NEWTON, LEIBNIZ AND ACTUARIAL SCIENCE

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ABSTRACT

In this bicentenary year of Gompertz (1825) we advance a conjecture; that the split between Newtonian and Leibnizian forms of calculus occurring in the 18th century had a major influence on the development of actuarial science in the 19th century. When the growing life insurance industry in Britain needed mathematical expertise, a profession with Newtonian roots was created to supply it, while elsewhere in Europe it was found in universities with Leibnizian roots. We consider the consequences up to the present day, when analysis in the Leibnizian branch has led via Kolmogorov (1933) to modern financial mathematics. The two branches appear to have nearly rejoined, and though having different foundations both embody an abiding desire for respectability and the search for certum ex incertis (motto of the former Institute of Actuaries).

KEYWORDS

Actuarial Science, Calculus, Gompertz, Leibniz, Newton

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1. Introduction

> "And as I use, from great preference, the fluxional notation of our great Newton, instead of the furtive notation used on the Continent and now much used in England, ..." Gompertz (1862)

Two hundred years ago, Benjamin Gompertz published his eponymous law of mortality (Gompertz 1825), a paper of wide scientific importance (Hooker 1965, Kirkwood 2015). It is timely, therefore, to consider the context in which it was introduced to actuarial science; a context in which Gompertz could write the words quoted above. This short paper outlines a hypothesis exemplified by those words, namely that the great split in calculus engendered by the dispute between Newton and Leibniz influenced the direction taken by actuarial science as it emerged in organized form, which happened first in Britain (Section 2). Having not much need of advances in mathematics beyond Newton, the actuarial profession there was shaped by social and institutional influences that have been well-described elsewhere (Section 3). Also, that while the Newtonian branch of mathematics stagnated in Britain, pace Gompertz above, the Leibnizian[1] branch of analysis flourished in universities in Continental Europe, leading by around 1900 to Lebesgue measure and

[1]We use the term 'Leibnizian' to refer only to the mathematics descending from Leibniz's calculus, and not to Leibniz's contributions to philosophy.

integration. Once Kolmogorov (1933) followed through and axiomatized the theory of probability, the venerable technique possessed by 'actuaries of the first kind' (Bühlmann 1987) was in due course overshadowed by modern financial mathematics[2] (Section 4). As far as life insurance is concerned, the gap in theory has been repaired up to a point, largely by a modern Scandinavian school, but where this leaves life insurance practice and mathematics today is an open question (Section 5). We do not in any sense prove the hypothesis in this note, but it is a conjecture of interest to actuaries and historians of actuarial science.

2. Newton versus Leibniz and its Aftermath in Britain

It is well-known that Isaac Newton invented the calculus, and that his laws of motion and gravitation launched mathematical physics. Some British mathematical physicists carried on the tradition, such as Hamilton, Maxwell and Kelvin[3]. Mathematicians interested in mathematics itself, however, were almost all to be found across the English Channel. Gottfried Leibniz had also invented the calculus, and the two inventors had been drawn into a vicious dispute over priority. Historians generally agree that Newton's loyal followers then marched British mathematics into a wilderness where it stayed, give or take, for 150 years (Bardi 2006, Gray 2006, Bellhouse 2014, Heard 2019).

The priority dispute was merely wounded national pride; of more consequence was the difference in notation. Newton denoted derivatives by dots, for example $\dot{x}$ and $\ddot{x}$, called them 'fluxions', and used a notation for integrals that only historians recall[4]. Leibniz denoted derivatives by $dx/dt, d^2x/dt^2$ and so on, and integrals by $\int x\,dt$. His dt notation made the connection between differentiation and integration much more transparent, and proved adaptable to multiple dimensions (partial derivatives, multiple integrals), variational calculus (functional derivatives) and probability (Lebesgue measure and integral).

Leibniz's superior notation was adopted by continental analysts such as Euler and the Bernoullis, whose legacy continued through Lagrange, Gauss, Cauchy, Riemann, Weierstrass et al. to Cantor, Lebesgue and Borel at the turn of the 20th century (see Gray (2015); Bell (1953) is still entertaining if sometimes romanticized). Meanwhile in Britain, mathematics other than mechanics and physics withered, and the introverted remains were mostly to be found in Cambridge (Craik 2008, Heard 2019). Recovery took generations. Writing two centuries on, Rouse Ball marked the start of the road back thus:

> "Towards the beginning of the last century the more thoughtful members of the Cambridge school of mathematics began to recognize that their isolation from their continental contemporaries was a serious evil." (Rouse Ball 1908).

