Time's arrow and Occam's razor (incomplete draft)

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1 Introduction

Although the microscopic laws of dynamics are symmetric under time reversal, many phenomena in Nature do not share this symmetry: they are asymmetric under time reversal. Among these are : our subjective sense of the flow of time, the fact that we remember the past and not the future, the fact that we feel we can affect the future but not the past, the irreversibility of thermodynamic processes as codified in the seond law of thermodynamics, the fact that spherical waves (or waves on the surface of a pond) spread out rather than contracting, the use of retarded rather than advanced potentials in electromagnetic theory, the expansion of the Universe, and the fact that black holes absorb matter but that there are apparently no white holes spewing out matter. Some of these asymmetries are related; for example the spreading out of electromagnetic waves is an immediate consequence of the use of retarded potentials, and (it can be argued) our feeling that we can alter the future but not the past is a consequence of the fact that we remember (and therefore know) the past but not the future.

In this note it will be argued that most of the asymmetries listed above are related, and the relationship will be expressed by deriving the time direction of these phenomena from a single hypothesis, to be called here the 'initial independence hypothesis' (IIH). One could say that the IIH is intended to provide an explanation of the time direction of the various phenomena. Of course, there is never any final explanation of anything, since the explanation itself then has to be explained in terms of something else, but if a number of apparently unrelated time asymmetries can be derived from the IIH then there is a sense in which it can be held to explain them. In a similar way Newton's law of gravitation, by showing that the fall of an apple and the orbit of the moon could both be derived from a single hypothesis, provided an explanation of both even though it left the origin and nature of gravitation itself unexplained.

The initial independence hypothesis will be formulated in probabilistic language. One may wish to interpret such language as saying that we are thinking of our Universe as being selected at random from an ensemble of universes. It is sometimes argued that one should not use probability in talking about such matters, because the Universe is a unique object; however the view taken here is that our universe can be thought of as a typical member of such an ensemble, in the sense that any property that is true with probability 1 in such an ensemble is surely a property of our own Universe. (In this respect we are following a similar philosophy to that of [2], a paper in which the use of probabilities that are not very close to either 0 or 1 is avoided altogether, and which includes the definition ' "typical" behaviour is that which occurs with large probability with respect to the given initial ensemble'. However we shall also be considering here probabilities that are not close to 0 or 1, in order to be able to describe correlations between different systems and different parts of space.)

It may be asked, how can we know the probabilities in this ensemble of universes (which is in any case only a mathematical construction, not an assumed reality), if we only have one of them to observe. The answer is that in general we cannot know them, but that we can under the conditions where we (as inhabitants of the Universe under consideration) normally use probability theory. Consider a statistical experiment (for example, tossing a coin) that is replicated a large number of times. As an idealization, suppose that it were replicated an infinite number of times. Then the law of large numbers, applied to the ensemble of universes, tells us that with probability 1 the limiting frequency of heads in the sequence of outcomes obtained in our Universe is equal to the probability of heads at a single throw. So, by measuring (or, rather, estimating) this limiting relative frequency, we can find out the probability of heads in the hypothetical ensemble of universes. (An alternative way of discussing this point would involve the assumption that the probability distribution over universes is ergodic under spatial translations, so that the probability of any event can be measured in our own universe by means of a space ensemble, as advocated in [3], but this assumption does not appear to be essential.)

The approach used here is based closely on ideas in [3]. However, that paper was influenced by the now defunct steady-state cosmological model, and was confined to classical mechanics. The present paper is compatible with 'big bang' cosmological models, and also with the use of quantum mechanics.

2 A classical model Universe

We represent the Universe in a space-time manifold, with a special space-like surface S_0 on which the IIH will be specified. The special surface, which is generally thought of as the time when the Universe came into existence, may be singular in the sense that the space-like distance between any two of its points is zero; such a singularity would correspond to the 'big bang'.

Given any space-like surface S we can, if we are using classical mechanics, specify the dynamical state of the Universe on that surface by giving the positions and velocities of all the particles and the values of all the field variables at every point on S. This dynamical state will be denoted by $\omega(S)$. The set of all possible dynamical states of S will be denoted by $\Omega(S)$. Given any two space-like surfaces S_1 and S_2, the laws of dynamics provide a one-to-one correspondence $C(S_1, S_2)$ between $\omega(S_1)$ and $\omega(S_2)$. The one-to-one character of this correspondence is a reflection of the deterministic character of the dynamical laws of classical mechanics.

