Problem Sheet 5

Module F13YT2

1. Three solutions of

$$y''(x) + a_1(x)y'(x) + a_0(x)y(x) = f(x) \qquad (1)$$

are $u_1(x) = x, \quad u_2(x) = x + e^x$ and $u_3(x) = 1 + x + e^x$
Determine the solution of (1) satisfying the initial conditions $y(0) = 1$ and $y'(0) = 3$.

2. Using the method of reduction of order, find the solution of the following initial value problems
$(i)\, y'' - 3y^2 = 0,\ y(0) = 2,\ y'(0) = 4;$
$(ii)\, y'y'' - x = 0,\ y(1) = 2,\ y'(1) = 1.$

3. Find the general solution of the equation

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y = 0.$$

4. Find particular solutions of
(a) $y'' + 4y' + 4y = x^2;$
(b) $y'' - 4y' + 3y = 2e^x;$
(c) $y'' + 2y' + 5y = \cos(2x);$
(d) $y'' + y = \sec(x).$

5. Find the general solution of the equation

$$y'' + 3y' + 2y = 5e^{-2t}$$

(i) using the method of undetermined coefficients,

(ii) using the method of variation of parameters.

6. Check that $y(x) = x$ satisfies the following equation, and hence find a fundamental set of solutions for it.

$$(1 + x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$