The Impact of DC Pension Systems on Population Dynamics

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Abstract

This study investigates the risk inherent in defined contribution (DC) pension plans on an individual and aggregate basis, based on U.S. data. Our aim is to gain insight into the consequences of a DC pension scheme becoming the predominant pillar of retirement income for an entire society. Using the stochastic simulated output of a DC flexible age-of-retirement model, we first determine the optimal investment strategies. We then examine the demographic retirement dynamics of an entire population of DC pension plan participants.

We observe that even for the most risk-averse plan members there is a high level of uncertainty in an individual's age at retirement. At the aggregate population level, we find that this uncertainty does not get dampened to any great extent by a diversification effect. Instead, the central role played by the market in determining retirement dates results in significant variation in the dependency ratio (the ratio of retirees to workers) over time. In addition, an attempt to ameliorate the outcome by introducing additional realistic features in the DC population modeling did little to dampen this volatility, which suggests that countries dominated by DC schemes of this type may, over time, be exposed to significant risk in the size of its labor force.

1. Introduction

The shift from defined benefit (DB) to defined contribution (DC) pension plans is a prevailing phenomenon throughout the pensions world. Generally the income support system for retirees is composed of three parts: personal savings, occupational pension plans, and government-provided social security. We define a DC pension plan as one that provides for each employee the deposit of a certain percentage of pay into an individual account that ultimately determines their retirement benefit. At retirement, individuals may wish to annuitize their savings or choose another medium of retirement funding. The DC pension design is, then, fundamentally an individual savings account, except that the contributions are made by more than the individual and employer contributions increase the likelihood that an annuity must be purchased after retirement. Therefore, the first pillar of retirement savings is inherently a DC plan. Second, among employer-sponsored pension plans in the United States, a strong trend toward DC pension plans has been in effect for over two decades (Ostaszewski 2001). The U.S. private DC plan market includes the popular 401(k) plan, which has grown at such a rate that over the next 30 years it could potentially become the largest source of retirement wealth across the nation (Poterba, Venti, and Wise 2000). The common hypotheses that have been put forth to explain the shift to DC plans include the following:

- The simplicity of DC plan designs
- The reduction in risk to employers when undertaking such a change in plan design
- The opportunity for plan sponsors to reduce their annual contributions
- The rising costs associated with the government's increased regulation of DB plans and

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- The superior portability properties of DC plans, which fit today's more mobile and independent workforce.

In addition to these hypotheses, Ostaszewski (2001) postulated that the weakening of real wage growth relative to real investment returns in the United States has created a macroeconomic incentive for individuals and plan sponsors to switch from DB to DC plans. Brown and Liu (2001) expanded on Ostaszewski's study by applying Canadian data to argue that the shift occurs when tax legislation and pension regulation increasingly favor DC plan designs over DB plan designs.

The third source of retirement income support in the United States is Social Security. It, too, is threatening to move toward a DC state pension system design (Cogan and Mitchell 2003).[1] In North America, financial security for the elderly is increasingly becoming a major concern as the Baby Boomers approach retirement, thus changing the demographic profile. For example, by 2025, Canada's aged dependency rate (the ratio of persons aged 65 and over to the population aged 15–64) is expected to reach 38%, and the expectation in the United States is not far behind at 34% (Turner and Watanabe 1995). As the population ages, stress on the economy would arise as unfunded pensions begin to be paid out, affecting Social Security and employer pension plans. To control costs, many plan sponsors have opted to switch their DB pension plans to DC pension plan designs; moreover, the United States has begun to debate seriously whether it should replace its current Social Security program to include DC-style personal accounts. The United States is not alone in considering this type of structural pension reformation; Mitchell (1998) concluded that the DC pension plan design caught the world's imagination during the 1990s. Approximately half of Latin America, as well as eight countries in Eastern Europe, have undertaken system restructuring to involve DC features (Gill, Packard, and Yermo 2004). A common purpose behind replacing a state system's unfunded DB with a funded DC plan, where the individual accounts are privately invested, is the country's desire to reduce the government's role in economic life and to increase reliance on market institutions (Turner and Watanabe 1995). Turner and Watanabe also argued that privatizing a country's pension system may serve to support the domestic financial market by increasing national savings (and hence drive up real capital investments), although the size of the effect is not agreed on by researchers. The problem in the United States, however, according to Gill and Tatucu (2005) in their study to draw lessons for the United States from the social security reforms in Latin America, is primarily the aging population. They wrote that the Latin American reforms reflected a loss of faith in governments to act responsibly in ensuring the promised pension benefits. This is not the case in the United States, where Social Security is well managed, and, although the current surplus is adequate to pay nearly 80% of the promised benefits over the next 75 years, the primary objective is to ensure the future financial solvency of the system (Gill and Tatucu).

Consequently, the DC pension plan has grown from being a single and relatively unimportant source of pension income to becoming two significant sources, with the potential of also becoming the third and final source of income for the nonworking elderly.

The practical application of a DC pension system has numerous drawbacks. DC pension plan schemes are notorious for the uncertainty in the level of pension that they can provide. Research and experience has shown that, with a fixed age of retirement, it is difficult to predict accurately the pension income under a DC pension plan design. Relative to a DB pension income benchmark, the accumulated wealth in a DC plan can be extremely risky (Blake, Cairns, and Dowd 2001). Chile, which completely reformed its state pension plan to a DC system two decades ago, affords its citizens some protection from the uncertainty in their retirement income by ensuring a minimum pension income. The provision of a minimum pension income, however, could be criticized as an expensive welfare system and, to a large extent, could expose the governmental pension sponsor to abuse and antiselection (Brown 1999). Furthermore, such a pension system could not be self-sustaining since a DC pen-

[1] The U.S. president's commission to strengthen Social Security by including personal retirement accounts can be found at www.csss.gov.

sion system can duplicate the benefits offered by a DB state pension systems only through further additional contributions of the state or the individuals concerned. Other ancillary benefits suitable for a DC system may include disability and death benefits, as well as a top-up for females, who would suffer an inherently lower pension income at the time of annuitization owing to their longevity. DC pension plan designs are frequently applauded for their portability properties, but this feature does not avail a state pension plan as there is usually no need for a pension to be portable, perhaps with the exception of emigrants. Under a DC pension plan, there is no redistribution of wealth. Finally, DC pension plan participants are responsible to pay onerous costs and fees, such as fund manager fees; in fact, administrative expenses are higher than in a socialized system (Brown 1999). A recent World Bank report (Gill, Packard, and Yermo 2004) investigated the outcome of the Chilean government's having implemented a DC pension system, and the Turner Report (Pensions Commission 2005) described the current situation in the United Kingdom, where the state pension plan also incorporates private accounts. Both commentaries confirm several of the predicted shortcomings of implementing a DC state pension system noted above, such as the rising costs of Chile's minimum pension income in consequence of low-income workers' preferring not to save rather than have their minimum pension reduced. Besides Latin America and Britain, additional international reform comparisons have been studied to provide perspective to the current U.S. Social Security debate. Simonovits (2005), while outlining the relevant lessons for the United States from the Hungarian reforms, concluded that partial privatization of Social Security is not helpful and does not solve the problems of Social Security.

The debate around privatization of Social Security has given rise to many arguments for and against the introduction of individual savings accounts. The arguments outlined above are generally drawn from analogies to employer pension plans (Blake, Cairns, and Dowd 2001; Brown and Liu 2001; Ostaszewski 2001), meaningful lessons drawn from the experiences of other countries that have undergone similar pension reforms (Gill and Tatucu 2005; Simonovitz 2005), and general observations that stem from the current political, economical, and regulatory arena (Cogan and Mitchell 2003; Mitchell 1998; Brown 1999). (Please note that some of these studies overlap in two or even three categories.) Although extremely beneficial and relevant, these papers do not provide a clear picture of the impact of a ''pure'' nationwide DC plan, that is, the aggregate effect of a DC plan without the constraints of additional regulations or the contemporary complexities that may exist in the country today. This study uses a bottom-up approach to emphasize and focus on the general effects of a pure DC pension plan monopolizing the income support system for the retired members of a population.

We assume that the individual will lengthen or shorten their working career depending on their accumulated pension savings in relation to their expected life span; in this way, participants can delay their retirement until a sufficient pension fund has been accumulated. Rather than focus on the accumulated wealth at a specified retirement age, we investigate the likely retirement age of DC participants if they hope to maintain a fixed standard of living at retirement, which would sustain them till death. The motivation behind this assumption is that it is necessary for the primary source of retirement income to provide a pension sufficient to offer financial security to the elderly and, therefore, facilitate the transition from employment to retirement. Owing to uncertainty in its accumulated wealth, such a requirement could not be fulfilled by a pure DC pension plan if the pension delivery date is fixed. If there were rigid restrictions on the worker's age of retirement, dictated by either statute or company policy, it would be difficult for a DC pension plan design to work on a large scale since inadequate pensions would be commonplace, rather than the exception, as is the case in a well-designed DB pension scheme. Second, in a pure DC pension plan, a participant's retirement would be entirely financed by their accumulated fund, and, aside from personal circumstances, there would be no embedded incentives to retire at any particular age. Lachance (2003) similarly justified a flexible retirement date by noting that a fixed retirement age in the context of a DC plan, although common and convenient, is inconsistent with the absence of structural incentives in the retirement decision as well as the mounting evidence that the performance of an individual's investments affects their decision to retire.

Given that our objective is to observe the full impact of a DC state pension system, the realistic existence of retirement age flexibility in a DC design is central to our analysis of the demographic implications of a nationwide DC plan. Furthermore, allowing a flexible retirement date provides the DC plan the unaided opportunity to fund fully an adequate level of pension income, without the use of top-ups or a minimum pension income. We regard results bearing too high a probability of retirement at very old ages (for example, above age 80) as a drawback of the DC pension design since, in reality, illness and other factors could influence an individual's decision to retire if sufficient funds have not built up before reaching an elderly age.

Realistic modeling of the retirement savings behavior for a population is an ongoing process, since there is always room to add elements that are more realistic. Accordingly, our current model includes simplifying assumptions as well as a variety of realistic features. One important assumption is that we have taken as given the dynamics of the stock market, which means that we do not attempt to model the macroeconomic effects of the mass actions of the DC plan members such as mass demand for equities or liquidation of a particular asset. Clearly, we expect that changing retirement patterns would impact the prosperity of a country by impacting tax revenue, labor force growth, social programs, national savings, and company profits. These outcomes, which we comment on in Section 3.2, in turn would affect the prices of financial assets along with wages. The effect could contribute to both positive and negative feedback; thus, without its inclusion, we are limiting ourselves to a slightly artificial model, and the results of this report could be described as some worst-case scenarios. Having said that, we hope this paper will provide an initial impression of what could occur, leading to a wider discussion of nationwide DC pension plan design.

The advantage of choosing a DC pension plan for the plan sponsor is that it shifts the risk of an inadequate pension from their hands to those of the individual. When a DC conversion occurs on the national level, we propose that there are intrinsic risks not only to the economic well-being of the plan participants, but as well to the entire nation. We hypothesize that introducing a national DC pension plan as the primary source of retirement income would result in the financial market's condition strongly affecting the retirement pattern of the citizens, which could be contrary to the interests of the society at large. Consequently, the proportion of pensioners and workers in the population could well be unpredictable and uncontrollable by the state, as well as possibly detrimental to the economy.

Section 2 outlines our assumptions in modeling the DC population, including a short description of the steps undertaken to build and execute the population retirement model. Section 3.1 explains how we initially use the stochastically simulated results to select efficient portfolios for the members of the population. We then discuss the retirement pattern behavior of the aggregate group of DC participants in Section 3.2. Since the results are contingent on the validity of the model, Section 4 investigates their sensitivity to the model's simplified assumptions by assessing the impact of adding more realistic features to the population retirement savings model. In Section 5 we conclude by examining a DC plan design feature that could potentially alleviate the possible risks that a DC pension system poses on the size of the labor force from one year to the next.

2. MODEL AND ASSUMPTIONS

In this study we aim to model the DC pension system in its "pure" form. The three sources of retirement income—personal savings, social security, and employer benefits—are treated as a single traditional DC pension plan. This is done for simplicity, but also to fulfill our aim to further the understanding of the impact of a traditional DC plan's sustaining the retirement income of an entire population. We regard any additional features that differentiate a hybrid DC plan from its pure form as clouding our understanding, and, consequently, they are removed. There is neither a requirement to purchase an annuity contract after retirement, a minimum pension guarantee, nor a mandatory age of retirement, either by statute or by company policy. Every member within the population adopts a retirement strategy that bases their retirement date on the size of the pension income that can be supported by their retirement savings. In our pure DC plan population model, annual contributions are made to each participant's individual retirement account, and the member can direct his or her investments to five different assets.

Despite the use of U.S. data, our DC population modeling assumptions are intended neither reflect the current state of pension plans in the United States, nor project the effect of the proposed U.S. Social Security reforms. As Section 1 discusses, the adoption of a state DC pension system is a growing reality in numerous countries, including the United States, and the shift within occupational pension plan scheme designs from DB to DC is also a prevailing phenomenon. We realize, nevertheless, that a collective shift in the United States among all DB providers to an identical DC plan design is unlikely unless there existed strict government regulation. We recognize that variety among the employer pension plan designs would likely remain even if all occupational pension schemes became the DC type, and that a DC state pension system would not have a pure design, since it generally contains ancillary features and limitations on its participants. Although not realistic in practice, the pure DC design approach in this study is intended to provide a clearer picture of the overall impact of individual accounts fueling the retirement of the masses.

A DC accumulated pension income depends on such factors as the pension portfolio's rate of return, salary growth, annuity discount rate, and relationship between each rate specified. We assume that the pension fund is invested across five assets: equities, fixed-income bonds, index-linked bonds, risk-free one-year bonds (cash), and index-linked cash. This study uses the Vasicek interest rate model (Vasicek 1977) to underpin the dynamics of all the asset returns. Appendix A describes the stochastic asset-return model. The participant's salary model incorporates a merit scale, the prevailing inflation rate, and real wage growth. The last two components are an integral part of the stochastic asset model. The salary modeling is elaborated on in the Appendix. With respect to the asset accumulation model, we have approached a macroeconomic problem using microeconomic tools given that we do not model the possible link among the retirement behavior of the working population, asset demand, wages, and the financial market returns, which is explained further in Section 1. Also in the asset accumulation model, there are neither taxes, expenses, nor allowances for profit in the financial assets' pricing and the management of the DC plan.

