Productive Corecursion in Logic Programming

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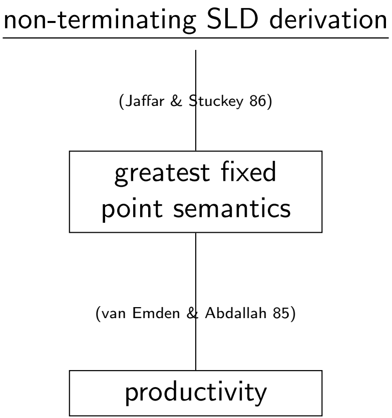
Outline

1. Motivation
   - Overview of motivation
   - Background knowledge for understanding motivation
   - Problem description
2. Productive Corecursion
   - Loop detection rule review
   - Productivity guarantee
3. Conclusion
4. Future Work & Implementation

non-terminating SLD derivation

(Jaffar & Stuckey 86)

greatest fixed point semantics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(Jaffar & Stuckey 86)
greatest fixed point semantics
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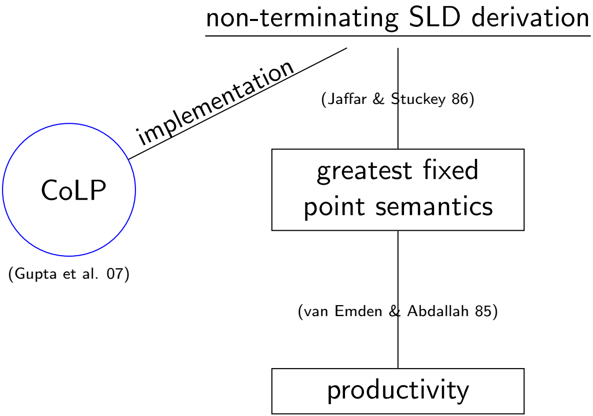
productivity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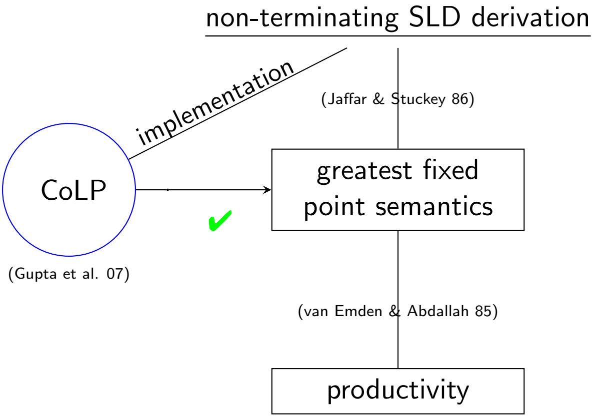
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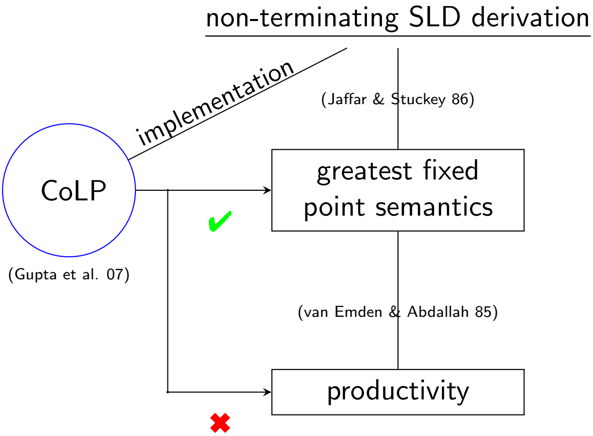

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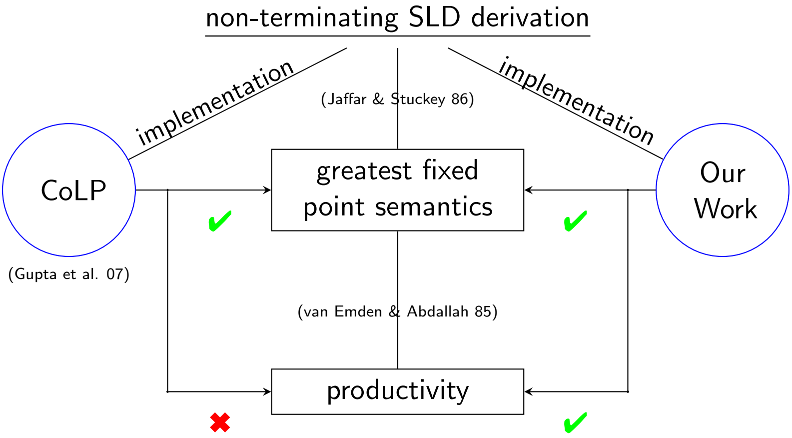

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Program:=Set of Definite clauses

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Formulae computed by non-terminating derivations are in greatest fixed points. (Jaffar & Stuckey 86; van Emden & Abdallah 85)

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Now consider finite implementation of non-terminating SLD derivations.