Since we are regarding our universe as having been selected at random from an ensemble of universes, the possible dynamical states $\omega(S)$ for a given S have a probability distribution p over $\Omega(S)$. We denote the probability of a given event (set of states) E from $\Omega(S)$ by $p(E)$. Given any two space-like surfaces S_1 and S_2, their probability distributions p_1 and p_2 are related by $p_1(E_1) = p_2(E_2)$, where E_1 and E_2 are any two events in $\Omega(S_1)$ and $\Omega(S_2)$ that map one to the other under the one-to-one correspondence $C(S_1, S_2)$.

In classical mechanics, we formulate the IIH as an independence condition relating to the special space-like surface S_0 :

Let R_1, R_2 be any two disjoint subregions of S_0, and let E_1, E_2 be any two events in $\Omega(S_0)$ such that the answer to the question whether or not the state ω lies in E_i depends only on the dynamical state of R_i, that is on $\omega \cap R_i$.

The IIH asserts that all such sets of events are statistically independent:

$$p(E_1 \cap E_2) = p(E_1)p(E_2) \quad \text{if} \quad R_1 \cap R_2 = \oslash \tag{1}$$

The implications of this hypothesis will be explored in section 3. It may be that other hypotheses about the initial correlations could also be used, but this one has the virtue of simplicity and is therefore compatible with the principle of Occam's razor.

IIH had its origins in Reichenbach's [5] 'principle of the common cause', (a similar point was emphasized by Feynman [1]) that if two events in different systems are correlated, this correlation must have been produced by some common cause in their past. Dynamically, a cause arises from an interaction, so Reichenbach's principle tells us that if two (spatially separated) systems are correlated they must have interacted in the past, either with each other or with some third system. At the initial time, there had been no past, and therefore no past interactions to produce correlations, so that it seems reasonable to assume that there were then no correlations between spatially separated systems.

A possible way of thinking about the IIH is to imagine that, prior to the time of S_0, the Universe was an ideal gas in thermal equilibrium, so that its spatially separated parts were independent of one another, and that at the time of S_0 the interactions were switched on; the subsequent history of the universe is then the history of the effects produced by these interactions. Another possibility is to use the hypothesis about the initial time used in [2], where the initial probability distribution is taken to be one for which the phase-space density is uniform over some appropriate macrostate: if this initial macrostate is a direct product of macrostates defined on some set of elementary regions, and if the interactions between particles in different elementary regions are neglected, then we arrive at (1), at least in the case where the regions R_i used there are both unions of some of the elementary regions used in defining the initial macrostate.

3 Some implications of the IIH

The time direction of thermodynamics, as codified in the law of increasing entropy, is derived in statistical mechanics from time-asymmetric assumptions about the initial conditions on an ensemble. Such derivations are given in many places, for example [4]. The time-asymmetric assumption, as formulated in [4], is that if a succession of observations are made on some system then the microscopic probability distribution used at any moment to describe the system depends only on what has happened in the past, not on what is going to happen in the future. In particular, if we prepare a system in some way, then its initial microscopic probability distribution depends only on the method of preparation.

To see how this time arrow is related to IIH, consider the idealized situation in which a system is prepared in the following way: first finding some far-away matter that has never interacted at all with the experimenter's part of the Universe, then making some observations on that piece of matter (the system to be investigated) to see whether or not it is suitable, then doing some mechanical operations on it which do not depend on its microscopic state (for example compressing it with a piston). Before the first step the matter is (by IIH) microscopically uncorrelated with (i.e. statistically independent of) the experimenter's part of the Universe. After the first step (finding and selection) it is still microscopically uncorrelated with the experimenter's part of the Universe, in the probability measure conditional upon the selection criteria used by the experimenter. After the second step the system and the experimenter are still uncorrelated in this conditional ensemble, since the system's dynamical evolution is independent of what state the experimenter and his part of the the Universe are in. So at the moment the experiment begins, the system under study is (conditionally) uncorrelated with the experimenter, and its probability distribution conditional upon the state of the experimenter is therefore independent of the state of the experimenter, and in particular independent of whatever experiments the experimenter is proposing to do on it. It is this (conditional) probability distribution that is the initial distribution for the system as used in all statistical mechanics calculations, and from which such things as the law of increasing entropy can be deduced (subject to the usual difficulties about the ergodicity of the system).

Most of the other asymmetric processes listed in the Introduction can have their time direction related to IIH in a similar way, but that is enough for the time being.

References

[1] R P Feynman, 'The character of physical law', chapter 5 MIT Press, Cambridge, Mass, 1967

[2] J L Lebowitz, 'Time's arrow and Boltzmann's entropy', Rutgers University preprint, 1992.

[3] O Penrose and I C Percival, 'The direction of time', Proc Phys Soc circa 1960

[4] O Penrose, 'Foundations of statistical mechanics', Rep Prog Phys circa 1978

[5] H Reichenbach, 'The direction of time', circa 1950.