The asset allocation strategy of the DC participants in our study is static, which means that participants maintain constant asset proportions in their portfolios throughout the accumulation phase. This results in constant portfolio rebalancing at the end of each year. Although a simplifying assumption, Blake, Cairns, and Dowd (2001) showed that a well-chosen static asset allocation strategy performs substantially better than various common dynamic strategies, such as the popular ‘‘lifestyle’’ strategy. The contributions made to the DC pension plan are regarded as being truly invested, a method of funding known as a funded stated pension system.

Furthermore, the model includes demographic assumptions such as retirement decision-making behavior, mortality, contribution rate, merit gains on salary (noted above), age of plan enrollment, and the population's age and gender distribution. Sections 2.1, 2.2, 2.3, and 2.4 are dedicated to four of these assumptions, while the rest are detailed below.

We carry out a series of analyses. The first DC population retirement model, in Section 3.1, is the most basic. The participants enter the DC pension plan on their 25th birthday, have no dependents, have average annual merit gains of 2%, and make annual contributions of 10% of salary at the beginning of each working year (see Section 2.3). Thus, the initial analysis assumes that all members enter the plan at the same age, pay the same contribution rate, follow the same career path, and adopt the same investment and retirement strategies. Such a level of uniformity is unlikely to exist among actual population members. The purpose of choosing a simple model is that it aids in identifying the efficient portfolios; nevertheless, its value in realistically portraying a population's retirement savings behavior is less obvious. Our intention in first investigating this extreme case is not to put this forward as a realistic representation of the risks we face in the future. Instead, our objective is mainly exploratory, and we present it here as a potential worst-case scenario. We will see that this extreme system results in a considerable degree of variability in the size of the working population.

In subsequent sections, we work away from this worst case by introducing heterogeneity into the system and examining its effect. We improve the model's level of sophistication by developing heterogeneity among the DC participants in their

- Investment strategy, in Section 3.2
- Entry age to plan, in Section 4.1
- Career flight path (high flyers/low flyers), in Section 4.2 and
- Contribution rate, in Section 4.3.

These realistic refinements are tested to assess their ability to dampen the severity of the simulated demographic outcomes that arise from the simulation of the DC population.

2.1 Retirement Decision Model

The reasons why people decide to retire on a particular date are many. Factors influencing the retirement date of a DC plan member include accumulated wealth, health, age, preference for leisure time over work, direct pressure from employer, and general peer pressure owing to social customs (Brothers 1998).

In our study we base retirement on the level of pension income that can be provided by the DC participant's accumulated wealth. Pension income is determined by dividing the pension fund on the retirement day by the annuity factor ($\text{Wealth}(t)/\ddot{a}_{e+t}(t)$). The pension income divided by the individual's salary at retirement ($\text{Salary}(t)$) produces the replacement ratio, $RR(t)$:

$$RR(t) = \frac{\text{Wealth}(t)/\ddot{a}_{e+t}(t)}{\text{Salary }(t)},$$

where t is time since entry into the plan and e is the individual's age of plan enrollment. The numerator and denominator are gross incomes and are not adjusted for taxes, making the replacement ratio a *pre-tax* measure. The annuity factor, $\ddot{a}_{e+t}\ (t)$, is the present value at the time of retirement, t, of one unit of an annuity for the remaining life of an annuitant aged $e + t$. Since we are not assuming mandatory annuitization, money may not actually be withdrawn to purchase an annuity at retirement. The philosophy, however, that an adequate amount of accumulated funds is necessary for retirement is the same no matter what medium of retirement funding is used, and the replacement ratio based on a fixed-income annuity is the selected measurement for adequacy. We also considered index-linked annuities as the funding medium benchmark since they offer the benefit of maintaining the purchasing power of the pension income. We chose fixed-income annuities since, to achieve the same initial pension, the higher cost of the index-linked feature would simply drive up the retirement ages and the general conclusions would be unaltered. In addition, index-linked annuities (or voluntary annuitization in general) are scarce among retirees in practice since people typically prefer higher initial pensions. Brown and Warshawsky (2001) further explored the explanation behind the reluctancy of individuals in the United States to annuitize their DC fund, despite the potential benefits argued in Davidoff, Brown, and Diamond (2005). Brown and Warshawsky (2001) explained that there is a lack of inflation protection in the few annuity contracts purchased, suggesting to us that a fixed-income annuity assumption is preferable.

This study considers the retirement age as a random stopping time, when the pension purchasable exceeds two-thirds the outgoing salary. In other words, at the beginning of each working year, the replacement ratio is measured so that

$$\text{Retirement Age} = \min \{e + t : RR(t) \geq \tfrac{2}{3}\}.$$

This decision rule, thus, incorporates the reasonable view that retirement will be deferred until such time as the member can afford retirement.

We model the retirement decision based on the accumulated pension wealth of the participant as well as their expected longevity. The two-thirds rule does not explicitly allow for age dependency in the decision, but age is taken into account indirectly through the annuity factor:

$$\ddot{a}_{e+t}(t) = \sum_{s=0}^{\infty} P(x_1(t),t,t + s)_s\text{P}_{e+t},$$

where $P(x_1(t),t,t + s)$ is the price at time t of a risk-free zero-coupon bond that matures at time $t + s$ and $x_1(t)$ is the instantaneous risk-free rate of interest at time t. The price formula for

$P(x_1(t),t,t + s)$, given in the Appendix, is based on the Vasicek model. The annuity factor decreases as the individual ages owing the consequential fewer number of payments expected to be made due to higher expected mortality. The lower the annuity factor, the higher the pension income and the likelihood that it exceeds 66.67% of the outgoing salary.

The two-thirds target is selected to represent an adequate level of income for retirement, a target value that is not largely different from the prevailing replacement ratio for singles in the United States and throughout the OECD countries. Disney, d'Ercole, and Scherer (1998), in an OECD study on aging, measured the replacement ratios across all OECD countries. Having only gross U.S. income data, they determined that the average pre-tax salary replacement ratio of singles in the United States is 62% when incorporating all sources of disposable income in both the numerator and the denominator. Our pre-tax replacement ratio benchmark is close to this actual average.

Moreover, the OECD study found that the replacement ratios across all the OECD countries exhibit a high degree of uniformity, with a typical average of 70%, when the calculations are carried out using all sources of retirement income net of direct taxes paid. According to the OECD study, a replacement ratio that is based on pre-tax income underestimates one that is net of taxes. It follows that, in the presence of a progressive tax schedule, the replacement ratio benchmark would exceed two-thirds once the taxes are deducted and draw nearer to the typical 70%.

In addition to our target's consistency with current data, the two-thirds replacement ratio also falls within the range of an adequate pension, given as a rough guide in *The Handbook of Canadian Pension and Benefit Plans* (Greenan 2002). Here an after-tax replacement ratio should fall between 60% and 70% to maintain a pensioner's preretirement standard of living. Further, the Canadian Institute of Actuaries (1996) reported that an individual can preserve their pre-retirement standard of living if their income replacement ratio falls between 60% and 74%, where the exact level depends on their unique earnings level since a higher rate is required for low-income workers to satisfy minimal needs.

2.2 Mortality Model

This study uses the United States Life Tables 2002 for females and males (Arias 2004), published by the National Center of Health Statistics. The data used to prepare these tables are, within the United States, final numbers of deaths for year 2002, postcensal population estimates for the year 2002, and data from the Medicare program of the Centers for Medicare and Medicaid Services.

The annual mortality rate for a participant aged x (q_x) is a fixed blend of 50% of the male mortality rate (q_x^m) and 50% of the female mortality rate (q_x^w), both determined from the U.S. life tables. To focus on total population dynamics, unisex rates are chosen to simplify the calculation procedure even though this does not replicate the population since females, having a longer life expectancy than males, would have a higher weighting as the two genders age. Dealing separately with the males and the females by applying sex-distinct mortality functions would have altered our results only very slightly and would not affect our general conclusions.

2.3 Contribution Rate

Like the contribution rate in our model, a 10% mandatory payroll tax pays for the individual account plans in Chile. In the United States, Poterba, Venti, and Wise (2005) summarized recent studies that measured the mean 401(k) contribution rate, including both employee and employer contributions. The reported values from their references are between 8.7% and 12.6%. Finally, Blake, Cairns, and Dowd (2001) assumed a 10% rate while simulating DC individual accounts since it is a typical contribution rate of DC plans in the United Kingdom.

2.4 Simulation of an Entire Population

We use a stationary and stable population model to simulate the demographics of a population. That is, there is no growth in the population size (a stationary population) and the population age distribution is identical from one period to the next (a stable population), (Bowers, Cairus, and Dowd 1997). The model has 81 cohorts in the population. At every point in time, the cohorts range in age from 20 to 100. The relative size of each cohort aged x is labeled lx:

$$l_x = e^{-\int_{20}^{x}\mu_t\,dt} = {}_{x-20}p_{20}.$$

For example, the size of the aged 20 cohort at time t is 1, without loss of generality, and is size ${}_sp_{20}$ by age $20 + s$ at time $t + s$.

From the output, each year of simulation calculates the dependency ratio (ratio of the number of retirees to the number of workers) based on the constant proportional sizes of each age group.

2.5 Dependency Ratio

We measure the retirement dynamics of the society of DC pension plan participants with the dependency ratio, defined as the ratio of the number of retirees to the number of nonretirees at or above the age of 20. This second group of individuals is referred to as workers, although they may or may not have entered the pension plan:

$$\text{Dependency Ratio} = \frac{\#\ \text{retired population}}{\#\ \text{working population}}.$$

(Note: this is not the only definition of a dependency ratio. In Section 1 the aged dependency rate is defined as the ratio of persons aged 65 and over to the population aged 15–64. The reason behind this deviation is that our purpose is to measure the demographic labor force dynamics rather than the age structure of the population.)

2.6 Stochastic Simulation

In this paper we use stochastic simulation to investigate the range of outcomes for a variety of quantities of interest. The steps followed in our model construction and simulation are as follows:

1. We begin with the asset model specification and calibration. Asset return modeling includes choosing an asset return model that is parameterized according to the realized U.S. returns and volatilities. Appendix A explains the derivation of the accumulation model, parameter estimates, and sources of data.
2. We then carry out the demographic model specification and calibration. In this step we select the mortality table, age of plan enrollment for the participants, design of the DC pension plan, and characteristics of the population model, including their retirement savings behavior. This aspect of the study is presented in Sections 2–2.4.
3. The simulation calculates the pension wealth for each DC participant so that, between time $t-1$ and t, an individual's pension account is accumulated forward in the following manner:

$$\text{Wealth}(t) = (\text{Wealth}(t-1) + 0.1 \times \text{Salary}(t-1))(1 + i(t)),$$

   where $i(t)$, the investment return between times $t-1$ and t and depends on the specified portfolio strategy. The wealth and salary are carried forward for each of the simulated years until the member retires, when $RR(t) \geq 66.67\%$. Section 2.1 gives the details of the retirement model. For computational efficiency and to aid comparisons among different strategies, we simulate the asset returns once and use this same set of sample paths for each of the investment strategies.
4. Given the various assumptions, the program generates an empirical distribution of possible retirement ages for each individual and dependency ratios for the entire population corresponding to each particular investment strategy. To accompany each simulated outcome, relevant information regarding the population and the financial market is also generated.

3. RESULTS FOR AN ENTIRE POPULATION

Our first step in analyzing the results is to calculate the efficient portfolios in Section 3.1, and, from among them, we select three asset mixes to represent the investment strategies of the members of the population in Section 3.2. In the subsequent simulations, we focus on relevant aspects of the population's retirement dynamics using cumulative distribution functions, time series plots, and scatterplots. These plots provide insight into the consequences of DC pension plan schemes. Our analysis suggests that a DC pension system could have a strong impact on the labor force stability of the population.

When measuring the value of the DC pension pension plan on an aggregate level, our indicator of success or failure is the dependency ratio, as Section 2.5 explains. The appropriate interpretation of the dependency ratio in our results may

Table 1
Dependency Ratio's Simulated Average Value and Standard Deviation, along with Equivalent Average Age of Youngest Group Retired for Each Asset

Asset	100% Index-Linked Cash	100% Cash	100% Index-Linked Bond	100% Bond	100% Equity
Dependency ratio					
Average	0.2510	0.2842	0.3139	0.3405	0.7677
Standard deviation	0.0557	0.0564	0.0507	0.0987	0.3051
Average equivalent age of youngest group retired	69.48	67.94	66.68	65.62	53.80

not be readily obvious. Our simulation assumes that the entire working population uniformly enrolls in a DC pension plan design and that a worker's retirement is triggered by an adequate accumulated pension income; thus, a low dependency ratio raises concern, as this indicates that elderly workers are financially unable to retire owing to the insufficiency of their DC pension fund account. On the other hand, a high dependency ratio signifies that the DC pension plan is allowing workers to retire at young ages. Normally a high dependency ratio is undesirable since it is a symptom of an aging population. A growing proportion of the elderly and nonworking members of the population would put a strain on economic programs such as Social Security and health care. Yet, considering that the distribution of ages is unchanging within our model, an increasing dependency ratio is a positive outcome that measures the financial ability of individuals to retire earlier. Since the plan is funded, a high dependency ratio would not incur costs for the working population to provide for the financial needs of the retirees. ''Dependency ratio'' could be a misnomer in our study since only those workers with a financially secured retirement become pensioners. Consequently, retired members do not require the financial welfare support of the working population; nevertheless, they do continue to rely on the workers to produce the necessary goods and services for their consumption.

3.1 Choosing Optimal Investment Portfolios

The choice of investment strategies plays an important role in the retirement outcome for an individual. In this section we evaluate the effects of the investment strategies on the individuals by observing their retirement age patterns and on the population dynamics by measuring how the dependency ratio varies over time. The results suggest that, among the efficient portfolios, equities perform impeccably well in the interest of individual members but pose a threat to the stability of the population's dependency ratio.

Using the initial DC population model whose homogeneous assumptions are explained in Section 2, we are able to investigate a range of different asset allocation strategies by assuming everyone in the population adopts the same strategy. In the simulations, we consider 581 different investment strategies, each containing a different combination of bonds, cash, index-linked bonds, index-linked cash, and equities. The portfolio's exposure to bonds, index-linked bonds, and equities is tried in increments of 10% of the total portfolio, but only 20% increments for the cash and index-linked cash. The less precise increments of cash and index-linked cash are inconsequential since they are absent from the resulting efficient portfolios.

