1. Find the general solution of the following differential equations

$$(2xy + y^2)\frac{dy}{dx} + y^2 - x^2 = 0$$

$$\frac{dy}{dx} + \frac{y}{x} = y^2$$

[12 marks]

2. Find all constant solutions of the equation

$$\frac{dy}{dx} = y^3 - 2y$$

Draw the direction field for the equation and sketch the graph of the solution of the equation satisfying the initial condition $y(1) = 1$.

[6 marks]

3. Solve the following initial value problems

(i) $$\frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 = 1, \qquad y(0) = 1, \quad y'(0) = 0,$$

(ii) $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4x^2 \quad y(0) = 0, \quad y'(0) = 0.$$

[18 marks]

4. Suppose that λ is a real parameter. Considering separately the cases where $\lambda > 0$, $\lambda = 0$, and $\lambda < 0$, find a fundamental set of solutions $\{y_1, y_2\}$ of the following differential equation

$$\frac{d^2y}{dx^2} + \lambda y = 0.$$

In each case calculate the Wronskian $W(y_1, y_2)(x)$ and show that the Abel's Theorem is satisfied.

[10 marks]

5. It is found experimentally that a 1kg mass hanging in equilibrium stretches a spring 0.2m. If the mass is pulled down an additional 0.25m and then released from rest with zero initial velocity, find the subsequent motion. Find the frequency, the period and the amplitude of the motion. Neglect the

air resistance and take $g = 9.8$m/s.

[**10 marks**]

6. Use the method of Frobenius to find the general solution of the following differential equation
$$2x\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0.$$

[**12 marks**]

7. Using the method of Laplace transform solve the following initial value problems

(a)
$$\frac{d^2x}{dt^2} + 16x = \cos 4t, \quad x(0) = 0, x'(0) = 1.$$

(b)
$$\frac{dx}{dt} + 2x = f(t), \quad x(0) = 0,$$

where
$$f(t) = \begin{cases} 2 \text{ for } 0 < t < 2 \\ t \text{ for } t \geq 2 \end{cases}$$

[**21 marks**]

8. Solve the following boundary value problem using an appropriate Green's function
$$\frac{d^2y}{dx^2} - y = f(x), \quad y(0) = 0, \ y(1) = 0.$$

[**11 marks**]

END OF PAPER