Problem Sheet 12

Module F13YT2/YF3 **2006**

1. Discuss the type and stability of the equilibrium point $(0, 0)$ for each of the following systems:

(i) $\dfrac{dx}{dt} = x - y;$ $\dfrac{dy}{dt} = x + y - 2xy.$

(ii) $\dfrac{dx}{dt} = x - xy + y^2;$ $\dfrac{dy}{dt} = x - 2y - x^3.$

2. Determine all the equilibrium points of each of the following systems and determine their nature.

(i) $\dfrac{dx}{dt} = 1 - xy;$ $\dfrac{dy}{dt} = (x - 1)y.$

(ii) $\dfrac{dx}{dt} = x + y^2;$ $\dfrac{dy}{dt} = x + y.$

3. Find all equilibrium points, determine their nature and sketch the phase plane for the second order differential equation

$$\ddot{x} + x - x^3 = 0.$$

4. For the following systems describing the interaction of two competing species find all the equilibrium points and determine the nature of each. Hence sketch the phase planes for the systems.

(a) $\dfrac{dx}{dt} = x(1 - x - y);$ $\dfrac{dy}{dt} = y(1.5 - y - x).$

(b) $\dfrac{dx}{dt} = x(1.5 - x - 0.5y);$ $\dfrac{dy}{dt} = y(2 - y - 0.75x).$

5. Draw the phase plane for the predator-prey system

$$\frac{dx}{dt} = x(1.5 - 0.5y); \qquad \frac{dy}{dt} = y(-0.5 + x)$$

and find the equations of the trajectories.

6. For the following system which describes the interaction of a prey (x) and predator (y) species find all equilibrium points and determine the nature of each.

$$\frac{dx}{dt} = x(1 - 0.5x - 0.5y); \qquad \frac{dy}{dt} = y(-0.25 + .5x).$$

Hence sketch the phase plane.