The homogeneity of the population with regard to their investment strategy results in there being no ''gaps'' in the ages of the retirees. More specifically, for each year of simulation, there is a single age in which everyone at or above is retired and everyone below is working. Every dependency ratio level has an equivalent age therefore, that indicates the age of the prevailing youngest group retired. For example, a dependency ratio of 35.65% indicates that everyone at or above the age of 65 is retired. Table 1 presents the average dependency ratios and their equivalent ages assuming a 100% investment in each of the five assets.

The results of the simulation suggest that the ideal investment strategy in terms of offering citizens the earliest retirement age would be to allocate the majority of the funds in equities, with the remainder in bonds and index-linked bonds, while avoiding both cash and index-linked cash. According to Table 1, the performance of a pure equity portfolio exceeds all other asset allocation strategies with a mean dependency ratio of 76.77%. The inverse relation between the dependency ratio and the mean age of retirement necessitates that the equity portfolio carries the youngest mean retirement age from among the investment portfolios, which is just under age 54. In comparison, a pure index-linked cash portfolio produces a mean dependency ratio of 25.1%, which corresponds to a mean retirement age above 69. Thus, from the individual investor's perspective, choosing equities would accelerate their potential retirement date.

An analysis of a DC member's simulated retirement age provides further insight into the benefit of equities for the individual. In Figure 1, the empirical cumulative distribution function (CDF) of a participant's retirement age is graphed for each asset. The CDFs illustrate the advantage of holding an equity portfolio for an investor who tolerates some risk. (Each point on the CDF curve shows the probability that a participant's retirement age will fall below a particular level. If one curve lies more to the right than the other curve, then that particular investment strategy is likely to produce higher ages of retirement than the other investment strategy.) The CDF generated from an equity portfolio is distinctively shifted to the left. The index-linked bond portfolio is a distant second in terms of producing a lower retirement age. On the right tail of the equity portfolio CDF, after crossing the other CDFs, is where it is more likely that the equity portfolio will deliver a later retirement. Furthermore, if a CDF curve rises more steeply between 0 and 1 than another, such as the index-linked bond's CDF relative to the equity's, then that strategy is less volatile. The high crossover point (age 70) between the two CDFs suggests that, although investing the majority of funds in equities creates less certainty in the age of retirement, it is still the best investment strategy since there is more opportunity for early retirement and, except for the worst-case scenarios, the individual would most likely retire before or at the age that an

Figure 1

Simulated Empirical Cumulative Distribution Function of Retirement Age for Each Asset

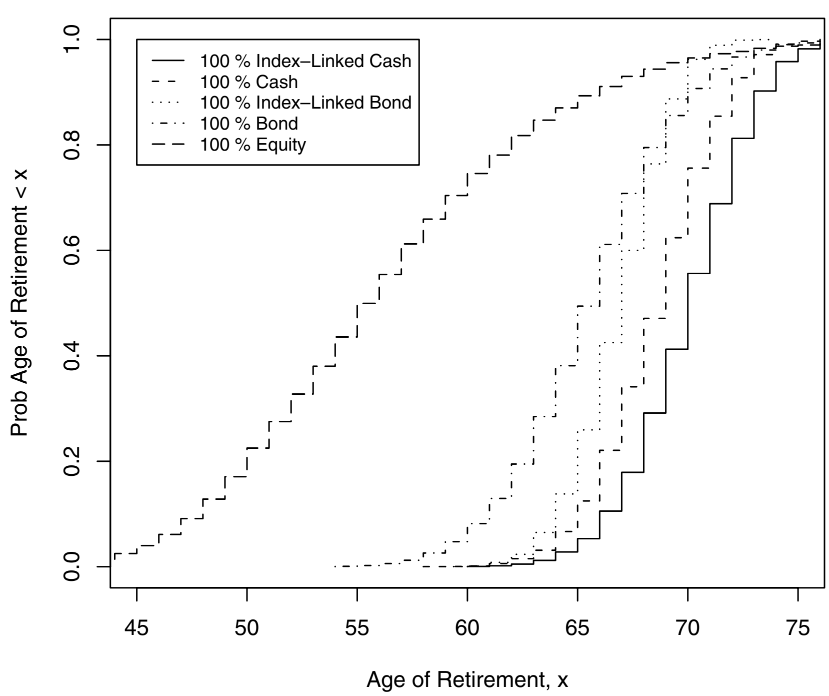
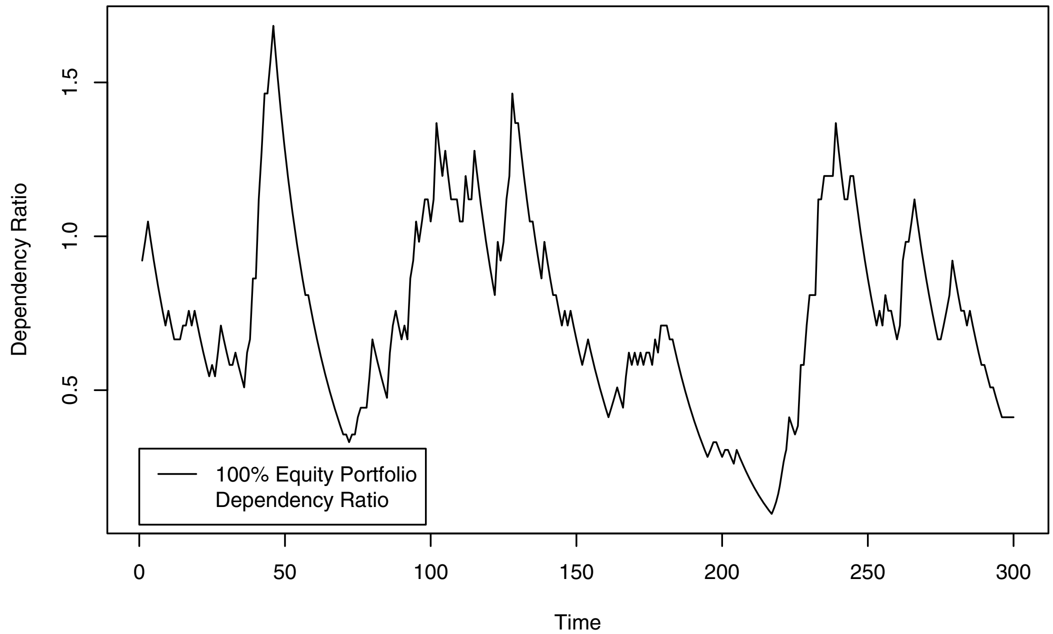


Figure 2
Simulated Time Series Plot of Dependency Ratio for a Population of Equity Investors



index-linked bond investment strategy would have permitted.

The benefit to a worker of increasing their risky asset allocation as a result of having a flexible retirement date has been ascertained in previous studies. Lachance (2003) examined how a worker's optimal portfolio choice is influenced by their capacity to adjust their retirement date as a function of market fluctuations. In her study she derived a closed-form solution for the optimal consumption and portfolio choice when a worker's retirement is flexible. Utilizing this solution, she showed that more investment risk can be assumed if a worker's retirement date is flexible instead of being fixed.

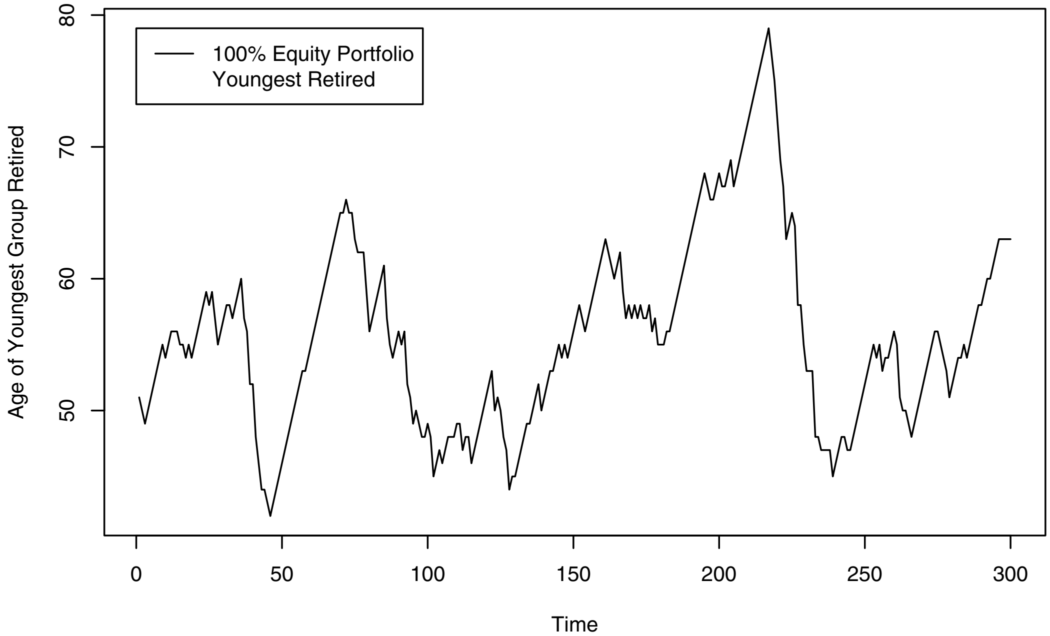
Interestingly, these results are also consistent with current asset allocation trends of DC pension plan participants in the United States, where DC pension plan investors are increasingly moving to equity investments. Between 1983 and 1996, U.S. members have increased the proportion of their DC pension plan assets in equities from 27% to 60% (Mitchell 1998).

Overall, an efficient investor should direct a large proportion of their funds to equities, despite the less certainty in the age of retirement.

The *fallacy of composition* (Brown 1997) argues that what could be good for the individual could possibly not be good in aggregate: that is, although an investment portfolio could be optimal for an individual DC participant over their lifetime, this same investment strategy could potentially not be the optimal solution for an entire population over many lifetimes. One concern on a public policy level is the instability of the dependency ratio, as it would cause an unstable economy. Therefore, although riskier investments are beneficial to the individual, the additional volatility that is incurred in the dependency ratio could be harmful to society and the economy as a whole. This is depicted by the erratic behavior of the dependency ratio in Figure 2, which tracks the dependency ratio for a population of equity investors over a 300-year simulation. Figure 3 displays the corresponding age of the youngest retired member in the population. It, too, is tremendously irregular, and its range spans 38 years. If we are to do what is best from an aggregate perspective, we require a risk measure that reflects the need for a stable dependency ratio. The standard deviation is an appropriate risk measure for our purposes since it describes the stability of the dependency ratio.

Figure 4 plots the mean of the simulated dependency ratio against its standard deviation, corresponding to each of the 581 asset strategies

Figure 3
Simulated Time Series Plot of Youngest Group Retired in a Population of Equity Investors



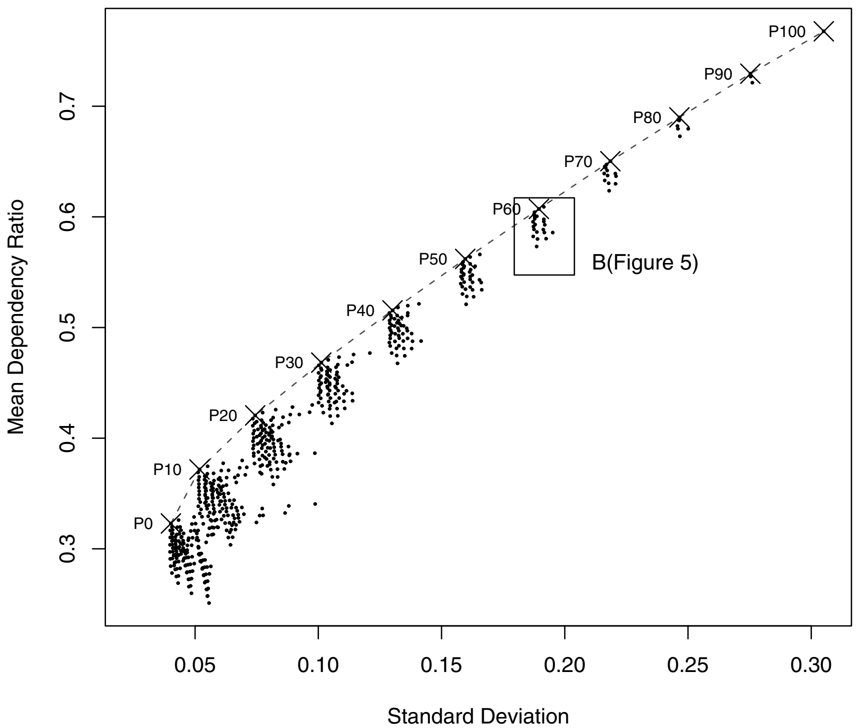
detailed at the beginning of this section. The return measure is the mean dependency ratio across 4500 years of simulation, while the risk measure is the standard deviation of the simulated dependency ratios. A long simulation run is preferred over multiple short runs since the latter would likely be biased by initial conditions. The plot gives an impression of the opportunity set based on this measure of risk, traced out by the 581 investment strategies. From the opportunity set, we can infer the efficient portfolios, which are the portfolios that carry the lowest risk for each given level of return.

It is useful to note several general aspects of the opportunity set plot in Figure 4:

- The indicators of success are high mean dependency ratios (members, from their perspectives, are happy because they are retiring earlier, on average) with low volatility (stable labor force); thus, higher values on the *y*-axis and lower values on the *x*-axis are the preferable portfolios. Specifically, the points in the opportunity set nearer to the top left are good.
- Since the population is discretized and all members follow the same strategy, the dependency ratio is restricted to a discrete set of values determined by the youngest retiree at any given time.
- Each cluster of points has a constant proportion invested in equities.
- The ''X''s mark the portfolios in each fixed-equity cluster on the efficient frontier. The compositions of the efficient portfolios, P0 to P100, are given in Table 2.

Figure 4 ranks the pure equity investment strategy as the most dispersed of the portfolios. It remains, nevertheless, as an efficient portfolio. Cash and index-linked cash appear to be a poor pension investment choice, as shown by their exclusion from every efficient portfolio listed in Figure 4. Figure 4 also suggests that, among the efficient portfolios, investing in equities would elevate the mean dependency ratio while both types of bond assets provide stability. Figure 5 describes the breakdown among the assets by extracting and enlarging the asset mixes with an equity exposure of 60% from the opportunity set in Figure 4 (box B). Looking from top to bottom, Figure 5a explains that increasing the portfolio's proportion of index-linked cash would typically diminish the mean dependency ratio without the benefit of lowering the standard deviation. In con-

Figure 4
Simulated Opportunity Set for a Population of DC Members Who Homogeneously Allocate Their Funds in Specified Investment Portfolio



Notes: This plot is generated for 581 different asset allocation strategies (small dots). The dependency ratio's standard deviation is plotted against its mean. The efficient portfolios are marked by an "X" and the compositions of these portfolios, P0 to P100, are given in Table 2. A dashed line indicates the efficient frontier. Figure 5 contains an enlarged box B.

