Productive Corecursion in Logic Programming

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Outline

non-terminating SLD derivation

(Jaffar & Stuckey 86)

greatest fixed point semantics

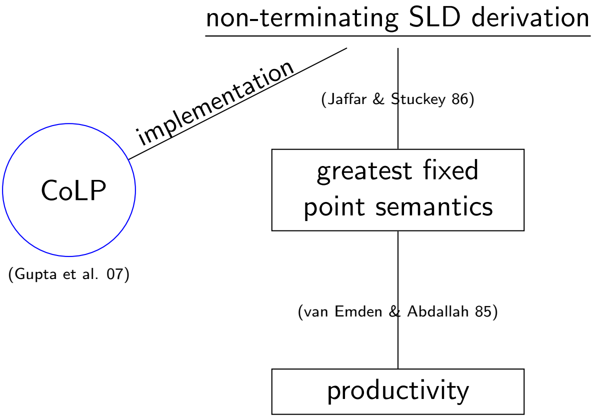
non-terminating SLD derivation

(Jaffar & Stuckey 86)

greatest fixed point semantics

(van Emden & Abdallah 85)

productivity


non-terminating SLD derivation
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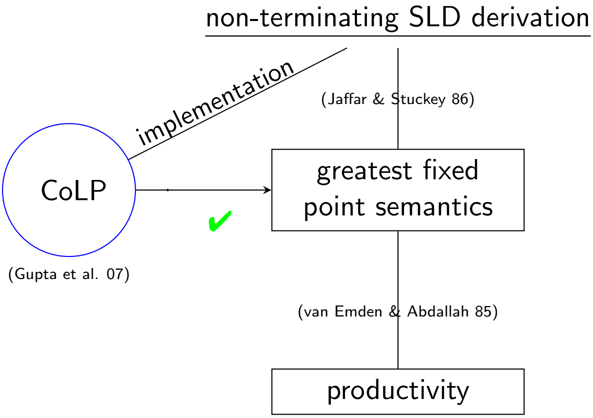
(Jaffar & Stuckey 86)
CoLP
greatest fixed point semantics
(Gupta et al. 07)
(van Emden & Abdallah 85)
productivity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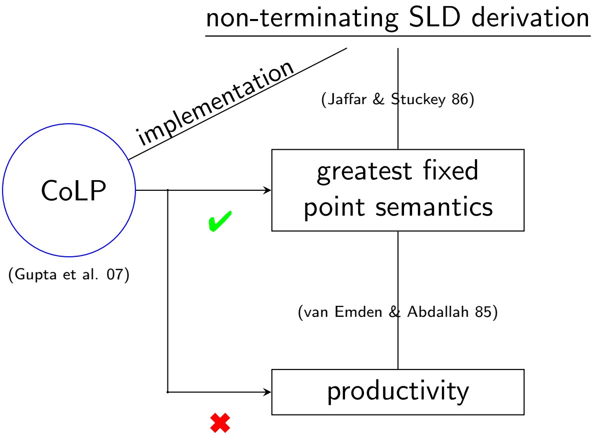
✔: sound



✔: sound

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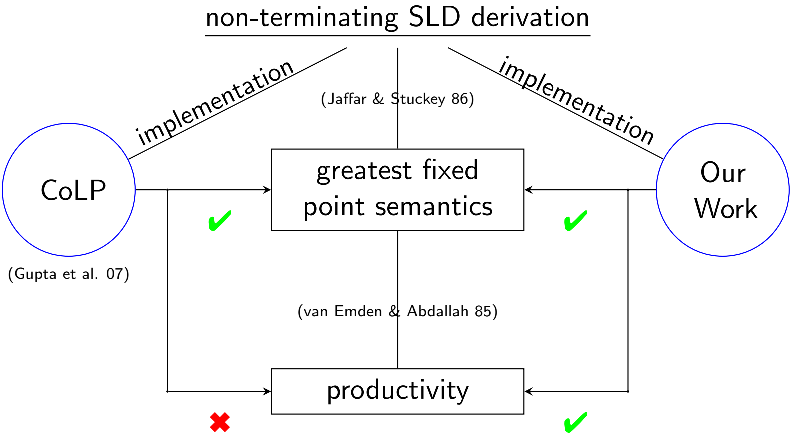

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Definition (Syntax of definite clause logic) (Lloyd 87)

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Program:=Set of Definite clauses

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The least fixed point is {nat(0), nat(s(0)), nat(s(s(0))), ...}.