Since regular formulae have cyclic derivations, finding a cycle (loop) is suffice for knowing the whole derivation.

Definition (Gupta et al. 07)

CoLP = SLD resolution + loop detection rule.

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A goal succeeds if it unifies with its ancestor goal.

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Successful coLP derivations only compute formulae in greatest fixed points.

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$\downarrow$ $X_2 \mapsto s(X_3)$

G_2 nat(X_3)

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However, coLP does not take good care of productivity . . .

Assume some successful coLP derivation that computes an infinite formula.

Problem 1: *It is not guaranteed that there exists a corresponding non-terminating SLD derivation.*

e.g. For program p(f(X),X) ← p(X,X) and goal p(f(X),X), coLP computes p(f(f(. . .)),f(f(. . .))) but here is no non-terminating SLD derivation.

CoLP derivation

G_0 : p(f(X),X)

↓

G_1 : p(X,X)

↓ $X \mapsto f(f(\ldots))$ by unifying G_1 with G_0

G_2 : □

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Problem 2: *There exists a corresponding non-terminating SLD derivation but it computes a different formula.*

e.g. For program q(f(X),Y) ← q(X,h(Y)) and goal q(f(X),Y), coLP computes q(f(f(...)),h(h(...))) but the corresponding non-terminating SLD derivation computes q(f(f(...)), Y).

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SLD derivation (non-terminating)

$$
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 & \downarrow \\
G_1 : & \mathrm{q(X,h(Y))} \\
 & \downarrow \; X \mapsto f(X_2) \\
G_2 : & \mathrm{q}(X_2,\mathrm{h(h(Y))}) \\
 & \downarrow \; X_2 \mapsto f(X_3) \\
G_3 : & \mathrm{q}(X_3,\mathrm{h(h(h(Y)))}) \\
 & \vdots
\end{array}
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G_3 : q(X_3,h(h(h(Y))))

⋮

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Can we have an implementation of infinite SLD derivation, that is not only sound regarding greatest fixed points, but also sound regarding productivity?

Our answer is affirmative.

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Definition (Loop detection rule) (Gupta et al. 07)

A goal succeeds if it unifies with its ancestor goal.

We also characterized a class of logic programs whose non-terminating SLD derivations, if any, are guaranteed to be productive.

Definition (Rewriting for LP) (Komendantskaya et al. 15)

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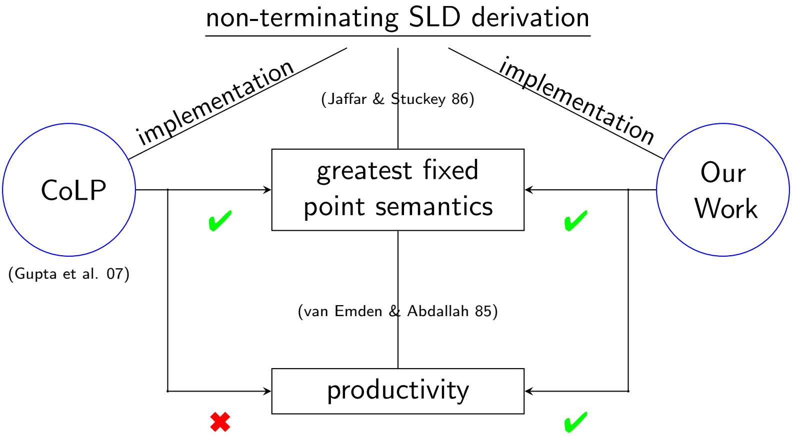
Theorem (our main result: Productivity Semi-decision)

Productivity is semi-decidable for programs characterized above, by SLD resolution combined with our loop detection rule.



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Streams of Fibonacci numbers, e.g. 1 1 2 3 5 8 ... or 10 4 14 18 32 ..., are defined by a corecursive clause that has an existential variable.
fibs(X,Y,[X|S]) $\leftarrow$ add(X,Y,Z), fibs(Y,Z,S).

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Implementation is available at
GitHub / coalp / Productive-Corecursion

Thanks!