Table 2
Portfolio Mix for Efficient Portfolios P0 to P100 in Figure 4

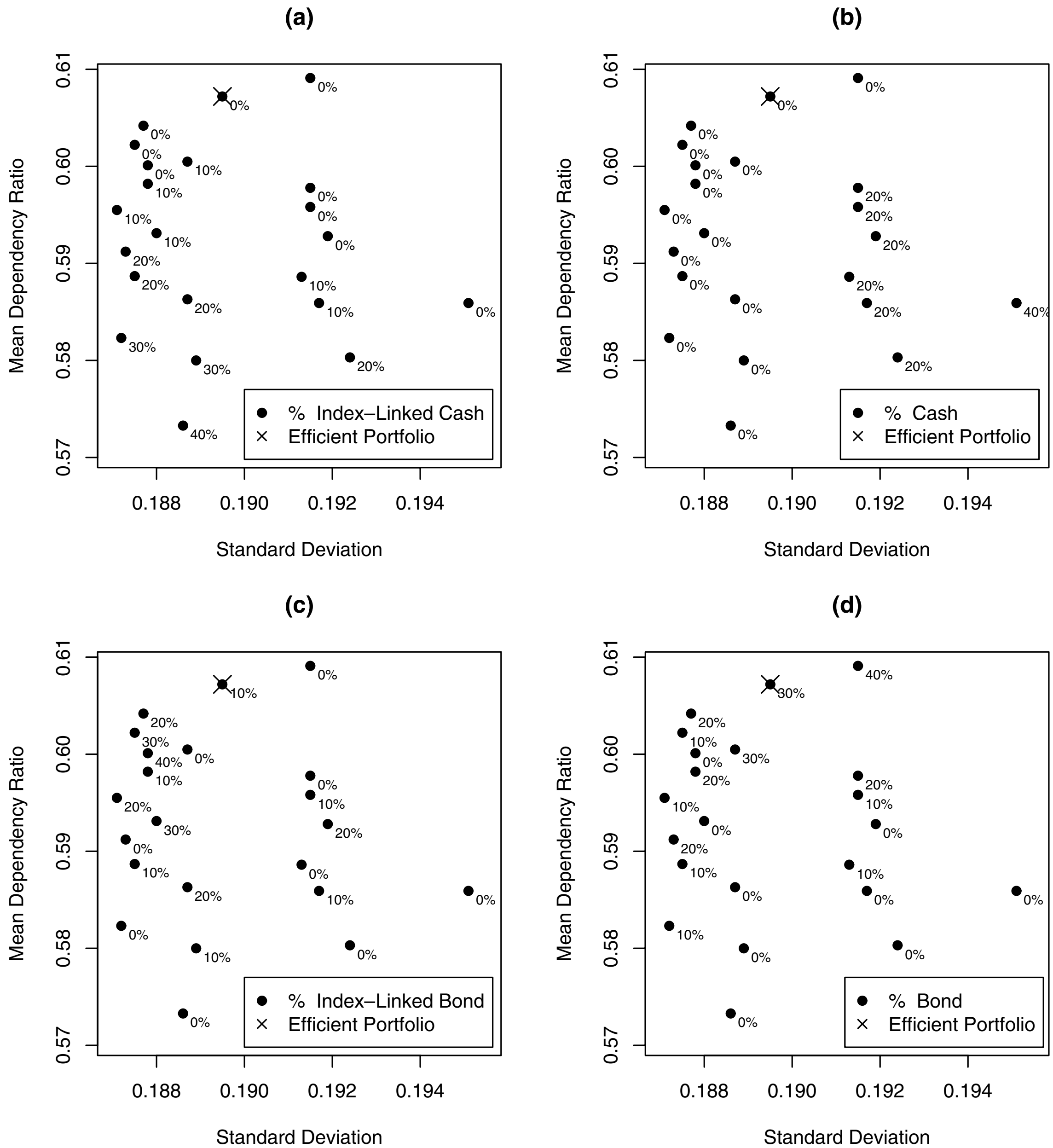
Minimum Risk Portfolio	% Index-Linked Cash	% Cash	% Index-Linked Bond	% Bond	% Equity
P0	0	0	70	30	0
P10	0	0	60	30	10
P20	0	0	50	30	20
P30	0	0	40	30	30
P40	0	0	30	30	40
P60	0	0	20	30	50
P60	0	0	10	30	60
P70	0	0	0	30	70
P80	0	0	0	20	80
P90	0	0	0	10	90
P100	0	0	0	0	100

trast, Figure 5b shows that (by looking from left to right) a heightened exposure to cash typically causes the dependency ratio's standard deviation to escalate without significant improvement in its mean. Increasing the allocation to bonds (Fig. 5c and d) shows an opposite, but less clear, effect. Raising the proportion of the index-linked bonds (c) vaguely lowers the standard deviation, and increasing the weight in fixed-interest bonds (d) typically raises the mean dependency ratio. One, perhaps obvious, conclusion that we can take away from this is that while cash (fixed or index-linked) could be a good low-risk short-term investment, our results suggest that it is not good as a long-term component of a DC pension plan fund.

3.2 Variety in the Population's Investment Strategy

We now move towards a more realistic scenario. Having established the efficient investment

Figure 5
Enlargement of Box B in Fig. 4



Notes: Each plot contains the same points; the numbers provided, however, tell us about the proportions invested in (a) index-linked cash, (b) cash, (c) index-linked bond, and (d) bond. The efficient portfolio (P60) is marked by an X.

strategies for the DC members of the population, we first determine three investment strategies appropriate for both the individual members and the entire population. We then examine the dynamics of the retirement behavior within a population whose members follow these three investment strategies. Our analysis shows severe volatility in the dependency ratio, which is primarily driven by the market's performance.

To add realism to the simulation, we expand the population's investment strategies from a single arbitrary homogeneous investment portfolio choice to three well-performing portfolios in line with current market trends. Specifically, in this second DC population model, we assume that equal proportions of each age group will allocate their wealth to a low-risk fund (Portfolio A), a medium-risk fund (Portfolio B), and a high-risk fund (Portfolio C); each fund is detailed below. Once Portfolios A, B, and C become the investment strategies for their respective one-third of new entrants, those members will maintain the same proportion of assets over their entire working life, classified earlier as a static asset allocation strategy. Thus, we assume that plan assets are rebalanced annually to maintain predetermined proportions in each asset class:

Portfolio A: P20 (20% Equities, 50% Index-Linked Bonds, 30% Bonds)
Portfolio B: P60 (60% Equities, 10% Index-Linked Bonds, 30% Bonds)
Portfolio C: P100 (100% Equities).

A perceived need of this study is to incorporate realistic features that would help minimize the volatility of the dependency ratio. This objective stems from the severity of the results presented later in this section. For this reason the selection of the asset mixes is based on their efficient portfolio status as exhibited in Figure 4 as well as the following:

- The portfolios are consistent with the current market trends for DC schemes in the United States according to Mitchell (1998), who pronounced equities as being the most popular investment choice with an average asset allocation of 60%, but with some diversification with bonds and other assets (Portfolio B).
- If a participant could tolerate risk, we establish in Section 3.1 that it is in their best interest to allocate their funds solely into equities (Portfolio C) on account of the large decrease in the expected retirement age.
- There could also exist investors who are extremely risk averse and whose concerns lie in the 99% percentile of the retirement age distribution. As Section 3.1 explains, investors who require elevated levels of assurance that they will retire prior to a particular age should increase their exposure to bonds, since portfolios with a high bond weighting deliver better results in the worst-case scenarios. Portfolio A satisfies the needs of such investors, as well as inserting additional diversity to the portfolio selection of the population.

It is useful to look at the whole of the dependency ratio distribution under the second population model, using histograms, CDFs, time series plots, and scatterplots. The standard deviation risk measure associated with each investment strategy is helpful in Section 3.1 in deciding between assets mixes; nevertheless, having decided on efficient portfolios, it is the plots in the coming sections that help us to penetrate the DC pension plan and to understand its impact on the workforce dynamics.

To examine various aspects of the population's retirement patterns, it is helpful to have a feel for what key factors influence the dynamics of the dependency ratio. We investigate its relationship with a simple geometrically weighted average of the past annual investment returns (specifically, we attach a weight of 5% to the most recent fund returns and add this to the 95% weight of the previous year's weighted average). We find this coefficient of 5% to be approximately optimal in terms of maximizing the correlation with the dependency ratio. Displaying the asset performance using this smoothing technique reflects the increasing importance of the most recent fund returns on the DC participants' accumulated wealth. The total pension fund of an entire population with a heterogeneous investment strategy (composed of Portfolios A, B, and C) is labeled "Portfolio ABC." Similarly, "Portfolio B" signifies the aggregated fund of the portion of the population who invest their funds in Portfolio B. The return on Portfolio ABC is equal, therefore, to one-third of the return on Portfolio A, one-third of the return on Portfolio B, and one-third of the return on Portfolio C.

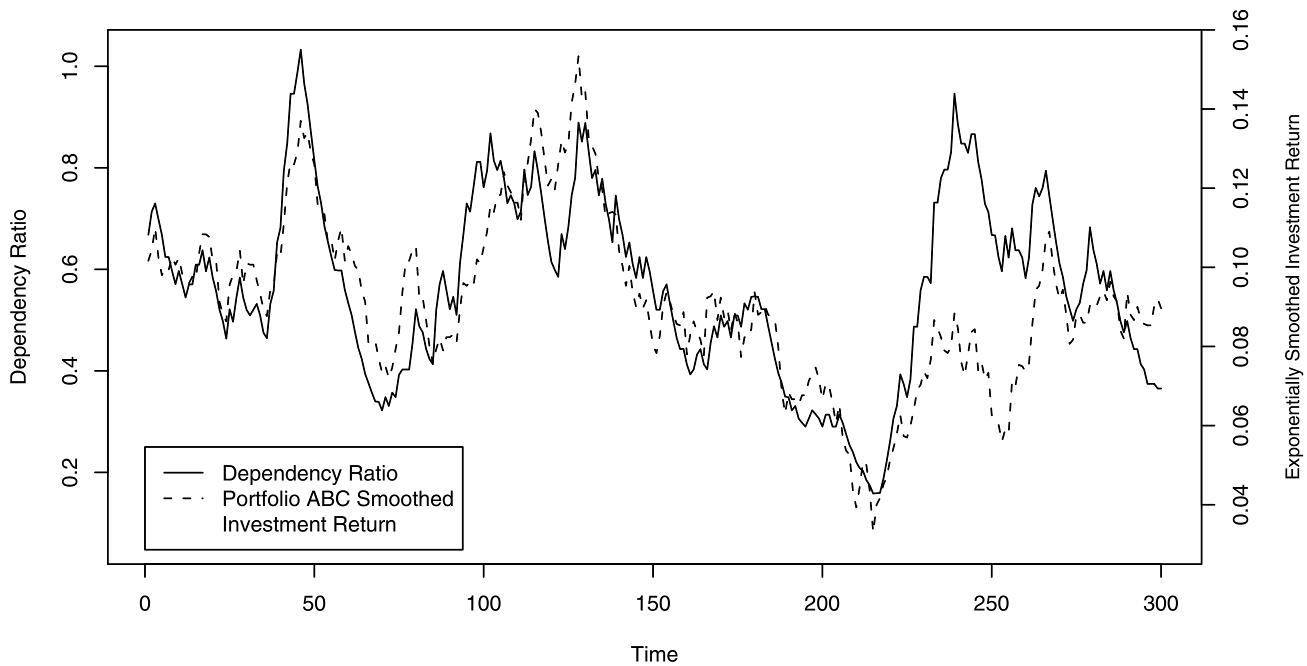
Over the span of 300 simulated years, Figure 6a tracks the volatile dependency ratio (solid line) for the heterogeneously invested population and the equally volatile smoothed investment return (dashed line) of Portfolio ABC. It is worth noting that since the time span of the simulation is relatively short, the time series plots will appear different from one simulation trial to the next. We will, therefore, focus on observations that are consistent across all the trials executed.

The results demonstrate the important effect the smoothed investment return has on the dependency ratio. Their harmonious movement is displayed in Figure 6a, where a double *y*-axis facilitates their comparison. Unsurprisingly, during

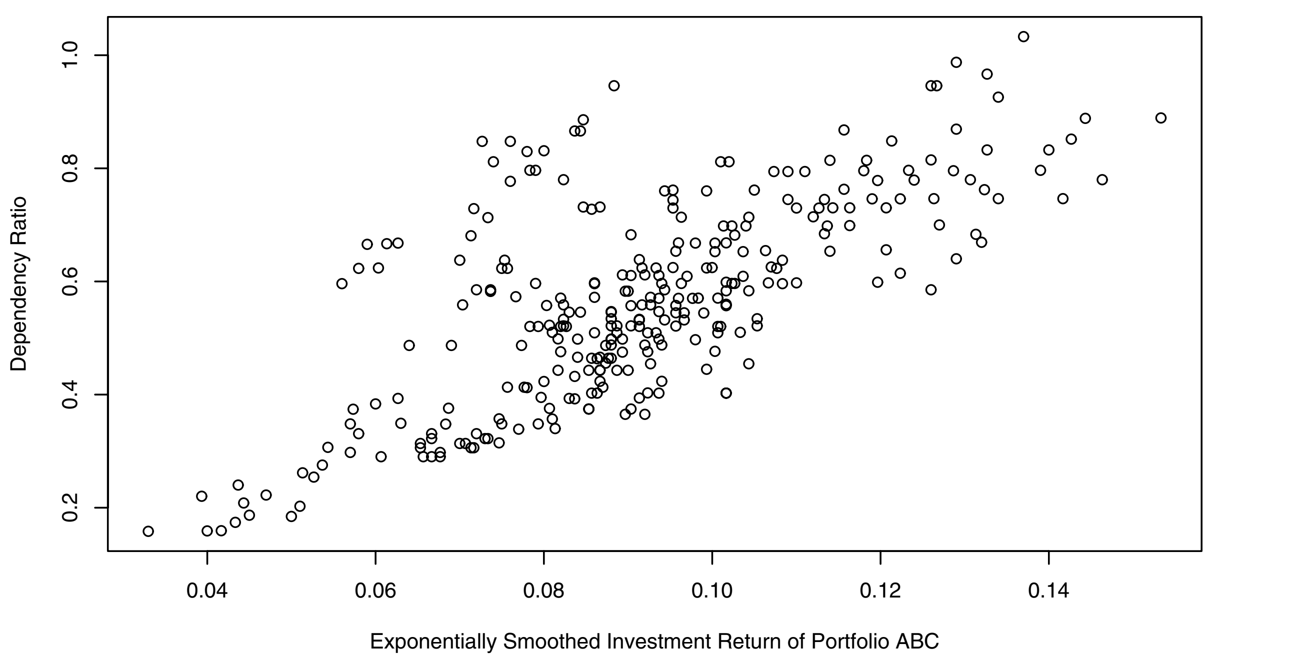
Figure 6

Simulated Time Series and Scatterplot of Dependency Ratio and Smoothed Investment Return

