

Problem Sheet 6

Module F13YT2

1. Find the Laplace transforms of the following functions of t :

$$(t+1)^2; \quad \sin^2(t); \quad t \sin(t); \quad e^t \sin(t); \quad te^{-t}.$$

2. Find the inverse Laplace transforms of

$$\frac{s+1}{s^2+2s+2}; \quad \frac{1}{s^2-3s+2}; \quad \frac{s}{(s-1)^2(s^2+4)}; \quad \frac{s}{s^2+4s+8}.$$

3. By using Laplace transforms solve the following initial value problem:

$$y'' - 2y' + 2y = \cos(t); \quad y(0) = 1, \quad y'(0) = 0.$$

4. Find the Laplace transforms of the following functions:

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \frac{\pi}{2} \\ \sin t & \text{if } t \geq \frac{\pi}{2} \end{cases} \quad g(t) = \begin{cases} 1 & \text{if } 0 \leq t < 2 \\ 2 & \text{if } t \geq 2. \end{cases}$$

5. Solve the initial value problem

$$y'' + y = f(t); \quad y(0) = 0, \quad y'(0) = 0$$

where

$$f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 1 \\ 0 & \text{if } t \geq 1. \end{cases}$$

6. Solve the following initial value problems by using Laplace transforms:

(i) $y'' + y' - 6y = 2\delta(t-1); \quad y(0) = 0, \quad y'(0) = 0.$

(ii) $y'' + 4y' + 4y = u_3(t); \quad y(0) = 2, \quad y'(0) = 0.$

7. Solve the following systems of equations by taking Laplace transforms:

(i) $\dot{x}_1 = x_1 - x_2; \quad \dot{x}_2 = 5x_1 - 3x_2; \quad x_1(0) = 1; \quad x_2(0) = 2.$

(ii) $\dot{x}_1 = x_1 + x_2 + e^{-2t}; \quad \dot{x}_2 = 4x_1 - 2x_2 + -2e^t; \quad x_1(0) = 0; \quad x_2(0) = 1.$

(iii) $\dot{x}_1 = x_2 + f(t); \quad \dot{x}_2 = -x_1 + f(t); \quad x_1(0) = 0; \quad x_2(0) = 0.$