describe the original information is less than that of original \( p \) axes. The principle components (PC) are the eigenvectors of the variance-covariance matrix.

Since the PCA is operated on the 2 dimensional matrix, the 3 dimensional feature matrix \( F(x, y) = \{F_k^i(x, y)\}_{i=1, 2, \ldots, 8,k=1, 2, 3} \) should be translated into a 2 dimensional matrix. We convert every feature \( \{F_k^i(x, y)\}_{M \times N} \) into a column vector \( [F_k^i(n)]_{M \times N \times 1} \), then the new 2 dimensional feature matrix is \( [\hat{F}(n, l)]_{M \times N \times 1k} \). The PCA algorithm is as following:

1) Compute the covariance matrix of feature matrix.
2) Compute the eigenvector and eigenvalue of the covariance matrix.

3) Select $q$ eigenvectors correspondent to $q$ largest eigenvalues to compose the projection matrix $\Phi$, the number of $q$ is determined by

$$\sum_{j=1}^{i+k} \lambda_j / \sum_{j=1}^{q} \lambda_j \geq \alpha \quad (10)$$

4) Project the original feature matrix $\tilde{F}$ to new axes $\Phi$ and get the new feature matrix $Y$.

$$[Y]_{MN \times q} = \tilde{F}\Phi \quad (11)$$

The new feature matrix $Y$ is decreased greatly in dimension while maintaining the main information of the original features. What’s more, the $q$ features in the new feature matrix are those have the largest variance among all the $i \times k$ features in the new feature matrix. The larger the variance of the feature, the effective the feature is. Therefore the new feature matrix is more effective than the original feature in segmentation while the size of the new feature matrix is much smaller than that of the original.

IV. EXPERIMENTAL RESULTS

As described above, our texture segmentation algorithm contains three stages: the wavelet frame packet decomposition, feature extraction and classification. First we use the wavelet frame packet to decompose the texture images and select out eight subimages according to the maximum variance standard. Then we extract features from these initial subimages and some improvement of the raw features is done. In this stage, the feature extraction window is $11 \times 11$ and smooth window is also $11 \times 11$. The PCA is provided to decrease the number of features while keeping the main information. At last the fuzzy c-means is applied to classify these features. In the experiments the test mosaics($128 \times 128 \times 256$)are made up of the Brodatz texture.

A. Experiment 1

In order to show the effectivity of the quadrant mean filter, we compared it with the quadrant ratio filter and the window mean filter. We abstracted the mean variance feature from the eight subimages and filtered these features with these three filters. The segmentation results (the number of the wrong segmentation pixels) are shown in Table I. In Table I the Q.M., Q.R., and W.M. are the quadrant mean, quadrant ratio, and window mean filter, respectively and the components of every test mosaic are shown in the bracket after the test mosaic. The segmentation error figures of T1 are shown in Fig. 4.

From Table 1 we found that the quadrant mean filter and window mean filter could decrease the segmentation error introduced by the original raw features, and the quadrant mean filter is better than the window mean filter. At the same time, we found that the quadrant ratio filter could not deduce the error. Comparing these error figures in Fig. 4, we find that the quadrant mean filter can smooth the noise in the texture region and preserve the boundary; the quadrant-ratio filter can make the boundary exact but enhance the noise, consequently the total number of the error pixels increased; the window mean filter smoothes the noise in the region but blurs the boundary.

These raw features, which are extracted from the subimages of wavelet packet frame decomposition, need to smooth the noise in the texture region and make the boundary exact as well. The quadrant mean filter combined the advantage of the window mean filter and quadrant ratio filter that can smooth the little noise in the texture region and make the boundary exact in certain level.

B. Experiment 2

In this experiment we applied the PCA to optimize the features, which are the mean variance, energy and entropy features extracted from every subimage and smoothed with the quadrant mean filter. And we compared the segmentation result of the features optimized by PCA with that of all the features. The parameter will influence the number of the features being kept in PCA algorithm and the segmentation result. So we also compare the segmentation result with different value of $\alpha$. Table 2 shows the number of the wrong segmentation pixels in the row named E and the feature’s dimension is in the row named D.

In Table II we find that the segmentation result of the optimized features is nearly same to that of the features without the PCA processing and the difference of segmentation error is no greater than 50 pixels. What’s more, the dimension of the optimized feature is decreased greatly. If we let the be $85\%$, the dimension of the new features is 4 or 3. So we can say that the main information of all the features is kept in very few new features after the PCA processing. In the new feature matrix the first feature has the largest variance.
and the last feature has the least variance. So if we remove some features with little variance, the segmentation result will not be influenced even sometimes improved. Comparing segmentation result listed in Table I and Table II, we find that the segmentation is improved, which proves that the multifeature can make segmentation more exact. In Fig.5 we gives out the segmentation result images of the optimized features with the parameter $\alpha = 95\%$.

V. CONCLUSION

A new method of segmentation texture images has been developed and demonstrated. We extract multifeatures and optimize the features with the normalization, smoothing, and decreasing dimension. The experimental results show the effectiveness of the method.

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REFERENCES


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<td></td>
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<td>D</td>
<td>24</td>
<td>12 9 7 6 6 5 4 4</td>
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Fig. 5. The segmentation result and error of all the test images. (a) to (l) Test Image T1 to T6’s segmentation result and error, respectively.