(a)

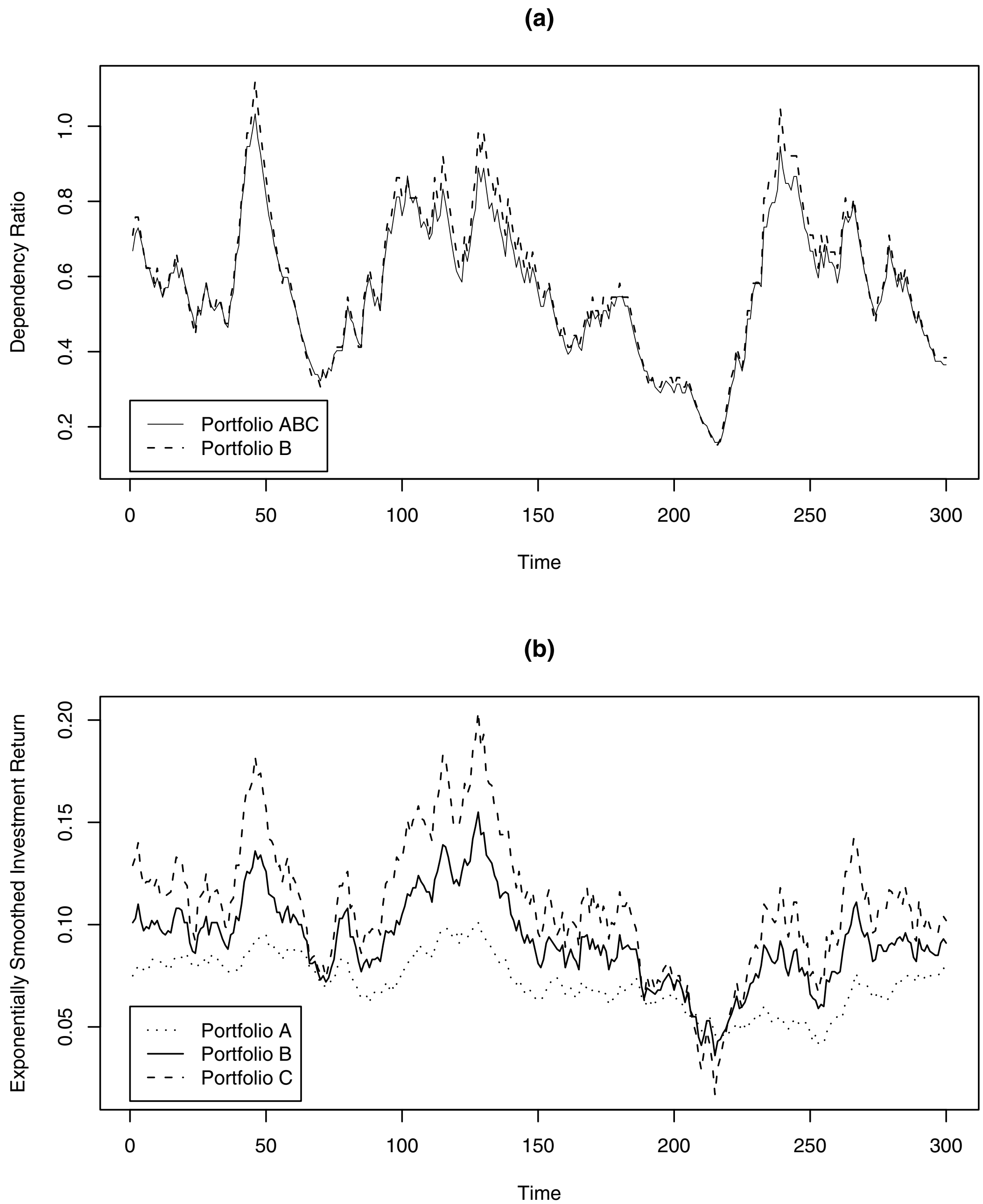


(b)



Notes: (a) Simulated time series plot of the dependency ratio (left-hand scale) for a population with a heterogeneous Portfolio ABC investment strategy and the exponentially smoothed investment return of Portfolio ABC (right-hand scale), and (b) a scatter plot of these two values for each simulated year.

Figure 7
Simulated Time Series Plots of Dependency Ratio and Smoothed Investment Return



Notes: (a) Simulated time series plot of the dependency ratio for a population invested in Portfolio ABC and for a population invested in Portfolio B. (b) Smoothed investment return plots of Portfolios A, B, and C.

bull markets, members are able to retire earlier, thus causing the dependency ratio to rise. Likewise, a bear market drives down the dependency ratio. The scatterplot in Figure 6b illustrates the highly positive correlation between the dependency ratio and the smoothed interest rate; specifically, the correlation coefficient is over 70%.

The next striking conclusion is that diversifying the asset allocation decisions among the participants does little to reduce the significant fluctuation of the dependency ratio, which is controlled primarily by the unpredictable performance of the market. From the same 300-year simulation, Figure 7a plots the dependency ratio for a

Table 3
Long-Term Correlation among Smoothed Investment Returns of Each Portfolio, Based on 8000-Year Simulation

	Portfolio A	Portfolio B	Portfolio C
Portfolio A	1	0.87	0.79
Portfolio B	0.87	1	0.99
Portfolio C	0.79	0.99	1

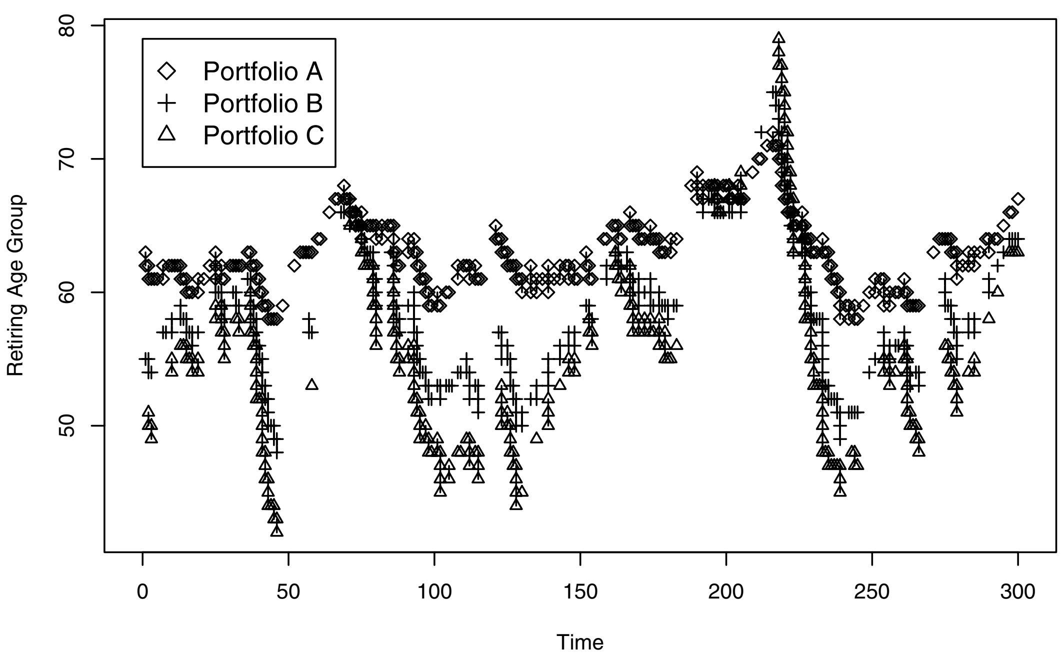
Table 4
Long-Term Correlation among Annual Log Returns of Efficient Investment Assets, Based on 8000-Year Simulation

Asset	Index-Linked Bond	Bond	Equity
Index-linked bond	1	0.40	−0.17
Bond	0.40	1	0.09
Equity	−0.17	0.09	1

population with a heterogeneous investment strategy of Portfolio ABC and for the members of the population with a homogeneous investment strategy of Portfolio B. The minimal impact on the dependency ratio is due to the similar movement of the smoothed investment return of each portfolio, which Figure 7b illustrates. Simulating over an extended time period reveals that the fluctuations of the *smoothed rates* are highly correlated. This is in spite of the low correlation between the *annual log returns* on index-linked bonds and equities of −17%. Table 3 lists the correlation coefficient among the smoothed investment returns of each portfolio, and Table 4 displays the correlation coefficients between the annual log returns of the relevant assets.

The inconsistency in the ages of retirement from one year to the next within every investment strategy is surprising when we consider the identical nature of each member; to be more precise, each retiree has identical investment portfolios, levels of contribution, employment histories, mortality statistics, and retirement decision-making behavior. Figure 8 displays the ages of the newly retired members within each year. In terms of offering a young age of retirement, the superior performance of the equity investment is ap-

Figure 8
Time Series Plot of Ages of New Retirees for Each Year of Simulation



Notes: Each symbol indicates an age group retiring at that particular time. Plotted age groups are divided according to their investment strategy through the use of a different symbol for each portfolio choice. To facilitate viewing, a thin line connects the age groups with the same investment strategy and who retire in the same year.

parent. The age plot of the portfolio with the least amount of equities is indeed more stable (diamonds), but it consistently delivers the highest ages of retirement, causing the stability to be a poor tradeoff for the near certainty of a delayed retirement. In contrast, the pure equity portfolio produces ages of retirement (triangles) much more variable, but consistently lower than the other investment strategies.

For each year of simulation in Figure 8, the vertical clusters of symbols, followed by gaps of no retirements, indicate that members collectively decide to retire or not retire at the same time because of their common dependency on the financial market's performance. The pattern of retirements revealed by Figure 8 suggests that there could be

- An unstable demand for different types of financial assets if individuals choose to revise their portfolios at retirement and adopt a less risky investment strategy and
- An unstable demand for financial assets in general owing to variation in the relative number of workers and retired members. A growing proportion of workers would create an increased demand for those assets that make up Portfolios A, B, and C. In contrast, a large population of retired members would result in a greater supply of assets as pensioners sell financial assets to support retirement consumption.

For example, there could be a greater demand for equities when there are fewer retirements. Figure 8 demonstrates this scenario between years 47 and 71. Here there is a gap of 11 years, which is almost immediately followed by a gap of 12 years, during which time no members of the Portfolio C group retire. A single triangle at year 58 represents the retirement of the only retiring age group holding Portfolio C during nearly a quarter of a century. Once retirement becomes affordable, there could be a massive shift from equities to bonds. In Figure 8 such an event occurs in our scenario beginning in year 218, where the steep vertical of triangles indicates a multitude of retirements among Portfolio C holders. During the following 10 years, the retirement age drops from age 80 to age 58. In year 227 alone, seven Portfolio C cohorts (ages 58–64) retire.

The lack of stability of the dependency ratio is worrisome and could have far-reaching effects, as is the DC pension system's inability to retire the participants at systematic and reasonable ages. The first concern is the late retirement risk for plan members. If such a case did occur in reality, factors other than finances could force retirement, such as illness or disability, thus causing insufficient pensions and hardship for such retirees. In other words, the DC pension scheme would fail these elderly participants. Second, the swings in the labor force could affect the country's economy. A comprehensive understanding of this is outside the scope of this study, but a few of the repercussions could include the following:

- According to the results, the participants make their retirement decisions in large numbers. Recall that the simulation model does not consider the interrelations among the sectors of the economy and their effect on asset prices. A successful market would generate the retirement of the masses, leading toward a rise in asset liquidation, while a suffering market would encourage workers to delay their retirement and continue saving. Market dynamics could be influenced by the irregular retirement patterns, and there could possibly be market equilibrium upset, as Section 1 briefly discusses.
- Second, the benefit of a high dependency ratio is that participants are able to retire early. A large number of retirements could, nevertheless, cause a labor shortage. It could also reduce tax revenue, because when somebody retires, his or her income would generally decrease.
- On the flip side, unemployment could increase during times of a low dependency ratio. To explain further, if the older citizens cannot afford to retire, then they would need to cling onto their jobs. If there was a fixed supply of jobs, this could cause unemployment to rise among younger members of society who cannot penetrate the workforce. If the elderly were forced to retire, they would not have a sufficient pension. This could create additional elderly poverty or reliance on state-funded welfare programs.
- Finally, the yo-yo effect of the dependency ratio insinuates that a DC pension design does an incredibly poor job in terms of balancing the economy's production with consumption. For

example, if the source of labor declines in size, then the society's production would suffer while the consumption would remain level. The anticipated repercussion of such a scenario is price inflation (Brown, Damm, and Sharara 2001).

Overall, it would be difficult for the government to manage a fluctuating dependency ratio. The production and consumption of goods, the tax revenue, and the necessary social programs would be as unpredictable as the stock market.

4. DAMPENING THE DEPENDENCY RATIO VOLATILITY

We recognize that the instability in the dependency ratio could be due to the combination of a simple model and a strict retirement rule. So our initial observations could portray a form of worst-case scenario. If this is the case, then refining the model and adding realistic features should hopefully dampen the volatility. We see in Section 3.2 that adding diversity in the participants' investment choice does not successfully dampen the dependency ratio owing to the long-term correlation among the assets. In this section we introduce further heterogeneity into the model to identify what, if any, aspects of a DC system contribute to greater stability in the dependency ratio.

We first discuss the theoretical basis for each of the additional features. Following this, we present the dependency ratio outcome resulting from the inclusion of each individual modification. We also experiment with the aggregate impact of incorporating the combination of modifications in Section 4.5. We find that none of the specified model modifications ameliorate the dependency ratio volatility.

4.1 Multiple Ages of Entry

A fixed age of plan enrollment for all members of the population is unrealistic since, although there could exist an age after which it is mandatory for working citizens to contribute to a state pension plan, it is unlikely that all participants would have entered the workforce by that age. Participants undoubtedly begin employment at a variety of ages that could exceed the mandatory age of pension plan enrollment. We now observe the impact of incorporating multiple ages of entry into the DC plan. To do so, we assume that one-third of the population enters the plan at age 20, one-third at age 25, and the final third at age 30.

