University of Sheffield

School of Mathematics and Statistics

The emergence of spatial segregation patterns from animal movements and interactions

Jonathan Potts - j.potts@shef.ac.uk

Research project 2

Inter-population taxis: two's company, three's chaos

In the paper by Potts and Lewis (2016), the authors modelled two populations who avoid one another via a spatial memory process. In dimensionless co-ordinates, the two populations, u_1 and u_2, obey the following diffusion-taxis equations (for $i \in \{1, 2\}$)

$$\frac{\partial u_i}{\partial t} = \frac{\partial^2 u_i}{\partial x^2} + \gamma \frac{\partial}{\partial x}\left[u_i \frac{\partial \bar{k}_i}{\partial x}\right], \tag{1}$$

where $\gamma > 0$ denotes the strength of avoidance. Here, $k_i(x, t)$ is the *conflict zone* of population i, which has its own dynamics as follows

$$a\frac{\partial k_i}{\partial t} = u_1 u_2(1 - k_i) - k_i(m + bu_i). \tag{2}$$

More details of the model and the specific parameter values are given in Potts and Lewis (2016). The task of this project is to investigate what happens when (a) there are more than two populations in the mix, (b) the mechanism of interaction is not necessarily one of mutual avoidance, but may be attractive and/or asymmetric. A model for this is given by

$$\frac{\partial u_i}{\partial t} = \frac{\partial^2 u_i}{\partial x^2} + \frac{\partial}{\partial x}\left[u_i \sum_{j \neq i} \gamma_{ij} \frac{\partial \bar{k}_{ij}}{\partial x}\right], \tag{3}$$

$$a\frac{\partial k_{ij}}{\partial t} = u_i u_j(1 - k_{ij}) - k_{ij}(m + bu_i), \tag{4}$$

where $i \in \{1, \dots, N\}$. Here k_{ij} can be thought of as the cognitive map of the presence of population j in the minds of those in population i. Depending on the relationship between i and j, γ_{ij} may be positive or negative.

The specific tasks are as follows.

1. Make sure you have read and understood the paper by Potts and Lewis (2016).

2. For the case $N = 2$, consider the sign of γ_{12} and γ_{21}. What does the situation $\gamma_{12}, \gamma_{21} > 0$ correspond to, biologically? What about $\gamma_{12}, \gamma_{21} < 0$ and $\gamma_{12} < 0 < \gamma_{21}$?

3. For the case $N = 3$, use linear pattern formation analysis to examine conditions under which you might see stationary pattern formation (Turing bifurcations) or oscillatory patterns (Turing-Hopf bifurcations). The latter occur when the dominant eigenvalue is non-real for some wavenumber, and at the same time the real part becomes greater than zero as we pass through the bifurcation point. [Hint: it may help to examine the case $a = 0$ first, which allows the system to be written in terms of just three equations. To make things even easier, you also can set $m = 0$ to begin with.]

4. Consider the case where $N = 3$ and also $\gamma_{ij} = \gamma_{ji}$ for all i, j. What can you say about the possible patterns that form in this case?

5. Examine the case $N = 3$ using numerical simulations in the pattern formation regimes. Have an explore and see what you find!

References

Potts, J. R. and M. A. Lewis. 2016. How memory of direct animal interactions can lead to territorial pattern formation. J Roy Soc Interface, **13**:20160059.