# Prob Sheet 2: Linear systems of ODES 1

**Module F13YT2**

**1.** Express the following equations as systems of first order equations

$$(i) \quad \ddot{x} + 4\dot{x} - 3x = \sin(2x) \qquad (ii) \quad \frac{d^4u}{dt^4} + u = 0$$

$$(iii) \quad \frac{d^2y}{dx^2} + y^2 = 2z + x, \quad \frac{d^2z}{dx^2} + y\frac{dz}{dx} + 2\frac{dy}{dx} = z^2 + y.$$

**2.** By making use of the generalisation of Picard's theorem, show that $x_1(t) = t, x_2(t) = t^2$ cannot be a solution of the initial value problem

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} a(t)x_1 + b(t)x_2 \\ x_1^2 - x_2 \end{pmatrix}, \qquad \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

if $a(t)$ and $b(t)$ are continuous functions.

**3.** Let $\boldsymbol{x}_0(t)$ be a particular solution of the inhomogeneous linear system

$$\dot{\boldsymbol{x}}(t) = A(t)\boldsymbol{x}(t) + \mathbf{b}(t). \tag{1}$$

Show that $\boldsymbol{x}$ is a solution of (1) if and only if $\boldsymbol{x}(t) = \boldsymbol{x}_0(t) + \boldsymbol{y}(t)$ where $\boldsymbol{y}(t)$ is a solution of

$$\dot{\boldsymbol{y}}(t) = A(t)\boldsymbol{y}(t).$$

**4.** Show that the following vectors are linearly independent

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Do they constitute a basis for $\mathbb{R}^3$? Could these be the eigenvectors of a real matrix with eigenvalues $1, 1+i, 1-i$?

**5.** Find the eigenvalues and eigenvectors of the matrices

$$A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & -1 \\ 2 & 0 \end{pmatrix}$$

What is $A^{-1}$?