We make the assumption that individuals will maintain consistency in their plans for retirement, irrespective of their ages of entry. All else being equal, a participant would expect to retire at an older (younger) age if they began saving for retirement at a later (earlier) age. We assume, therefore, that an individual will increase their contribution rate if they begin saving at a later age than the norm or will reduce their contributions if they enter the plan at a younger age. The implication of this assumption is that the average dependency ratio and retirement age of each entry age group are approximately equal.

To target the same average retirement age among the three groups, participants who enter the plan at age 20 should reduce their savings to 8.25% of salary per annum (a 1.75% decrease), those who begin at age 25 should continue to contribute 10%, and 30-year-old entry aged participants should save at the higher rate of 12.5% (a 2.5% increase).

4.2 Multiple Levels of Merit

We could upgrade the authenticity of the wage simulation by incorporating additional variety in the career paths. This could be done by varying the average annual growth that accounts for merit increases, which is currently fixed at 2%.

The participants' wage growth modeling is outlined in Appendix A. In summary, the wage growth is made up of two components: general wage inflation and merit increases. General wage inflation is simulated stochastically and affects each member of the population in an identical manner. We model merit increases deterministically and as a function of the individual employee's years of service, granting more significant merit increases during the early years of employment.

In our first DC population model, the merit contribution to salary growth amounts to an average annual rate of 2%. If we raise this value to 3%, we could introduce employees with flourishing careers (high flyers). Similarly, we could include less successful workers by decreasing the assumption to 1% (low flyers). We could test the effect of multiple career paths by having one-third of the population fall under each category (low flyers, medium flyers, and high flyers). Figure 9 plots the merit model function at the three assessed levels.

4.3 Multiple Contribution Rates

Bringing in several contribution rates among the participants could also enhance the retirement savings portion of the model. In reality, a vastly different retirement savings pattern prevails across the U.S. population. Wise (2003) discussed the great variation in the savings behavior of U.S. residents on account of public and employer policies, as well as their social and economic environment. We strengthen the model by experimenting with three contribution levels across the population: 9%, 10%, and 11%.

4.4 The Effects on the Dependency Ratio

The model refinements do not appear to improve the stability of the dependency ratio. Figure 10 contains the opportunity sets of each of the population simulations involving the three model improvements. Section 3.1 outlines the method of simulation, except we are now considering a reduced number of investment portfolios. There are 20% increments for each available asset, totaling 126 portfolios. Figure 10a is the benchmark plot; that is, it is a less detailed version of Figure 4 since it is based on the original set of assumptions from the first model except with a reduced number of executed investment strategies. The efficient frontier from Figure 10a acts as a point of reference when assessing the effect of the modifications; therefore, it is represented by a thin solid line in Figures 10b, c, and d.

Adding variety to the pension plan entry ages in plot b, the contribution rates in plot c, and the merit scales in plot d produces dependency ratios that are virtually identical in shape and value to that produced under the homogeneous scenario, as shown in plot a. This does not imply, however, that the dependency ratio's standard deviation is unmoved by each individual modification. For example, under the bond and equity investment strategies, the dependency ratio is directly correlated with the entry age. Table 5 lists the standard deviations for each entry age generated by the bond and equity portfolios, including the standard deviation of the aggregate population's dependency ratio. Comparing the three entry age outcomes under each portfolio, it appears that increasing the population's entry age reduces the dependency ratio's volatility. This is a reasonable

Figure 9

Merit Scale Function When Average Annual Merit Growth Is 1%, 2%, and 3%

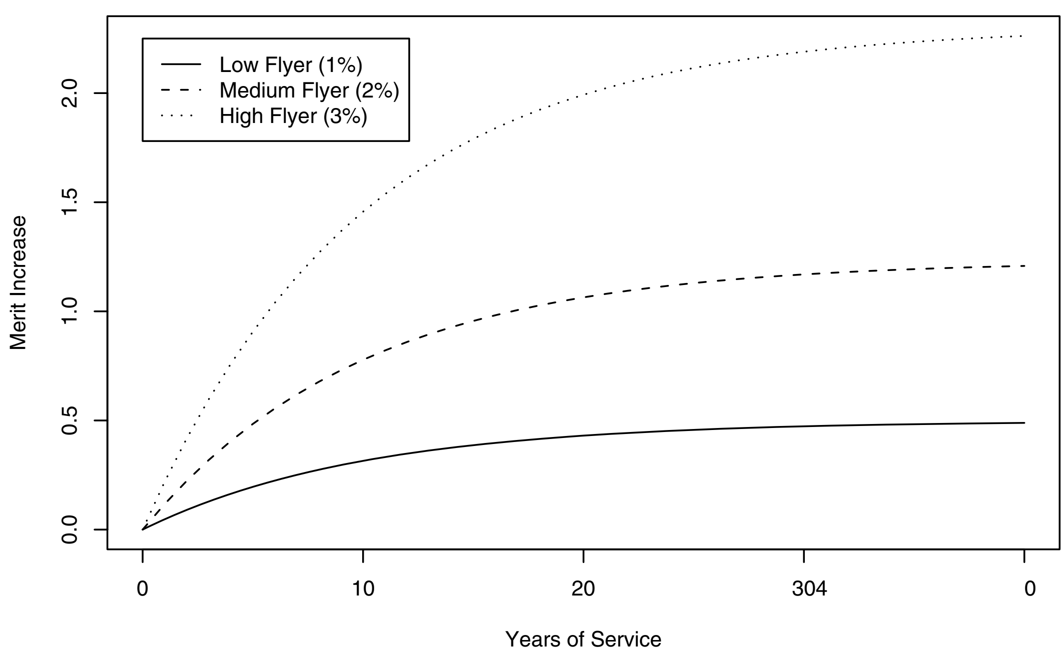
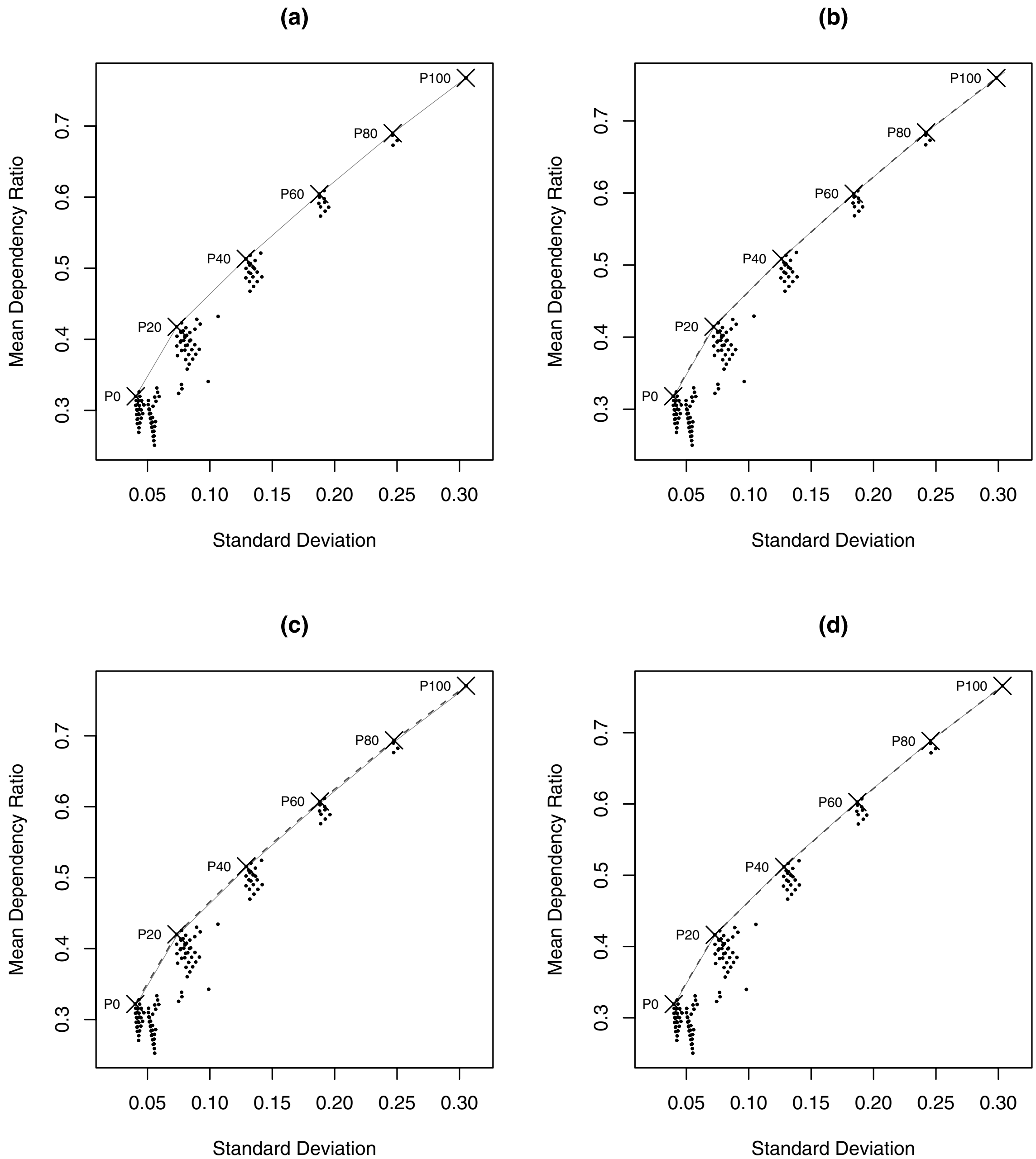


Figure 10
Fig. 4 with Three Separate Model Improvements



Notes: Similar to Fig. 4, except considering a reduced number of investment portfolios in plot (a) and including the following model improvements: (b) multiple ages of entry, (c) multiple contribution rates, and (d) multiple merit scales. To facilitate comparison, the efficient frontier from plot (a) (thin solid line) is drawn in each of the plots (b), (c), and (d), along with their respective efficient portfolios (dashed line).

result since an older age of plan enrollment, accompanied by a larger contribution rate, should shorten the participants' exposure to the stock market fluctuations. For example, consider the extreme case in which all participants begin saving at a very late age (for example, age 60), at which time they make an enormous contribution. If the participants make a large enough contribution so that they are able to retire immediately, they would have no exposure to the stock market fluctuations. In this exaggerated example, there is no fluctuation in the dependency ratio since

Table 5
Simulated Dependency Ratio Standard Deviation of Aggregate Population and of Each Individual Entry Age Group within the Population

Asset	Entry Age of 20	Entry Age of 25	Entry Age of 30	Total Population
100% bond	0.106	0.099	0.086	0.099
100% equity	0.358	0.305	0.248	0.305

everyone above age 60 is consistently retired while everyone below is not. The results also show that it is a population of equity investors whose dependency ratio volatility is most sensitive to the age of enrollment assumption.

Increasing the population's entry age and contribution rate to achieve stability in the dependency ratio is contrary to the objective of this study, since our aim is to understand the impact of a realistic DC pension plan over the careers and generations of an entire population. The relevance of this exercise is to examine whether including multiple ages of entry would reduce the overall volatility of the aggregate population's dependency ratio. The conclusion is that the three model enhancements do not diminish the severity of the dependency ratio's volatility.

Figure 11 further illustrates the unchanging volatility of the dependency ratio by revisiting the results of the second model in Figure 6a and including the upgrades. We continue to assume that the population invests across the three portfolios that Section 3.2 describes. Heterogeneity is incorporated within the entry ages in Figure 11a, contribution rates in Figure 11b, and career paths in Figure 11c. The dependency ratios resulting from the adjusted assumptions are plotted with dotted lines. The original path of the dependency ratio, which Figure 6a shows, continues to be represented by a solid line as a benchmark for comparison. The dependency ratio's behavior is nearly identical despite the improvements.

4.5 Collective Impact of All Model Improvements

The final test is the simultaneous incorporation of all the model refinements. As with each individually applied modification, we evaluate their aggregate impact by first assuming that one-third of the population invests in Portfolios A, B, and C. Thereafter, we incorporate the model improvements in the following manner:

- The members within each third of the population are subdivided into three groups, where one group makes contributions of 9%, another of 10%, and the last of 11%.
- After this, nine groups exist in the population, each of which is further subdivided into three entry age groups, where

  The first group enters the pension plan at age 20, while reducing their contribution rate by 1.75%

  The second group enters at age 25, while maintaining their contribution rate and

  The remaining group enters at age 30, while increasing their contribution rate by 2.5%.

  To summarize the three contribution rate levels associated with each entry age, the three groups of investors are each divided into the following nine categories:

Contribution Level	Entry Age of 20	Entry Age of 25	Entry Age of 30
Low	0.0725	0.0825	0.0925
Medium	0.09	0.10	0.11
High	0.115	0.125	0.135

- Finally, each of the 27 groups is further partitioned into three career path groups: high flyers, medium flyers, and low flyers.