While Woodhouse, one of Newton's successors as Lucasian Professor, campaigned to wean his colleagues off fluxions, Gauss was enjoying his most brilliant years in Hanover (Rouse

[2]Notably, when financial mathematicians needed a theory of credit risk, which is essentially the survivorship over time of corporations in various states of creditworthiness, they had no need whatever of any existing actuarial theory of insuring biometric risks (Bielecki & Rutkowski 2002).

[3]Respectively Irish working in Ireland, Scottish working in England and (Northern) Irish working in Scotland.

[4]Dotted fluxional notation is still used in mechanics, a subject in which 19th century British mathematicians excelled. Whatever its benefits, it never contributed to the elucidation of probability.

Ball 1908, Dunnington 2004). Unlike Gompertz, Cambridge was largely won over to the 'continental notation' by 1850[5] (Grattan-Guinness 2011), but by then the split was more deep-seated. Heard (2019) noted, even of the later 19th century, that:

> "Complex analysis had arrived late because of Cambridge's lengthy domination of the British mathematical scene and the closed nature of the community, which together resulted in one generation of Cantabrigians teaching the next, with very little influx of new blood." (Heard 2019, p.117)

Gray (2006), in polemical mood, surveyed the state of mathematics in Victorian Britain in unflattering terms, saying inter alia: "[what] strikes me too often about these people is their belief that they are the equals of their continental peers when no such comparison can be entertained" (p.179). Not until the 20th century did G.H. Hardy drag the notorious Mathematical Tripos at Cambridge out of the long shadow cast by Newton, saying it had "... effectively ruined serious mathematics in England for a hundred years" (Snow 1967).

So it happened, that when actuarial science was called into being by practical necessity in Britain in the early 19th century, mathematics had split into a continental, Leibnizian mainstream, and a British, Newtonian backwater. Neither front-line mathematicians nor universities would play any part. This was the world in which Gompertz introduced his mortality law in 1825, and would be described as "one of the greatest mathematicians of Europe" (Woolhouse 1862), "the last of the learned Newtonians" (de Morgan (1865) cited in Hooker (1965)) and "one of the best mathematicians of the day" (Schooling 1924), see Hooker (1965) for further references.

3. The Growth of the (Respectable) British Actuarial Profession

Much has been written on the early history of statistical thinking, the emergence of life insurance, and the rôle of the life table in both (Hald 1990, Johnson 2017, Turnbull 2017, Daston 2023). As far as actuarial science in Britain is concerned, the usual story starts with Equitable Life in 1762 and proceeds through the pragmatic invention of sound reserving, surplus measurement and bonus distribution[6] in which Dr Richard Price was

[5] Thanks to his Jewish religion, Gompertz had no formal links with Cambridge, but he remained faithful to Newton and fluxions all his life. He justified his view quoted at the start of this paper on the moral ground that the furtive differential notation: "... appears to give Leibnitz a greater claim to originality, to the prejudice of Newton, than I think he is justly entitled to" (Gompertz 1862). In fairness, Gompertz's dislike was based on genuine doubts about infinitesimals, shared by others, though these doubts were largely cleared up by Cauchy before Gompertz's death (Gray 2015, Chapter 5). In the same paper Gompertz wrote: "But whilst I am endeavouring to clear away the shadowing clouds which may obstruct the brilliant light of Newton's lamp from being duly perceived by the scientific eye, I am willing to acknowledge that Leibnitz's differential d has, in many instances, done great service in his own hands, in the hands of Euler, Lagrange, Laplace, and of a host of scientific men whose names cannot be pronounced without gratitude and reverence".

