Advanced PDE II: Hyperbolic PDE

(MIGSAA Ph.D. course & SMSTC advanced course)

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Course websites: https://www.smstc.ac.uk/supplementary_modules/advanced_pde_2

Lecture: 9:00-11:00 Wednesday (10 Jan. 2018 - 14 Mar. 2018)
Room 1.07 (on the ground floor) in ICMS, 15 South College Street

Assessment: 2 homework assignments to be submitted in weeks 7 and 10. (The problem sheets will be handed out in weeks 6 and 9.)

This course is dedicated to the study of hyperbolic PDEs in Sobolev spaces. We will mainly focus on nonlinear wave equations. The course will begin with some classical material: a classification of PDEs, d'Alembert's formula and consequences, a review of the Fourier transform, solving the linear wave equation using the Fourier transform.

Next, we will discuss the local well-posedness of the cubic Klein-Gordon equation in Sobolev spaces $H^s(\mathbb{R}^d)$, $s > \frac{d}{2}$. This will require the use of Sobolev embeddings, energy estimates and of the Picard iteration scheme.

We will then turn to briefly discussing the Strichartz estimates for the wave equation. We will use these estimates together with energy conservation to obtain local and global well-posedness of energy-subcritical semilinear wave equations in the Sobolev space $H^1(\mathbb{R}^d)$. We will then study the global well-posedness in $H^1(\mathbb{R}^3)$ of the much more delicate *energy-critical* defocusing quintic nonlinear wave equation. In addition to Strichartz estimates, this will require us to use monotonicity formulae.

The course will then turn to Sobolev space methods in the context of quasilinear wave equations. These include energy estimates, Klainerman-Sobolev inequality, the vector field method, and well-posedness of quasilinear wave equations using Sobolev estimates.

References

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