## Civil Engineering 2 Mathematics Autumn 2011

## **Revision sheet**

The following questions all use techniques that you were taught last year (and have possibly now forgotten!) We will be using these ideas extensively this year, so now is the time to refresh your memory. Try and complete this sheet before the next lecture.

1. Show, using the integrating factor method, that the solution of

$$y' + (5/x)y = 1/x^2$$

is  $y = 1/(4x) + C/x^5$ , where C is an arbitrary constant.

**2**. Show that the general solution y(x) of

$$y'' + y = 0$$

is  $y = A \cos x + B \sin x$ . In a similar way, show that the general solution of y'' - y = 0 is  $y = Ae^x + Be^{-x}$ . Demonstrate also that this latter solution may be written in the alternative form  $C \cosh x + D \sinh x$ .

3. Use the method of separation of variables to show that the solution to

$$\frac{dy}{dx} = y$$

subject to the boundary condition y = 1 when x = 0 is  $y = e^x$ .

4. Use integration by parts to show that

$$\int_{-\pi}^{\pi} x \sin nx \, dx = \frac{2(-1)^{n+1}\pi}{n},$$

where n is a positive integer.

5. Integrate by parts twice to show that

$$\int_{-\pi}^{\pi} \sinh x \, \sin x \, dx = \sinh \pi.$$

6. Show that

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\tan x}{x} = 1.$$

7. Find eigenvalues and eigenvectors of the matrix A.

$$A = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{vmatrix}.$$

Determine wether A is diagonalizable and in that case find the matrix C through which it diagonalizes. Verify that  $C^{-1}AC$  is the desired diagonal matrix.