[6] This short paper necessarily omits many interesting topics such as, in the Anglophone world, earlier activity around individual annuities and reversions (Bellhouse 2017), the prior claim to actuarial eminence of the Scottish Ministers' Widows' Fund and its connection with Colin Maclaurin (Dunlop 1992, Milevsky 2024) and the turn towards contribution bonus in the USA (Homans 1863). Similarly in Germany, actuarial activity around pensions and widows' funds was strong in the mid-19th century independently of any life insurance industry, see footnote 19 and the references in Seal (1977) for example.

the key figure (Ogborn 1962, Cox & Storr-Best 1962). Any survivorship bias lingering in this account[7] ended abruptly in 2002 when the society failed to meet its liabilities to certain policyholders (Penrose 2004).

Much else shaped the nascent actuarial profession (Turnbull 2017), but we should not overlook the isolation of British mathematics post-Newton, the moribund state of mathematics in British universities, and an environment described thus by Gray (2006):

> "Now, I think we must admit that there was a striking degree to which British culture in the nineteenth century just didn't like mathematics, except as an aid to science. There is an astonishing degree of philistinism, or, at the very least, rampant utilitarianism about British life in the period (perhaps more in the first than in the second half)." (Gray 2006, p.182)

Accounts of the foundation of the Institute of Actuaries in 1848 (Simmonds 1948) and the Faculty of Actuaries in 1856 (Davidson 1956) make it clear, mainly by default, that universities were irrelevant. Fairly minimal knowledge of calculus would be required even up to the time that King's textbook (King 1887) was revised in 1902[8].

British society enjoyed stable laws and politics, relatively speaking, and after Waterloo in 1815 would not face another convulsive war for 99 years. Economically, she was open to commercial enterprise, about to be transformed by the twin motors of manufacturing and railways, and had a growing middle class with wealth (and female relatives) to protect. Spurned by the universities, new professions emerged to meet new needs.

A more critical assessment is that professions emerged chiefly to claim rights over well-remunerated activities (Porter 1995). For example, describing evidence given to the Parliamentary Select Committee on Assurance Associations in 1853:

> "Still, several witnesses allowed that the government might reasonably require publication of financial records. This, the actuaries argued, should take the form of a straightforward factual presentation. It should not include summary numbers standing for assets and liabilities, which could imply that solvency could be determined by anyone capable of noticing which was the greater." (Porter 1995, p.112)[9]

A desire for respectability and status was surely also present. From Porter again:

> "The main object of the new institute of actuaries [sic] was to secure the recognition befitting a profession. This entailed more than a demonstration of technical competence, and the oft-expressed disdain for mere calculation must be understood as part of a strategy of legitimation in a society that gave little respect to mere technical experts" (Porter 1995, p.110)

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[7]See, for example, Babbage (1826) on the 'gene pool' out of which the Equitable Life model was to emerge as the dominant variety, with plenty of forgotten failures along the way.

[8]Menzler (1960, p.1) described King (1887) as "a monumental tome ... in which the calculus of finite differences was still paramount" and noted (p.31, in a chapter entitled 'The Stony Path to Textbooks') that "... in later days the calculus was embarked on at school at an earlier age than was usual in 1900", clearly indicating the expected level of entry to the profession.

[9]The actuaries had a point, despite Porter's scepticism. In a later age, unease about just comparing two numbers in a balance sheet, exemplified by Redington (1952), has led to increasingly elaborate statutory solvency regimes for life insurers.

As Paxman (1991, p.106) put it: "... the professional tag was so sought after that the rush to respectabilize was unstoppable. By the end of the century, architects, accountants and engineers had all created their own professional institutions." Ironically, 'respectability' might be roughly translated as a status equalling that of university degrees. This desire perhaps achieved its most momentous result with the establishment of business schools in the USA in the late 19th and early 20th centuries, to cement the status of business at the apex of credentialed society alongside law, divinity and medicine (Khurana 2007).

With awareness of professional status (as opposed to professionalism) came an inbuilt conservatism, guarding against intruders like an immune system[10]. A few examples are given below, to add to Gompertz's words at the start of this note.

