Civil Engineering 2 Mathematics Autumn 2011

Exercise Sheet 7

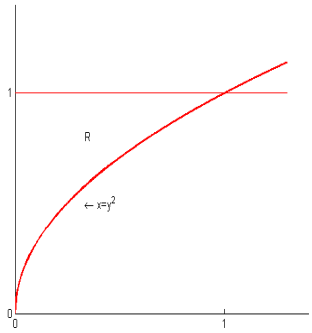
1. In each of the following cases (a) sketch the region of integration; (b) evaluate the integral; (c) write down the integral with the order of integration reversed; (d) evaluate the integral again (if possible) and compare with (b):

(i) for $a > 0$, $\int_0^a \left(\int_0^{a-x} dy \right) dx$; (ii) for $a > 0$, $\int_0^a \left(\int_0^x (x^2 + y^2)\, dy \right) dx$;
(iii) $\int_0^1 \left(\int_x^{\sqrt{x}} xy^2 dy \right) dx$; (iv) $\int_0^1 \left(\int_0^x e^{-x^2}\, dy \right) dx$.

2. Evaluate the double integral

$$\int\int_B \frac{y}{(1+x)(1+y^2)} dx\, dy$$

on the region R represented by



In the figure above the horizontal axis is the x-axis.

3. Use the transformation $u = x - y$, $v = x + y$ to evaluate the double integral

$$I = \iint_R (x+y)^2 \cos(x^2 - y^2)\, dxdy$$

where R is the region in the $xy-$plane enclosed by the lines $y = 0$, $x = 0$ and $y = 1 - x$.

4. Using the change of variables of exercise 3, evaluate the integral

$$\iint_R (x^2 + y^2)\, dxdy$$

over the interior of the square bounded by $y = \pm x$ and $y = \pm(x - 2)$.

5. Evaluate the integral

$$\int\int_R (x^4 y)\, dxdy,$$

where R is the circular disc $x^2 + y^2 \leq 1$ (use polar coordinates).

6. (A little harder...) Evaluate the integral $\int\int_D 2dxdy$ over the region D enclosed by the line $y = 0$ and the curves $y = x^2$ and $y = (x-2)^2$. Evaluate it both by integrating first w.r. to x and by integrating first w.r. to y .

Answers

1. (i) $a^2/2$, (ii) $a^4/3$, (iii) $1/35$, (iv) $(1 - e^{-1})/2$; part (d) is not possible for (iv);
2. $\frac{1}{4}\log^2(2)$ **3.** $(1 - \cos(1))/2$; **4**. $8/3$; **5.** 0; **6.** $\frac{4}{3}$.