Problem Sheet 4

Module F13YT2

1. A 2nd order linear ODE has a FSS $y^{(1)}(x) = \sin(2x)$, $y^{(2)}(x) = \sin(x)$. Find the Wronskian.

2. A 3rd order linear ODE has solutions $y^{(1)}(t) = e^t$, $y^{(2)}(t) = e^{-t}$ and $y^{(3)}(t) = e^{2t}$. Find the Wronskian and show that the solutions are linearly independent.

3. Use the method of variation of parameters to find a particular solution of the ODE $\dot{\boldsymbol{x}} = A\boldsymbol{x}+\boldsymbol{b}$, where

$$A \;=\; \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}, \quad \boldsymbol{b}(t) = e^t \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

4. Show that the method of variation of the parameters applied to the equation

$$\frac{d^2y}{dx^2} + \omega^2 y = f(x),$$

where $\omega > 0$ is a real constant, leads to the particular solution

$$y(x) = \frac{1}{\omega}\int_0^x f(t)\sin[\omega(x-t)]\,dt.$$

5. Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}.$$

6. Find the general solution of the following equations

$$\begin{aligned}
&(i) \quad \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0 \\
&(ii) \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0 \\
&(iii) \quad \frac{dy^3}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0 \\
&(iv) \quad \frac{dy^3}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0.
\end{aligned}$$