- The Institute of Actuaries' efforts to obtain a Royal Charter were frustrated until 1884 by the opposition of many of the most senior actuaries, who objected to certificated learning and refused to join the new body[11] (Simmonds 1948).
- An early sign of universities awakening in England was the development of mathematical statistics, centred not on Cambridge but on University College London (Porter 1986, Lehmann 2011, Magnello 2011). Its tortuous passage into the actuarial syllabus is described with dry humour by Menzler (1960), including 'l'affaire Seal' and the mock trial Rex versus Seal[12].
- When Sidney Benjamin presented a paper to an Institute of Actuaries meeting in 1971 showing computer simulations of a maturity guarantee, the reception was such that the chair "... closed the meeting and classed the discussion as confidential so that its content would never be published" (Turnbull 2017, pp.195–196). A version of the paper later appeared as Benjamin (1976) and today looks mainstream, largely thanks to the later influence of 'the Wilkie model' (Wilkie 1986, 1995).
- The further reaction of the British actuarial profession to modern financial mathematics from roughly 1980–2000 is related in Turnbull (2017) and **?**. Its significance here is that it represented the first real incursion of the Leibnizian branch of mathematical history onto territory regarded by the profession as its own[13]. The key event was probability theory being made part of 'proper' mathematics by Kolmogorov (1933). That is a topic of Section 4.

Such conservatism is not necessarily to be deprecated; actuaries are not primarily research scientists and enthusiasm often rings alarm bells; a 'wait and see' approach may often be

[10] Other bodies of actuaries were just as capable of fierce disputation, see Seal (1977, p.434) on the arguments among 19th century German actuaries over multiple decrement theory.

[11] The Faculty of Actuaries in Scotland was granted its Royal Charter in 1868.

[12] Hilary Seal (1911–1984) was a graduate statistician and Fellow of the Faculty of Actuaries. He was one of several actuaries who worked in operations research in the Admiralty during the Second World War, who found that they "were not by divine right the outstanding practical statisticians which we had cosily thought ourselves to be" (anonymous, cited by Menzler (1960, p.73)).

[13] One can argue about when and where the Leibnizian branch announced itself to traditional actuaries. One candidate would be Hoem's introduction of Markov processes to problems of life and health insurance (Hoem 1969, 1988), following Sverdrup (1965). However, the Markov assumption preserved the emphasis on expected values (Section 4), via Kolmogorov's and Thiele's differential equtions.

wise. The catch is that, by the time a rather small profession has waited and has seen, it may have been left behind, and then be unable to effect a 'reverse takeover' of the novel body of thought (if anyone should think that desirable).

The brief conclusion of this section is that the stated purpose of the British actuarial professional bodies, to apply mathematical skills to benefit society, need not be doubted, and Royal Charters bestowed the ultimate seals of approval. Behind that shield, though, professional prerogatives could be defended against trespasses including, from time to time, new mathematics. One of the forces shaping the profession, a lasting consequence of the great calculus split, was that the defenders were the intellectual heirs of Newton, as represented (rather extremely) by Gompertz.

4. From Leibniz to Kolmogorov's (Respectable) Model of Probability

We must briefly consider the social situation in Continental Europe in the 19th century. After 1815, the scientific Great Powers were France and (collectively) the German states and 'science' meant the Napoleonic Grandes Écoles in one, and the Humboldtian research universities in the other. Both kinds of institution were, to an extent unknown in Britain, organs of the state regardless of the state's current position between the extremes of monarchy and republic. Official bodies knew where to find mathematical experts as needed, and there was no vacuum for professional bodies to fill. When industrialization and conditions conducive to a life insurance market followed along, fifty to a hundred years behind Britain, there was a supply of mathematicians to meet the demand.

Just as mainstream mathematics had been neglected in Britain, probability and statistics stood outside mainstream mathematics. The only results of real mathematical interest were the Law of Large Numbers and the Central Limit Theorem, computation was tedious, and applications were directed towards a search for expected values, not the study of random variables (Daston 2023). Reducing a game of chance to its expectation as a 'fair' outcome was a moral action (Johnson 2017). Even in the later 19th century, the idea of randomness as natural variation rather than measurement error was not universally accepted (Porter 1986); rather it was something best reduced to a Gaussian distribution or, later, the fewest possible moments (Magnello 2011). This view of probability was summed up in the motto of the Institute of Actuaries, certum ex incertis, and thus by Hans Bühlmann: "Contrary to [the Actuary] of the First Kind in life assurance, whose methods were essentially deterministic, [the Actuary of the Second Kind] had to master the skills of probabilistic thinking" (Bühlmann 1987).

