Correction to ‘Statistical mechanics of nonlinear elasticity’ (Markov Processes and Related Fields 8, 351-364 (2002))

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In Theorem 1 part (ii) it is stated that \( f(\alpha a, \beta b) \) is a convex function of the positive real numbers \( \alpha, \beta \). I believe this to be true, but the proof given is erroneous; when corrected, it shows only that \( f(\alpha a, \beta b) \) is convex in either one of the variables \( \alpha, \beta \) when the other is held constant, but not in both variables together. (For example the function \( \alpha^2 - 3\alpha \beta + \beta^2 \) is convex in \( \alpha \) and \( \beta \) separately, but not in both together.)

In the proof given for Theorem 1, eqn (5.6) should read

\[
\frac{\log Z_{el}(a, b; 2^{k+1}, 2^{k+1})}{4^{k+1}} \geq \frac{1}{4} \left( \frac{\log Z_{el}(a', b', 2^k, 2^k)}{4^k} + \frac{\log Z_{el}(a', b'', 2^k, 2^k)}{4^k} + \frac{\log Z_{el}(a'', b', 2^k, 2^k)}{4^k} + \frac{\log Z_{el}(a'', b'', 2^k, 2^k)}{4^k} \right)
\]

and eqn (5.7) should read

\[
f_{el} \left( \frac{1}{2} [a' + a''], \frac{1}{2} [b' + b''] \right) \leq \frac{1}{4} [f_{el}(a', b') + f_{el}(a', b'') + f_{el}(a'', b') + f_{el}(a'', b'')]
\]

in which \( a' \) must be parallel to \( a'' \) and \( b' \) parallel to \( b'' \). By setting \( b'' = b' \) we see that \( f \) is convex in \( \alpha \) at fixed \( \beta \), and similarly it is convex in \( \beta \) at fixed \( \alpha \); but the corrected formula (5.7) does not show that \( f \) is convex in both variables together.