1. Find the solution of the following differential equation

$$\frac{dx}{dt} = (1 - x^2)t$$

satisfying the initial condition $x(0) = 3$.

[8 marks]

2. Find the general solutions of the following equations

(i) $\dfrac{dx}{dt} = x + t,$

(ii) $\dfrac{dy}{dx} = \dfrac{x}{y} + \dfrac{y}{x},$

(iii) $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + y = 0.$

[26 marks]

3. Using the variation of parameters method find the general solution of the following equation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = \frac{\mathrm{e}^t}{t}.$$

[13 marks]

4. A mass $m = 1$kg is attached to a spring. It is known that the system is in resonance when the external force $F(t) = 2\sin 4t$ Newtons is applied. Neglecting resistance to the motion, find the spring constant k. Find the general solution of the equation of motion.

[10 marks]

5. Using the method of power series find two independent solutions of the following equation

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0.$$

[13 marks]

continued overleaf

6. Use the Laplace transform method to solve the following initial value problem

$$\frac{d^2x}{dt^2} + 4x = w(t), \qquad x(0) = 0,\ x'(0) = 0,$$

where

$$w(t) = \begin{cases} 4 & \text{for} \quad t \geq \pi \\ 0 & \text{otherwise.} \end{cases}$$

[12 marks]

7. Solve for $x(t)$ using the convolution theorem

$$x(t) = 1 + \int_0^t x(\tau)\sin(t-\tau)d\tau.$$

[8 marks]

8. Find the Green's function for the boundary value problem

$$\frac{d^2y}{dx^2} + y = f(x), \quad y(0) = 0,\ y(\pi) - y'(\pi) = 0,\ \ 0 \leq x \leq \pi,$$

and hence write down a solution for the problem.

[10 marks]

END OF PAPER