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Formulae computed by non-terminating derivations are in greatest fixed points. (Jaffar & Stuckey 86; van Emden & Abdallah 85)

Definition (Productivity)
(LP: van Emden & Abdallah 86; Komendantskaya et al. 16;
FP: Sijtsma 89; Endrullis et al. 08)

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$$\begin{array}{c} \text{nat}(X) \\ \downarrow \; X \mapsto s(X_2) \\ \text{nat}(X_2) \\ \downarrow \; X_2 \mapsto s(X_3) \\ \text{nat}(X_3) \\ \vdots \end{array}$$

Now consider finite implementation of non-terminating SLD derivations.

Since regular formulae have cyclic derivations, finding a cycle (loop) is suffice for knowing the whole derivation.

Definition (Gupta et al. 07)

CoLP = SLD resolution + loop detection rule.

Definition (Loop detection rule) (Gupta et al. 07)

A goal succeeds if it unifies with its ancestor goal.

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Successful coLP derivations only compute formulae in greatest fixed points.

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G_0 & \mathrm{nat}(X) \\
 & \downarrow\ X \mapsto s(X_2) \\
G_1 & \mathrm{nat}(X_2) \\
 & \downarrow\ X_2 \mapsto s(X_3) \\
G_2 & \mathrm{nat}(X_3) \\
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However, coLP does not take good care of productivity . . .

Assume some successful coLP derivation that computes an infinite formula.

Problem 1: *It is not guaranteed that there exists a corresponding non-terminating SLD derivation.*

e.g. For program p(f(X),X) $\leftarrow$ p(X,X) and goal p(f(X),X), coLP computes p(f(f(. . .)),f(f(. . .))) but here is no non-terminating SLD derivation.

CoLP derivation

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\begin{array}{rl}
G_0: & \text{p(f(X),X)} \\
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Problem 2: *There exists a corresponding non-terminating SLD derivation but it computes a different formula.*

e.g. For program q(f(X),Y) ← q(X,h(Y)) and goal q(f(X),Y), coLP computes q(f(f(. . .)),h(h(. . .))) but the corresponding non-terminating SLD derivation computes q(f(f(. . .)), Y).

CoLP derivation

G_0 : q(f(X),Y)

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G_1 : q(X,h(Y))

↓ $X \mapsto f(f(\ldots)), Y \mapsto h(h(\ldots))$ by unifying G_1 with G_0

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G_2: & \mathrm{q}(X_2,\mathrm{h(h(Y))}) \\
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G_3: & \mathrm{q}(X_3,\mathrm{h(h(h(Y)))}) \\
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& \downarrow\ X \mapsto f(X_2) \\
G_2: & \mathrm{q}(X_2,\mathrm{h(h(Y))}) \\
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G_3: & \mathrm{q}(X_3,\mathrm{h(h(h(Y)))}) \\
& \vdots
\end{array}
$$

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Our answer is affirmative.

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instead of

Definition (Loop detection rule) (Gupta et al. 07)

A goal succeeds if it unifies with its ancestor goal.

We also characterized a class of logic programs whose non-terminating SLD derivations, if any, are guaranteed to be productive.

Definition (Rewriting for LP) (Komendantskaya et al. 15)

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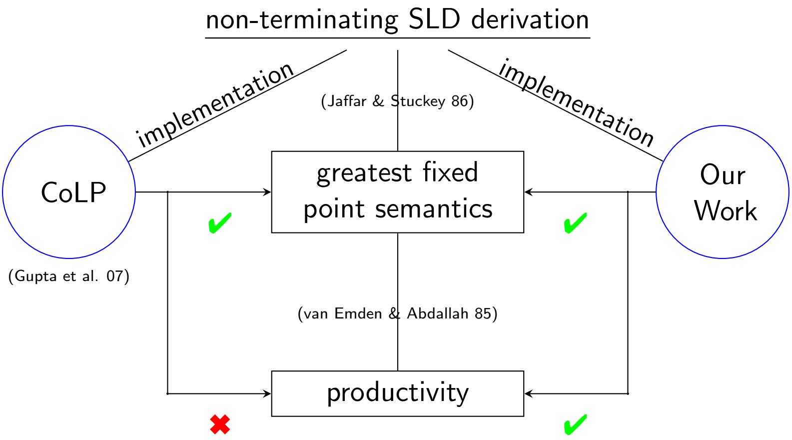
Theorem (our main result: Productivity Semi-decision)

Productivity is semi-decidable for programs characterized above, by SLD resolution combined with our loop detection rule.

1. Motivation
   - Overview of motivation
   - Background knowledge for understanding motivation
   - Problem description
2. Productive Corecursion
   - Loop detection rule review
   - Productivity guarantee
3. Conclusion
4. Future Work & Implementation



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fibs(X,Y,[X|S]) ← add(X,Y,Z), fibs(Y,Z,S).

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Implementation is available at
GitHub / coalp / Productive-Corecursion

Thanks!