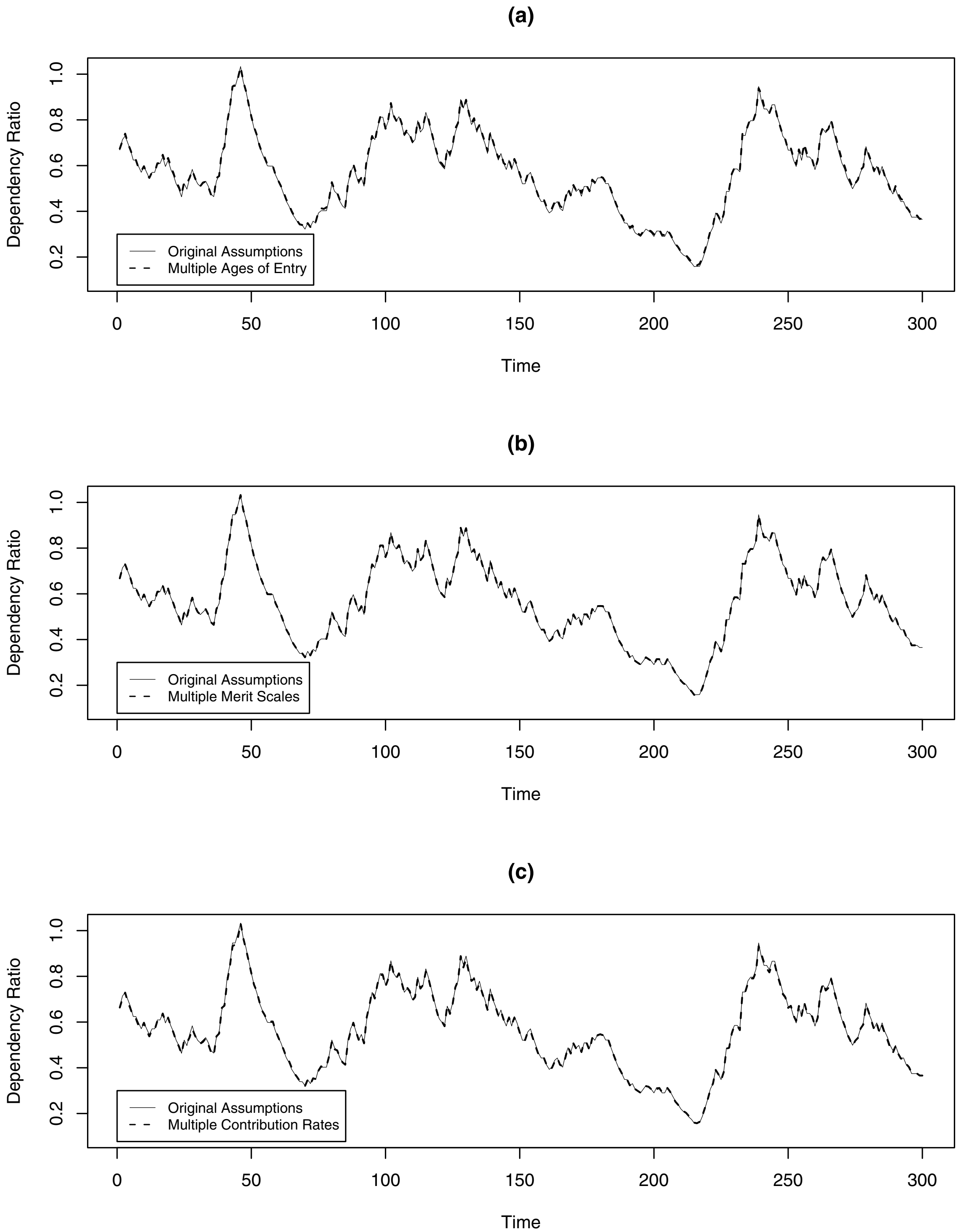
The result is 81 groups, each possessing different retirement savings characteristics. Figure 12 displays the simulated outcome. Once more, the dependency ratio behavior is nearly unchanged.

5. RISK MANAGEMENT WITHIN THE DC PLAN DESIGN

In this section we briefly examine a feature that could be included in the DC system design to reduce the potential dependency ratio fluctuations. A method of possibly adding stability to the dependency ratio is to put a restriction on equities in the personal accounts, but we find that there would be several drawbacks of such a policy.

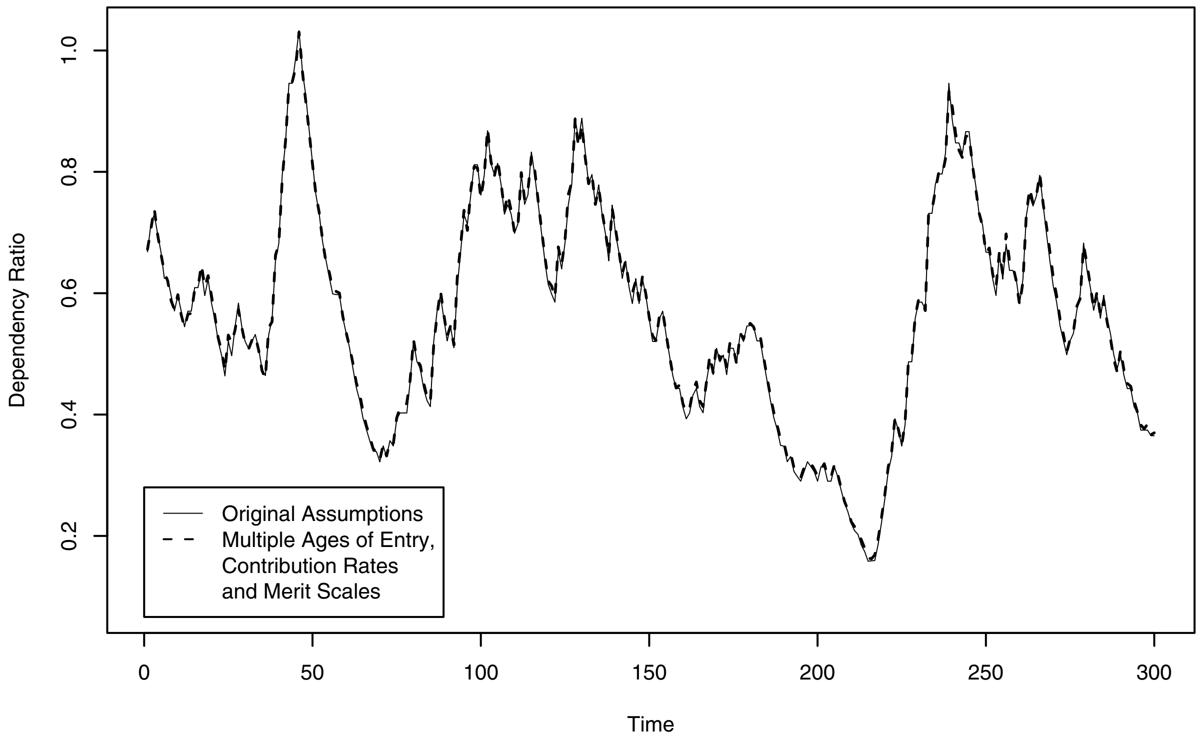
Figure 11

Fig. 6(a) with Three Separate Model Improvements



Notes: Simulated time series plot of the dependency ratio from Figure 6(a) (thin solid line), with the following separate model improvements (dashed line): (a) multiple ages of entry, (b) multiple contribution rates, and (c) multiple merit scales.

Figure 12
Fig. 6(a) with Three Additive Model Improvements



Notes: Simulated time series plot of the dependency ratio from Figure 6(a) (thin solid line), with the collective addition of multiple ages of entry, multiple contribution rates, and multiple merit scales (dotted line).

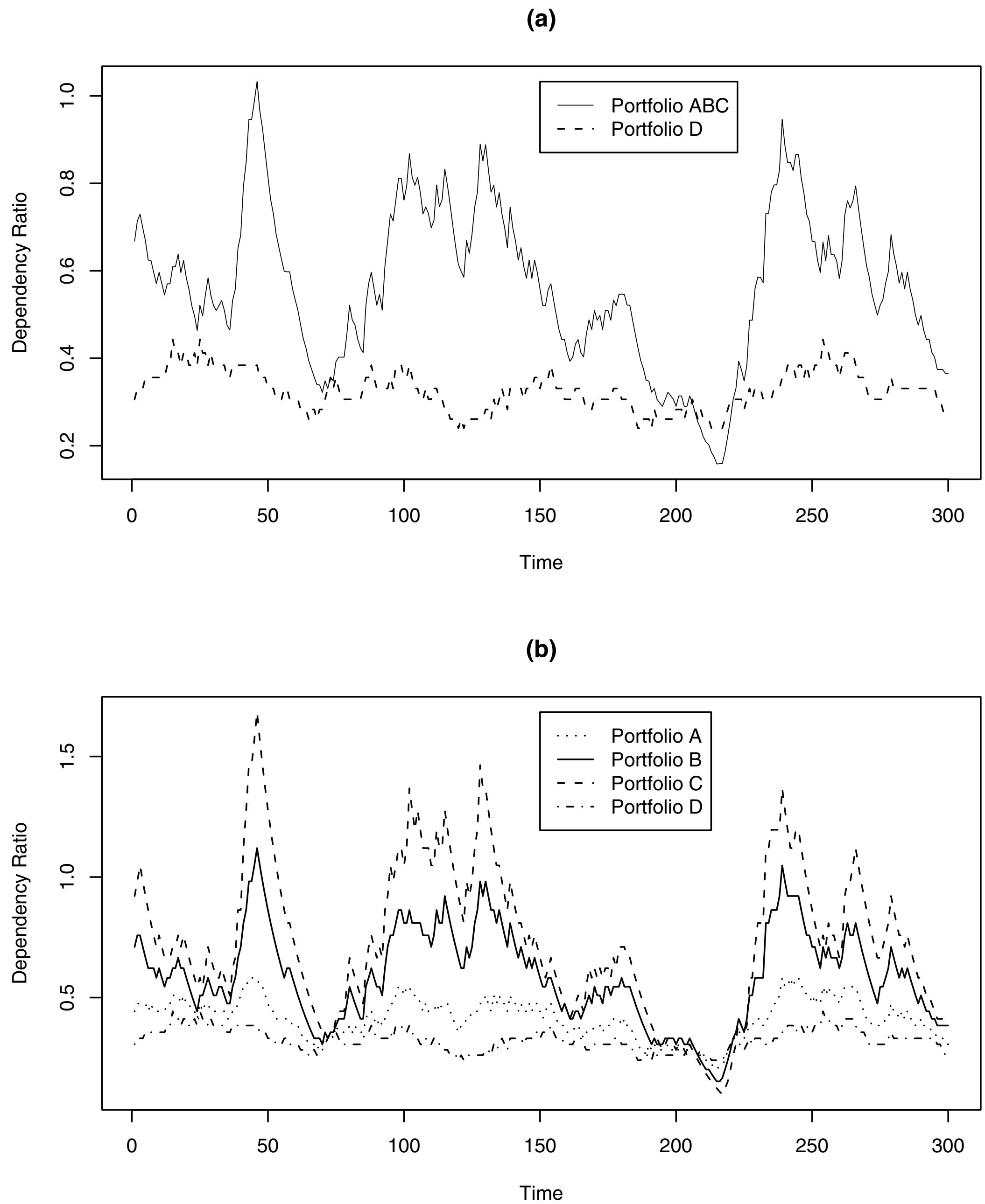
Figure 4 ranks equities as the largest source of dependency ratio volatility. We also discover in Figure 7a that the instability of the dependency ratio is not diminished when using equities as a tool for diversifying the asset allocation decisions among the participants (this is on account of the long-term correlation of the assets). In view of these two findings, a straightforward method to reduce the dependency ratio volatility would be to eliminate equities from the permitted investments in the individual DC accounts. Such a feature could be implemented with government policies that show preferential tax treatment toward fixed-income bonds and index-linked bonds only.

Over the same 300-year simulation in Figure 6a, Figure 13 presents the dependency ratio for a population that does not allocate their pension savings to equities. Assuming that they continue to invest efficiently, the appropriate portfolio according to Figure 4 is P0, which we now refer to as Portfolio D. As expected, there is an impressive improvement in the behavior of the dependency ratio relative to the heterogeneously invested population (Fig. 13a) and to each of the other homogeneously invested populations (Fig. 13b). The stability in the dependency ratio created by such a policy would likely be accompanied by numerous negative repercussions, including the following:

- A delay in the nationwide average retirement age: Eliminating equities should tighten the dependency ratio's movement about its mean, but the downside is that the mean exhibited in Figure 4 is relatively low and corresponds to an average retirement age just under age 67.
- A rise in the overall costs: Increased contributions is a means of reducing the average retirement age, but this would create a more expensive pension plan.
- Inadequate supply of bonds: There would also need to be a large supply of high-quality bonds, which may not be the case since companies would always need equity capital.

Figure 13

Simulated Time Series Plots of Dependency Ratio for a Population of Portfolio A, B, C, D or ABC Investors



Notes: Simulated time series plot of the dependency ratio for a population invested in Portfolio D, and for a population invested in (a) Portfolio ABC, (b) Portfolio A, B, or C.

If a restriction on the permissible assets was included in the DC plan design features, there could potentially be a reduction to the dependency ratio oscillations. Nevertheless, this could be tied in with several negative aspects that outweigh the benefits.

6. CONCLUSION

The drawbacks of implementing a DC pension system at the state level have been discussed and documented in numerous studies. In a DC plan design, investment, inflation, and mortality risk are transferred completely to individual workers rather than being shared across the population and over generations. Our study tackles the huge unknown effect of a national retirement savings pool on the economy and the labor force. We began with an extreme scenario that gave rise to considerable instability in the proportion of workers from one year to the next. We then considered realistic model improvements to determine which aspects of a DC pension system add stability. In reality, there exists great variation in the retirement savings behavior across a population; therefore, we introduced additional heterogeneity in the modeling of the population's investment strategies, contribution rates, entry ages into the pension plan, and career paths. We found that none of these dampened the volatility. We observed that restricting the participants' investment options to low-risk portfolios would provide some stability; this option, nevertheless, has significant shortcomings, including increased costs and problems with the supply of relevant assets.

Our flexible age of retirement model results suggests that, if a DC pension system were introduced to an entire society to serve as their principal salary replacement in retirement, the financial market would have an exceptional impact on the proportions of retirees and workers from one year to the next. Hence, we propose that the significant fluctuation in the market's performance could produce corresponding swings in the population's workforce demographics. Further to the detriment of the society's labor force structure, the unpredictability of the financial markets could produce ambiguous and unmanageable retirement ages, which could lead to personal hardship and anxiety for the individual DC member. The volatile demand for financial assets could potentially upset market equilibrium, while the unstable aggregate retirement pattern could be disastrous not only at the individual level, but also for the economic health of the entire population.

APPENDIX: ACCUMULATION MODEL

In this section we describe the arbitrage-free stochastic model used in this study for modeling the dynamics of the wage growth and the asset rates of return. The specific model chosen to simulate some of the economic processes is the Vasicek model (Vasicek 1977). This is a one-factor model for the term structure of interest rates within a continuous-time framework. We first describe the underlying economic processes. Following this, we present the wage model and the stochastic differential equations for the available assets for investment.

A.1. UNDERLYING ECONOMIC PROCESSES

Throughout this section we use the general notation $x_1(t)$ to ensure notational compactness of the stochastic differential equations (SDEs). The economic processes are numbered as follows:

1. $x_1(t)$ Instantaneous risk-free nominal rate of interest at time t
2. $x_2(t)$ The log total return on equities from time 0 to time t
3. $x_3(t)$ Instantaneous risk-free *real* rate of interest at time t
4. $x_4(t)$ The consumer price index (CPI) log growth from time 0 to time t
5. $x_5(t)$ The log *real* return on wages from time 0 to time t.

