DEPARTMENT OF MATHEMATICS - FIFTH-YEAR COURSES 2021/2022

Brief Descriptions: Please note that these courses run subject to demand and we might have to ask you to make an alternative selection at the start of the semester if the class is too small. You cannot take the courses with *'s if you took the equivalent course in year 4. You cannot take the course with a ** if you took F19NB.

Semester 1

MODELLING AND TOOLS, F11MT, [Semester 1]
The module aims to provide students with fundamental techniques of deterministic and probabilistic mathematical modelling. Model problems will be used to develop and illustrate these techniques. To investigate the problems, Python programs will be developed and implemented. The course will combine the presentation of theoretical material in lecture style and practical analytical and numerical studies of application problems.

APPLIED MATHEMATICS E, F11AE (Thermodynamics & Statistical Mechanics), [Semester 1,]
This course teaches the fundamental concepts of thermodynamics and statistical mechanics and describes applications to the real world. Topics covered include temperature, work, energy, heat, entropy, the four laws of thermodynamics, thermodynamic potentials, Maxwell's relations, applications to ideal gases and thermal radiation, the fundamentals of statistical mechanics, statistical entropy and equilibrium distributions, thermodynamics of the atmosphere and the greenhouse effect.
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MATHEMATICAL BIOLOGY A, F11AM*, [Semester 1]
The course aims to teach the application of ordinary differential equations and difference equations to problems in ecology. It will provide an understanding of the mathematical modelling methods that describe population dynamics, epidemiological (infectious disease) processes and evolutionary processes in ecological systems and instruction in the biological interpretation of mathematical results.

FUNCTIONAL ANALYSIS, F11FM*, [Semester 1]
This course introduces measure theory and functional analysis. The course includes measure and integration, monotone and dominated convergence theorems, application to evaluation of integrals, normed and inner product spaces, operators and their adjoints and inverses.

OPTIMISATION, F11MM*, [Semester 1]
The main aim of this course is to present different methods of solving optimization problems in the three areas of linear programming, nonlinear programming, and classical calculus of variations. The course includes analytic techniques for analyzing functions, strong and classical Lagrangian classical techniques, linear programming and dynamic programming.

NUMERICAL ANALYSIS C, F11NC*, [Semester 1]
Ordinary differential equations (ODEs) are central to mathematics and a multitude of different applications. In almost all cases, solutions to ODEs can only be approximated and usually this is done using methods from numerical analysis on a computer. This module provides an introduction to the derivation and analysis of techniques to do this both accurately and efficiently. The course includes the derivation and analysis of the properties of single-step and multi-step approximation methods for ODE initial value problems. There is also a practical programming mini-project (using Python) to implement and test various approximation methods.

PURE MATHEMATICS E, F11PE (Topology), [Semester 1]
Topology studies properties of mathematical objects that are unchanged under continuous transformations. The foundations of the subject begin with a framework in which continuity can be generally defined, but this course will focus on applications of topological ideas in particular settings. Topological properties include: connectedness of a network; whether you are living in a 3-dimensional or a 2-dimensional space; the interlacing properties of a knotted pattern. The course is motivated by these ideas and will study: the topology of finite graphs, and some algebraic applications such as the fundamental group of a graph or a surface; the classification of all the 2-dimensional spaces; the mathematical theory of knots.

MODELLING AND SIMULATION IN THE LIFE SCIENCES, F11MS, [Semester 1]
The aims of this course are to develop techniques of computational and differential equation modelling in biology, ecology and medicine. The course will introduce a number of modelling approaches that are widely used in applications to the life sciences, including reaction-diffusion equations, age-structured models, multi-scale modelling, and integral representations of dispersal. The course will teach practical implementation of these modelling approaches in the context of computer simulations, which will be illustrated by prototype applications from biology, ecology and medicine. This will be done by a mixture of lectures on basic methodology, case studies, and group-based modelling exercises.

<u>Semester 2</u>

MATHEMATICAL BIOLOGY B, F11AN*, [Semester 2]
This course develops models of biological, medical and physiological processes including wound healing, cancer growth, heart disease, nerve impulses and disease spread. This course will teach the application of ordinary differential equations to simple biological and medical problems, the use of mathematical modelling in biochemical reactions, the application of partial differential equations in describing spatial processes such as cancer growth and pattern formation in embryonic development, and the use of delay-differential equations in physiological processes.

PARTIAL DIFFERENTIAL EQUATIONS, F11MP*, [Semester 2]
The course aims to provide knowledge in the theory of partial differential equations. The course includes classification of linear second order equations, Cauchy problems, well posed problems for PDEs, the wave equation, the heat equation, Laplace's equation and Green's functions.

NUMERICAL ANALYSIS D, F11ND*, [Semester 2]
PDEs are used as the basis for modelling many phenomena. In almost all real-world cases exact formulae for solutions to PDEs cannot be found, and approximations are used instead. This is done using methods from numerical analysis on a computer. This course provides an introduction to numerical techniques and analysis required to find the approximate solution of partial differential equations. The course covers approximation methods for parabolic (including the time dependent heat equation), hyperbolic (including time dependent wave equations) and elliptic equations (including steady state heat equations). The course mainly covers finite difference methods, but also introduces finite element methods for elliptic problems. There is also a practical programming mini-project (using Python) to implement and test various approximation methods.

P**URE MATHEMATICS F, F11PF (Solitons)**, **[Semester 2]**
Solitons are localized travelling wave packets that maintain their shapes as they travel, and emerge unaltered after interacting with each other. Due to this “particle-like” nature solitons are of mathematical significance and have a plethora of realistic applications including optical fibres, water wave & tsunami formation, Bose-Einstein condensates and chemical reactions among others. In fact the soliton is part of Heriot-Watt history being discovered by J. S. Russell who noticed a solitary “wave of transition” that travelled for great distance on the Union canal just 1 mile from campus. In this course we will develop the methods to study and understand solitons. To achieve this we will first introduce basic quantum mechanical notions and in particular Schrödinger’s equation and use this as a background to study non-linear systems that display soliton behaviour.

GEOMETRY, F11PG*, [Semester 2]
This course develops methods of multidimensional calculus to investigate geometrical properties of smooth curves and surfaces. The topics covered include curves in Euclidean space, vector fields and differential forms, moving frames and structure equations, surfaces in Euclidean space, curvature and geodesics.

STOCHASTIC SIMULATION, F11SS, [Semester 2]
The inclusion of random effects in simulations is growing more and more important to try and quantify uncertainty in real world systems. The aims of this module are to develop understanding of continuous random variables and their practical numerical simulation. The course discusses simulation of random numbers and examines basic Monte-Carlo methods. We will look at Brownian motion its properties and simulation. We introduce stochastic integrals and stochastically forced ordinary differential equations. We show how to perform numerical simulations and analysis.

APPLIED LINEAR ALGEBRA, F11AL,[Semester 2]**
This course aims to provide a toolkit of modern techniques in applied linear algebra. As well as introducing algorithms, it aims to provide ways to evaluate their accuracy and efficiency. It will consist of a combination of background theory, practical applications and case studies.

DATA ASSIMILATION, F11DA, [Semester 2]
The aims of this course are to introduce a number of data assimilation approaches, including basic regression analysis, variational approaches, Kalman filtering, extended and ensemble Kalman filtering and the Bayesian inference approach. The course will teach practical implementation of these data assimilation techniques in the context of computer simulations, which will be illustrated by prototype applications from geophysical fluid dynamics and climate change models. Students will work on a particular problem (an extended project) involving modelling, data assimilation and simulation in a project related to biology, climate change or finance.