

Example of the Widrow-Hoff Rule

Training a McCulloch-Pitts Unit to produce the Logical AND of its inputs

Initial conditions :

$$W_1 = 0.2$$

$$W_2 = -0.5$$

$$\text{Threshold} = 0.1$$

$$\text{Firing Margin} = 0.1$$

$$\text{Learning Rate} = 0.5$$

Step	Input (X_1, X_2)	Desired Output	Sum of $X_i \cdot W_i$	Error Size	50% of Error	New W_1	New W_2	New Threshold
1	(0,0)	0	0.0000	0.0000	0.0000	0.2000	-0.5000	0.1000
2	(0,1)	0	-0.5000	0.0000	0.0000	0.2000	-0.5000	0.1000
3	(1,0)	0	0.2000	0.1000	0.0500	0.1500	-0.5000	0.1500
4	(1,1)	1	-0.3500	0.6000	0.3000	0.4500	-0.2000	-0.1500
5	(0,0)	0	0.0000	0.1500	0.0750	0.4500	-0.2000	-0.0750
6	(0,1)	0	-0.2000	0.0000	0.0000	0.4500	-0.2000	-0.0750
7	(1,0)	0	0.4500	0.5250	0.2625	0.1875	-0.2000	0.1875
8	(1,1)	1	-0.0125	0.3000	0.1500	0.3375	-0.0500	0.0375
9	(0,0)	0	0.0000	0.0000	0.0000	0.3375	-0.0500	0.0375
10	(0,1)	0	-0.0500	0.0000	0.0000	0.3375	-0.0500	0.0375
11	(1,0)	0	0.3375	0.3000	0.15000	0.1875	-0.0500	0.1875
12	(1,1)	1	0.1375	0.1500	0.0750	0.2625	0.0250	0.1125
13	(0,0)	0	0.0000	0.0000	0.0000	0.2625	0.0250	0.1125
14	(0,1)	0	0.0250	0.0000	0.0000	0.2625	0.0250	0.1125
15	(1,0)	0	0.2625	0.1500	0.0750	0.1875	0.0250	0.1875
16	(1,1)	1	0.2125	0.0000	0.0000	0.1875	0.0250	0.1875
17	(0,0)	0	0.0000	0.0000	0.0000	0.1875	0.0250	0.1875
18	(0,1)	0	0.0250	0.0000	0.0000	0.1875	0.0250	0.1875
19	(1,0)	0	0.1875	0.0000	0.0000	0.1875	0.0250	0.1875

Notes:

Training ceases when the error for each input pattern has decreased to an acceptable level. Steps 16-19 above indicate that a zero error has been achieved for each input pattern so the training process can stop at Step 19 and the AND function was actually learnt in 15 steps or 4 "epochs". An **epoch** constitutes presentation of each input pattern once.

At step 4 the actual output was not sufficient to exceed the threshold and so the unit does not fire when it should. We cannot simply use the threshold value minus the actual value to determine the error because achieving the threshold is not sufficient to fire the unit - the threshold must be actually exceeded to fire. We therefore use an arbitrary **Firing Margin**, slightly above the threshold, as our target value in these circumstances.