

Civil Engineering 2 Mathematics Autumn 2011

Sketch solutions of Revision sheet

1. The integrating factor is

$$\exp\left(\int \frac{5}{x} dx\right) = e^{5 \log x} = x^5.$$

Multiplying both sides of the ODE by x^5 gives

$$(x^5 y)' = x^3 \quad \Rightarrow \quad x^5 y = \frac{x^4}{4} + C$$

hence the result.

3. Separate the variables

$$\frac{dy}{y} = dx$$

and then integrate both sides using the given boundary conditions

$$\int_1^y \frac{dy}{y} = \int_0^x dx \quad \Rightarrow \quad \log y = x$$

(I do realize that the notation in the line above can be slightly misleading)

4. Use the following

$$\sin(nx) = \left(-\frac{\cos(nx)}{n} \right)'.$$

6. Multiplying and dividing by $\cos x$ gives an indeterminate form of the type " $\frac{0}{0}$ ", hence you can apply de L'Hôpital's theorem.

7. Eigenvalues $\lambda = 0, 2, 3$. Eigenvectors $(-1, 2, 1), (1, 0, 1), (-1, -1, 1)$. The matrix C that has these eigenvectors as columns is the diagonalizing matrix.