In 1900, Hilbert set out his famous problems (Yandell 2002) the sixth of which included finding satisfactory axioms for probability theory. This was achieved by Kolmogorov (1933) based on measure theory, the lineal descendent of the analysis begun by Leibniz's calculus[14,15] (Gray 2015). Kolmogorov added, inter alia, a model of information

[14]Readers may judge for themselves the place of probability in mathematics from the account in Yandell (2002), quoted here in its entirety: "Axiomatizing the theory of probabilities was a realistic goal; Kolmogorov accomplished this in 1933" (p.160). Probability is noted only as an application of integration in Bourbaki (1994), and is not mentioned at all in Temple (1981).

[15]Kolmogorov's axioms of probability did not stand entirely alone. von Plato (1994) identifies two other major lines of thought, mainly concerning epistemic questions of inference, by von Mises and de Finetti. The latter was an Italian actuary.

and a definition of conditional expectation, therefore allowing the future to be conditioned on the past[16], and giving solid ground to the stochastic processes begun by Markov and the Swedish actuary Lundberg. Above all, Kolmogorov made probability respectable[17].

Over the next few few decades probability, theoretical and applied, found its rightful place[18]. Almost inevitably, it would engulf financial risk and, just as inevitably, actuaries would be in no position to lead the way.

[16]The idea of a filtration originated with Doob (Meyer 2009). Of his own student days in the 1920s Doob said: "In those days there were very few advanced mathematics books in English, and those in French and German were too expensive for most students" (Snell 1997, p.302). Halmos recalled beginning his PhD studies with Doob in 1938: "What he originally had in mind was that I apply modern, high-powered measure-theoretic methods to the statistical ideas that R.A. Fisher was promulgating. He had already started to 'clean up Fisher' with his work on the method of maximum likelihood, but there was a lot left to do." (Halmos 1985, p.61)

[17]Respectability did not come easily. Meyer (2009) reported resistance to the Lebesgue integral in some quarters and "a certain bad mood, quite noticeable particularly in the United States" (p.12). Of a 1957–58 paper by Hunt he said: "This result unifying analysis and probability contributed to making the latter a respectable field" (p.9). Doob recalled that "many probabilists were complaining that measure theory was killing the charm of their subject without contributing anything new" (Snell 1997, p.306); in 1984 he received the Career Prize of the American Mathematical Society for "his fundamental work in establishing probability as a branch of mathematics ..." (Snell 1997).

[18]The growth of 'modern' probability is sufficiently recent that it is barely yet a matter for historians. Meyer (2009) and Mazliak & Shafer (2022) are notably helpful sources.

