Solutions 2

Module F13YT2

1.

$$(i)\quad \begin{pmatrix}\dot x_1\\ \dot x_2\end{pmatrix} = \begin{pmatrix} x_2\\ 3x_1-4x_2+\sin(2x_1)\end{pmatrix},\quad \text{where } x_1 := x$$

$$(ii)\quad \begin{pmatrix}\dot u_1\\ \dot u_2\\ \dot u_3\\ \dot u_4\end{pmatrix} = \begin{pmatrix} u_2\\ u_3\\ u_4\\ -u_1\end{pmatrix},\quad \text{where } u_1 := u$$

$$(iii)\quad \begin{pmatrix}\frac{dw_1}{dx}\\ \frac{dw_2}{dx}\\ \frac{dw_3}{dx}\\ \frac{dw_4}{dx}\end{pmatrix} = \begin{pmatrix} w_2\\ -w_1^2+w_3+x\\ w_4\\ w_1-2w_2+w_3^2-w_1w_4\end{pmatrix},\quad \text{where } w_1 := y,\ \ w_3 = z.$$

2. Picard's theorem tells us that there is a unique solution to this initial value proble. There is one soln $x_1(t) = x_2(t) = 0$, so there can not be another solution $x_1(t) = t^2$, $x_2(t) = t^2$.

3. The statement that $\boldsymbol{x}_0(t)$ is a particular solution means that

$$\dot{\boldsymbol{x}}_0(t) = A(t)\boldsymbol{x}_0(t) + \mathbf{b}(t). \tag{1}$$

If $\boldsymbol{x}(t) = \boldsymbol{x}_0(t) + \boldsymbol{y}(t)$ is a solution then

$$\dot{\boldsymbol{x}}_0(t) + \dot{\boldsymbol{y}}(t) = A(t)(\boldsymbol{x}_0(t) + \boldsymbol{y}(t)) + \mathbf{b}(t).$$

It then follows from (1) that

$$\dot{\boldsymbol{y}}(t) = A(t)\boldsymbol{y}(t).$$

On the other hand if $\dot{\boldsymbol{y}}(t) = A(t)\boldsymbol{y}(t)$, then it follows that

$$\dot{\boldsymbol{x}}_0(t) + \dot{\boldsymbol{y}}(t) = A(t)(\boldsymbol{x}_0(t) + \boldsymbol{y}(t)) + \mathbf{b}(t).$$

4. The 3 vectors are linearly independent since

$$\begin{vmatrix} 2 & 1 & 1\\ 2 & 0 & 1\\ 1 & 1 & 1\end{vmatrix} = 2.(0-1) - 1.(2-1) + 1.(2-0) = -1 \neq 0.$$

The two eigenvectors of a real matrix corresponding to $1 \pm i$ eigenvalues have to be complex conjugates of one another. Thus these 3 could not be eigenvectors.

5. The eigenvalues of A are 3 and -1, and the corresponding eigenvectors are

$$\begin{pmatrix}2\\1\end{pmatrix},\quad \begin{pmatrix}-2\\1\end{pmatrix}.$$

The eigenvalues of B are $-1+i$ and $-1-i$, and the corresponding eigenvectors are

$$\begin{pmatrix} -1+i \\ 2 \end{pmatrix}, \quad \begin{pmatrix} -1-i \\ 2 \end{pmatrix}.$$

A^{-1} is given by

$$-\frac{1}{3}\begin{pmatrix} 1 & -4 \\ -1 & 1 \end{pmatrix}.$$