We will sketch the stochastic differential equation (SDE) for each economic process under two probability measure models—the risk-neutral and the real world—since the risk-neutral measure is relevant for pricing bonds and the real world is needed for simulation purposes.

Following the assumption that the instantaneous risk-free rate of interest, $x_1(t)$, follows the Vasicek model, its SDE under the risk-neutral measure Q is as follows:

$$dx_1(t) = \alpha_1(\tilde{\mu}_1 - x_1(t))\, dt + \sum_{j=1}^{5} \sigma_{1j}\, d\tilde{W}_j(t), \tag{A.1}$$

where $\tilde{W}_1(t), \ldots, \tilde{W}_5(t)$ are independent standard Brownian motions under the risk-neutral probability measure, Q. In the model:

- σ_{1j}: the local volatility associated with risk j (i.e., $\tilde{W}_j(t)$)
- $\tilde{\mu}_1$: the risk-neutral long-term mean rate
- α_1: the rate at which the rate of return reverts back to its long-term mean
- δ_j: the market price of risk associated with the source of risk j (see eq. [4])
- μ_1: the real-world long-term mean rate, where $\mu_1 = \tilde{\mu}_1 + \sum_{j=1}^{5} \frac{\sigma_{1j}\delta_j}{\alpha_1}$ (see equation A.1).

Without loss of generality, we choose to parameterize the model so that $\sigma_{ij} = 0$ for $j > i$. The general parameters in the following equations maintain, however, these parameters for notational convenience.

Returning to equation (A.1), we continue by transferring from Q to the real-world measure P by replacing $d\tilde{W}_j(u)$ with $dW_j(u) + \delta_j dt$. Therefore, the SDE for $x_1(t)$under measure P is as follows:

$$\begin{aligned} dx_1(t) &= \alpha_1(\tilde{\mu}_1 - x_1(t))\, dt \\ &\quad + \sum_{j=1}^{5} \sigma_{1j}(dW_j(t) + \delta_j dt) \\ &= \alpha_1(\mu_1 - x_1(t))\, dt \\ &\quad + \sum_{j=1}^{5} \sigma_{1j} dW_j(t), \end{aligned} \quad \text{(A.2)}$$

where $W_j(t)$ is a standard Brownian motion under the real-world probability measure P and

$\mu_1 = \tilde{\mu}_1 + \sum_{j=1}^{5} \frac{\sigma_{1j}\delta_j}{\alpha_1}$. If $\sigma_{11} > 0$ (and $\sigma_{12} = \cdots = \sigma_{15} = 0$), then typically δ_1 is less than zero. This ensures that investments in fixed-interest bonds (which are risky in the short term) attract a positive premium.

The SDE for $x_2(t)$ under Q is

$$dx_2(t) = \left[x_1(t) - \frac{1}{2}\left(\sum_{j=1}^{5} \sigma_{2j}^2\right)\right] dt + \sum_{j=1}^{5} \sigma_{2j} d\tilde{W}_j(t),$$

where σ_{2j} is the local volatility associated with risk j.

The value of equities as the price of a tradable asset, meaning that the asset pays no dividends, is represented by $S(t)$, where $S(t) = S(0)e^{x_2(t)}$. The SDE for $S(t)$ under Q is

$$dS(t) = S(t)\left(x_1\ (t)\ dt + \sum_{j=1}^{5} \sigma_{2j}\ d\tilde{W}_j(t)\right).$$

We transfer from Q to the real-world measure P by replacing $d\tilde{W}_j(u)$with $dW_j(u) + \delta_j\, dt$. Therefore, the SDE for $x_2(t)$ under measure P is

$$dx_2(t) = \left[x_1(t) + \sum_{j=1}^{5}\left(\sigma_{2j}\delta_j - \frac{\sigma_{2j}^2}{2}\right)\right] dt + \sum_{j=1}^{5} \sigma_{2j}\ dW_j(t), \quad \text{(A.4)}$$

where $\sum_{j=1}^{5} \sigma_{2j}\delta_j$ is the equity risk premium over the risk-free rate of interest.

The SDE for $S(t)$ under measure P is

$$dS(t) = S(t)\left[\left(x_1(t) + \sum_{j=1}^{5} \sigma_{2j}\delta_j\right) dt + \sum_{j=1}^{5} \sigma_{2j}\ dW_j(t)\right].$$

Under the assumption that the instantaneous risk-free real rate of interest, $x_3(t)$, follows the Vasicek model, its SDE under the risk-neutral measure Q is as follows:

$$dx_3(t) = \alpha_3(\tilde{\mu}_3 - x_3(t))\ dt + \sum_{j=1}^{5} \sigma_{3j}\ d\tilde{W}_j(t).$$

Similar to $x_1(t)$, the following are the parameters used in the SDE for $x_3(t)$:

- σ_{3j}: the local volatility associated with risk j
- $\tilde{\mu}_3$: the risk-neutral long-term mean real rate
- α_3: the rate at which the rate of return reverts back to its long-term mean
- μ_3: the real-world long-term mean real rate, where $\mu_3 = \tilde{\mu}_3 + \sum_{j=1}^{5} \frac{\sigma_{3j}\delta_j}{\alpha_3}$.

Under measure P, the SDE for $x_3(t)$ is

$$\begin{aligned} dx_3(t) &= \alpha_3(\tilde{\mu}_3 - x_3(t))\, dt \\ &\quad + \sum_{j=1}^{5} \sigma_{3j}(dW_j(t) + \delta_j\, dt) \\ &= \alpha_3(\mu_3 - x_3(t))\, dt + \sum_{j=1}^{5} \sigma_{3j}\, dW_j(t), \end{aligned}$$

where $\mu_3 = \tilde{\mu}_3 + \sum_{j=1}^{5} \frac{\sigma_{3j}\delta_j}{\alpha_3}$.

The SDE for the log growth of the consumer price index, $x_4(t)$, under the risk-neutral measure Q is

$$\begin{aligned} dx_4(t) &= \left[x_1(t) - x_3(t) - \frac{1}{2}\left(\sum_{j=1}^{5} \sigma_{4j}^2\right)\right] dt \\ &\quad + \sum_{j=1}^{5} \sigma_{4j}\, d\tilde{W}_j(t), \end{aligned}$$

where σ_{4j} is the local volatility associated with risk j.

Similarly, the SDE for $x_4(t)$ under P is

$$\begin{aligned} dx_4(t) &= \left[x_1(t) - x_3(t) + \sum_{j=1}^{5}\left(\sigma_{4j}\delta_j - \frac{\sigma_{4j}^2}{2}\right)\right] dt \\ &\quad + \sum_{j=1}^{5} \sigma_{4j}\, dW_j(t). \end{aligned}$$

The value of the consumer price index (CPI) at time t, represented by $C(t)$, is $C(t) = C(0)e^{x_4(t)}$.

We model the logarithmic real return on wages in a similar fashion as the other four assets. That is, under the risk-neutral measure Q:

$$dx_5(t) = \tilde{\mu}_5\, dt + \sum_{j=1}^{5} \sigma_{5j}\, d\tilde{W}_j(t),$$

where σ_{5j} is the local volatility associated with risk j and $\tilde{\mu}_5$ is the risk-neutral long-term mean real salary growth.

We arrive at the SDE for $x_5(t)$ under the real-world measure P by setting $\mu_5 = \tilde{\mu}_5 + \sum_{j=1}^{5}\sigma_{5j}\delta_j$ and replacing $d\tilde{W}_j(u)$ with $dW_j(u) + \delta_j\, dt$, which gives

$$dx_5(t) = \mu_5\, dt + \sum_{j=1}^{5} \sigma_{5j}\, dW_j(t).$$

A.2. THE WAGE GROWTH

The growth in an individual's wage is generally made-up of general wage inflation and merit increases. We assume that general wage inflation is decomposed into two parts: price inflation and real wage growth. The wage level at time t for an individual who begins employment at time 0 is:

$$\begin{aligned} Y(t) &= \frac{m(t)}{m(0)} Y(0)e^{x_4(t)+x_5(t)-x_4(0)+x_5(0)} \\ &= \frac{C(t)m(t)}{C(0)m(0)} Y(0)e^{x_5(t)-x_5(0)}, \end{aligned}$$

where $Y(t)$ is the individual's wage level and $m(t)$ represents the merit component of the wage growth and is a function of the worker's length of employment.

Between times $t - 1$ and t, this model assumes that the general wage inflation depends on that year's annual inflation, $x_4(t) - x_4(t - 1)$ (Wilkie, 1995), a long-term mean real rate μ_5, and a random component that is independent of the first four sources of risks. In other words, we assume that $\sigma_{5j} = 0$ for $j = 1, 2, 3, 4$.

We also add to the salary growth a deterministic merit increase that depends on the individual's years of service. We assume that an average employee will receive merit increases that amount to an annual average of 2% over their career. If an average working career is 40 years and the annual overall merit increase is 2%, an individual's salary would be expected to grow by 121% from the merit increases alone. It is generally assumed that merit increases are more significant during the earlier part of one's career and level off as one becomes more senior. Following from this, we model the merit scale using an exponential function with an exponential coefficient of -0.1 to create a relatively gradual rate of change. We need to translate the function on the y-axis to allow for a total lifetime merit increase of 1.81. The merit function is plotted in Figure 9 and is given by

$$m(t) = 1.81 - e^{-0.1t},$$

where t is years of service.

A.3. AVAILABLE ASSETS FOR INVESTMENT

In addition to equities, the available assets for investment are the following:

- Cash: a one-year zero-coupon risk-free bond
- Index-linked cash: a one-year index-linked zero-coupon risk-free bond
- Fixed-interest bond: an irredeemable bond, which pays 1 at the end of each future year and
- Index-linked bond: an irredeemable index-linked bond, which pays $C(T)$at the end of each future year T.

If one unit is invested in each of the assets at time 0, then their values at time t are represented by

- Cash fund, $G(t)$
- Index-linked cash fund, $B(t)$
- Fixed-interest bond fund, $F(t)$ and
- Index-linked bond fund, $Q(t)$,

with the reinvestment of any coupon income.

According to the Vasicek model, the price at time t of a risk-free zero-coupon bond that matures at time T is given by

$$P_1(x_1(t),t,T) = e^{(A_1(t,T) - B_1(t,T)x_1(t))},$$

where

$$B_1(t, T) = \frac{1 - e^{-\alpha_1(T-t)}}{\alpha_1},$$

$$A_1(t, T) = (B_1(t, T) - (T - t)) \left(\tilde{\mu}_1 - \sum_{j=1}^{5} \frac{\sigma_{1j}^2}{2\alpha_1^2}\right) - \sum_{j=1}^{5} \frac{\sigma_{1j}^2 B_1(t, T)^2}{4\alpha_1}.$$

Similar to $P_1(x_1(t),t,T)$, the price at time t of a risk-free zero-coupon bond that matures at time T and yields a real rate of return is given by

$$P_3(x_3(t),t,T) = c^{(A_3(t,T) - B_3(t,T)x_3(t))},$$

where

$$B_3(t, T) = \frac{1 - e^{-\alpha_3(T-t)}}{\alpha_3},$$

$$A_3(t, T) = (B_3(t, T) - (T - t)) \left(\tilde{\mu}_3 - \sum_{j=1}^{5} \frac{\sigma_{3j}^2}{2\alpha_3^2}\right) - \sum_{j=1}^{5} \frac{\sigma_{3j}^2 B_3(t, T)^2}{4\alpha_3}.$$

At time t, the price of a zero-coupon index-linked bond that matures at time T is

$$C(t)P_3(x_3(t),t,T),$$

where $C(t)$ is the value of the consumer price index at time t and $P_3(x_3(t),t,T)$ signifies the expected risk-neutral growth component of the bond that is attributed to the guaranteed real rate of return. Therefore, an investment of $\$P_3(x_3(t),t,T)$ at t will return $\$\frac{C(T)}{C(t)}$ at T.

The rate of return on each of the funds between time t and $t + 1$ is as follows:

$$G(t + 1) = \frac{G(t)}{P_1(x_1(t),t,t + 1)},$$

$$B(t + 1) = B(t) \frac{C(t + 1)}{C(t)P_3(x_3(t),t,t + 1)},$$

$$F(t + 1) = F(t) \frac{1 + \sum_{T=t+2}^{\infty} P_1(x_1(t + 1), t + 1, T)}{\sum_{T=t+1}^{\infty} P_1(x_1(t),t,T)},$$

$$Q(t + 1) = Q(t) \frac{C(t + 1)}{C(t)} \left(\frac{1 + \Sigma_{T=t+2}^{\infty} P_3(x_3(t + 1), t + 1, T)}{\Sigma_{T=t+1}^{\infty} P_3(x_3(t), t, T)}\right).$$

Additionally, the annuity factor for retirement age $e + t$ at time t, ignoring expenses, is given by

$$\ddot{a}_{e+t}(t) = \sum_{s=0}^{\infty} P_1 \; (x_1(t),t,t + s) \; {}_s\mathrm{p}_{e+t}.$$

One disadvantage in the Vasicek model is that it allows for the instantaneous risk-free rate of interest to become negative, which is somewhat unrealistic, particularly when calculating the price of an annuity. Therefore, the instantaneous risk-free rate of interest, $x_1(t)$, used in calculating the fair annuity value has a floor of 0%.

A.4. PARAMETER ESTIMATES AND DATA

This model has been calibrated using U.S. data provided partly by Professor David Wilkie and partly from the following public sources:

- U.S. Federal Government Treasury nominal securities, which can be found at the U.S. Government Federal Reserve Web site (www.federalreserve.gov/releases/h15/data.htm) and
- The Bureau of Economic Analysis, U.S. Department of Commerce (these tables can be accessed from www.bea.doc.gov/bea/dn/nipaweb/SelectTable.asp).

The limited U.S. index-linked bond data have been supplemented by U.K. data, which are available online at the Heriot-Watt/Faculty and Institute of Actuaries Gilt Database (see www.ma.hw.ac.uk/~andrewc/gilts/).

The model parameter estimates are the following:

Parameter	μ_1	α_1	σ_{11}	δ_1	σ_{21}	σ_{22}	δ_2
Estimate	0.051	0.15	0.0185	−0.152	−0.011	0.156	0.328

Parameter	μ_3	α_3	σ_{31}	σ_{32}	σ_{33}	δ_3
Estimate	0.027	0.56	0.0075	0	0.0086	−0.419

Parameter	σ_{41}	σ_{42}	σ_{43}	σ_{44}	δ_4	μ_5	σ_{55}
Estimate	0	−0.013	0	0.0022	−0.066	0.01	0.0205

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