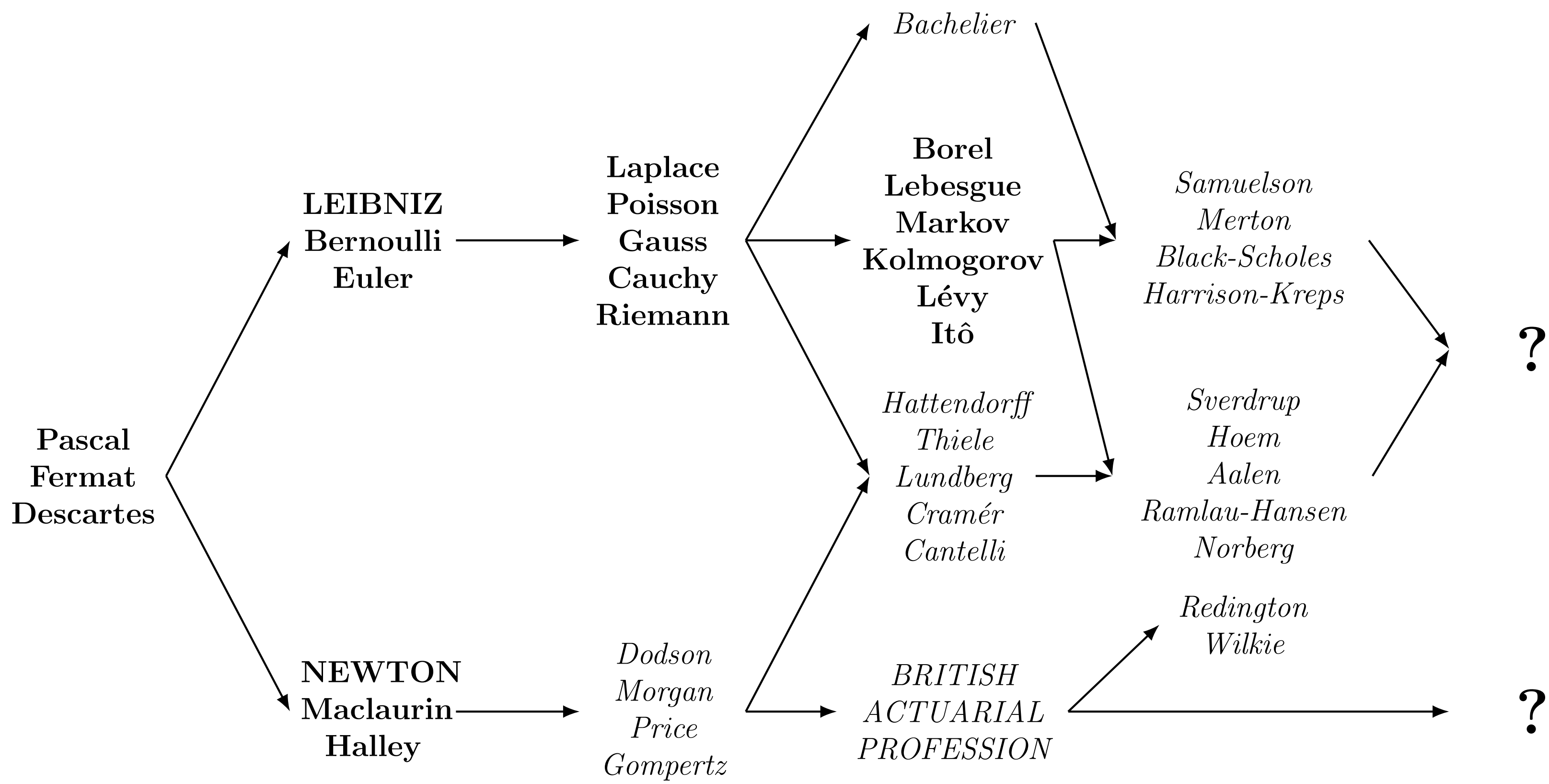

Figure 1: Schematic representation of the Leibnizian and Newtonian branches of history, as they are relevant to the development of the actuarial profession from the mid-18th century in Britain. **Bold** indicates the progress of theory, *italics* the application of theory to actuarial and financial problems. Sources are cited in the text.

A century after the foundation of the Institute of Actuaries, Frank Redington, the outstanding British actuary of his age, wrote a deservedly famous paper including, among other things, his theory of immunization (Redington 1952). With hindsight, it might seem that he was equipped with the wrong (non-stochastic) calculus. However, it would be absurd to suggest that Redington, an actuary working in the senior echelons of a British insurance company, should have known about a stochastic calculus still confined to research journals. Recall the Leibnizian pedigree of stochastic 'immunization' in Black & Scholes (1973), two decades after Redington. Financial economists trained in US universities (heirs to Humboldt's vision) rediscovered an old Parisian thesis (Bachelier (1900), examined by Poincaré no less), connected it with a Japanese paper on stochastic analysis (Itô 1944) directly descended from Lebesgue and Borel (France) via Kolmogorov (USSR), and linked everything up with recent non-arbitrage ideas.

Figure 1 illustrates what followed the great calculus split; the early stage at which Britain entrusted actuarial mathematics to a professional body, and the continuing development of mathematics in the Leibnizian branch of history. The figure highlights actuarial mathematics and its antecedents, and the Newtonian branch looks rather bare. It is fair to argue that if we drew a version highlighting actuarial practice, the Newtonian arm would be full of bulging muscle. While lacking professional bodies like Britain's, the Leibnizian arm would still include mathematicians of the calibre of Gauss[19] and Klein[20] (Germany), Thiele (Denmark), Cramér (Sweden), and Cantelli and de Finetti (Italy)[21].

