1. (a) Find the general solution of the linear differential equation

$$\frac{dy}{dx} + y = x^2. \qquad (*)$$

(*b*) For the equation $(*)$ determine the points in the xy-plane where

$$(i)\ \frac{dy}{dx} < 0, \quad (ii)\ \frac{dy}{dx} = 0 \quad \text{and} \quad (iii)\ \frac{dy}{dx} > 0.$$

Hence draw the direction field.

(*c*) Find the solution of $(*)$ satisfying $y(0) = 2$ and sketch it in the diagram you made for (*b*).

[12 marks]

2. Show that

$$xe^{xy}\frac{dy}{dx} + 2x + ye^{xy} = 0$$

is an exact differential equation and find its general solution.

[10 marks]

3. Find the general solutions of the following second order equations

$(i)\ y'' + 4y' + 13y = 0;$

$$(ii)\ \frac{d^2y}{dx^2} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2 = 0.$$

[17 marks]

4. By using the method of variation of parameters give the general solution of the following differential equation

$$y'' + 4y = \frac{1}{\cos(2x)}.$$

[13 marks]

continued overleaf

5. A particle of mass 1 kg is attached to a spring with spring constant 9 N/m and is acted on by an external force $\cos(3t)$ N in the downward direction at time t. Ignoring air resistance, write down the differential equation describing the motion of the particle and find its general solution. If the particle is initially in the equilibrium position and at rest show that its displacement at time $t = \dfrac{3\pi}{2}$ sec is $\dfrac{\pi}{4}$ m. What is the total force acting on the particle at that moment?

[11 marks]

6. By using the method of Frobenius find a nonzero solution of the equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0.$$

Is the solution analytic at $x = 0$?

[12 marks]

7. By using the method of Laplace transforms solve

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = e^{-3t}; \quad y(0) = 0, \quad \frac{dy}{dt}(0) = 1.$$

[8 marks]

8. Solve for $x(t)$ using the convolution theorem

$$x(t) + \int_0^t x(t-\tau)e^{\tau}d\tau = t.$$

[7 marks]

9. Find a Green's function for the boundary value problem

$$y''(x) = f(x); \quad y(0) + y'(0) = 0, \;\; y(1) - y'(1) = 0.$$

Use this result to solve $y''(x) = x^2$ subject to the same boundary conditions.

[10 marks]

END OF PAPER