Since Black & Scholes (1973), modern financial mathematics has often seemed able to make its own reality, at least temporarily, see Mackenzie (2006). Of course, this has happened during a revolution in computing power that rivals the industrial revolution in scale. At the same time, spurred on by the economics emanating from business schools (see Section 3), society has moved towards a model of contracts between individuals (counting corporations as such), and desirable risk is that whose management is believed to be understood. With echoes of the past, though, every 'Greek' hedged away in an incomplete market is in the spirit of early statisticians reducing probability distributions to their moments[22]. The aim and the illusion of 'certum ex incertis' spring once more to mind, this time mainly in the service of minimizing private capital deployed.

The Scandinavian influence on modelling biometric risks, including survival models,

[19]Gauss stabilized the Göttingen Professors' Widows' Fund in 1845–1851, saying of part of the work that took him a month: "In itself such a work is demanding, over 100,000 figures (according to my method of writing, where half or more are calculated in the head) ..." (Dunnington 2004, pp.233–234).

[20]Klein made Göttingen, the university of Gauss and Riemann, into the world's leading centre of mathematics, a status that lasted until 1933. In 1895, with support from the Prussian Parliament, Klein launched an actuarial science seminar at Göttingen "intended to train mathematicians and upper-level administrators for careers in the public and private insurance industry" (Tobies 2021, Section 7.6).

[21]The only comparable figures in Britain might be Maclaurin (footnote 6) and Sylvester. Sylvester was a founding Vice-President of the Institute of Actuaries, but for much of his life he was excluded from British universities by his Jewish religion, as Gompertz had been (Simmonds 1948, Bell 1953, Gray 2006). Of his employment as an actuary Bell said: "Such work for a creative mathematician is poisonous drudgery, and Sylvester almost ceased to be a mathematician." Gray's opinion was less positive: "Well known to his contemporaries for his brilliance, his boundless enthusiasm, and his total failure to communicate to lesser mortals, his work was even more scattershot and error prone than Cayley's" (p.179).

[22]For example: "An installment option (a fifth-order compound option), we will see, requires an analysis that takes into account at least nine moments for hedging stability." (Taleb 1997, p.203)

is present in Figure 1, in the node headed by Sverdrup. Here, mainly, life insurance mathematics has been given new foundations consistent with modern financial mathematics (Hoem & Aalen 1978, Ramlau-Hansen 1988a,b, Norberg 1991, 1992) and in that sense the two branches may be called convergent. However, nobody has yet truly conjoined the ancient law-of-large-numbers paradigm ruling independent risks and the newer hedging paradigm ruling singular financial risk, see Asmussen & Steffensen (2020) for example. Since that is a frontier of research, it is a suitable point at which to draw this historical note to a conclusion.

5. Certum ex Incertis All Over Again?

The conjecture outlined here is that the split triggered by the dispute between Newton and Leibniz over the invention of the calculus exerted an influence that has been overlooked in histories of actuarial science. It caused British mathematics to be cut off from the mainstream, just when other social forces in 19th century Britain, including moribund universities, led actuarial science down the path of professionalization. Thus British actuaries took officially sanctioned 'ownership' of an intellectual corpus isolated from academia locally and mathematics globally. The basis of this expertise was the contemporary certum ex incertis model of probability and statistics in which randomness was largely reduced to the expectation.

In continental Europe, the same social purpose was served by university-trained mathematicians, including some of eminence. Thus it was accommodated alongside the flourishing Leibnizian branch of mathematics, at home in Humboldtian universities.

When professionalized expertise in Britain met competition from mathematical statistics in the early 20th century, its response was to assimilate it, against some resistance. Nevertheless, Bühlmann (1987) could still describe life assurance actuaries as 'essentially deterministic'. Later in the 20th century, Kolmogorov's model of probability burst out of academia and into finance, and this further challenge to professionalized expertise (now worldwide) has also been met by assimilation, with varying degrees of local difficulty.

The conjecture is consistent with two intriguing parallels, neither of which is in itself an original observation. The first is the rather human search for respectability. British actuaries sought it in a profession, probabilists felt its absence until being rescued by Kolmogorov. The second is that the risk-neutral price represents a search for certum ex incertis as much as the actuarial present value ever did; it just rests on a different model, arising from the other branch of Figure 1. We may yet see Newtonian and Leibnizian branches truly join up, if dynamic hedging and avoidance of risk can be married to the law of large numbers and sharing of risk in a joint theory of valuation.

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Conflicts of Interest

None.

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