NEWTON, LEIBNIZ AND ACTUARIAL SCIENCE

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ABSTRACT

In this bicentenary year of Gompertz (1825) we advance a conjecture; that the split between Newtonian and Leibnizian forms of calculus occurring in the 18th century had a long-term influence on actuarial science in the 19th and 20th centuries. When the growing life insurance industry in Britain needed mathematical expertise, a profession with Newtonian roots was created to supply it, while elsewhere in Europe it was found in universities with Leibnizian roots. We consider the consequences up to the present day, when analysis in the Leibnizian branch has led via Kolmogorov (1933) to modern financial mathematics.

KEYWORDS

Actuarial Science, Calculus, Gompertz, Leibniz, Newton

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1. Introduction

> "And as I use, from great preference, the fluxional notation of our great Newton, instead of the furtive notation used on the Continent and now much used in England, . . ." Gompertz (1862)

Two hundred years ago, Benjamin Gompertz published his eponymous law of mortality (Gompertz 1825), a paper of wide scientific importance (Hooker 1965, Kirkwood 2015). It is timely, therefore, to consider the context in which it was introduced to actuarial science; a context in which Gompertz could write the words quoted above. This short historical essay examines a conjecture exemplified by those words, in brief that the great split in calculus engendered by the dispute between Newton and Leibniz influenced the direction taken by actuarial science as it emerged in organized form, which happened first in Britain; that this had consequences in the 20th century; and that this has been overlooked in histories of actuarial science.

The conjecture and the plan of the paper are as follows. There is good evidence that British calculus (as distinct from physics) was held back for a century and more, first by adherence to Newton's fluxional notation and the geometric philosophy that evolved alongside it, then by pursuit of an obsolete approach using Leibniz's notation (Section 2). At the same time, but separately, the isolation of British universities from changes wrought by the industrial revolution led to new applied expertise being located in professional bodies. Thus, we will argue, the actuarial profession founded in 19th century Britain entered the 20th century strong in practical matters but losing touch with mathematics and with academia (Section 3). Neglected in Britain, the Leibnizian[1] branch of calculus was transformed into analysis in universities in Continental Europe, leading by around 1900 to Lebesgue measure and integration. Once Kolmogorov (1933) followed through and created a new 20th century theory of probability, the venerable technique possessed by 'actuaries of the first kind' (Bühlmann 1987) was in due course overshadowed by modern financial mathematics[2] (Section 4) with consequences that have been documented elsewhere. As far as life insurance is concerned, the gap in theory has been repaired up to a point, largely by a modern Scandinavian school, but where this leaves life insurance practice and mathematics today is an open question (Section 5).

We do not in any sense prove the conjecture in this essay, and it is not a contribution to the historiography of science, but it ought to be relevant to actuaries interested in the origins of their subject.

2. Newton versus Leibniz and its Aftermath in Britain

It is now generally accepted that Isaac Newton and Gottfried Leibniz invented the calculus independently. However, that was not accepted fact at the time. After both had published their ideas, they were drawn into a vicious dispute over priority, which Newton pursued obsessively until his death in 1727 (Bardi 2006, Palomo 2021).

The priority dispute was unfortunate; of more consequence for the development of mathematics were differences of underlying philosophy and of notation (Sonar 2018). Newton's viewpoint tended to the geometrical[3], and ultimately he distrusted the use of infinitesimals (see also footnote 9). He denoted derivatives by dots, for example $\dot{x}$ and $\ddot{x}$, called them 'fluxions', and used a notation for integrals that only historians recall[4]. Leibniz's viewpoint was analytical, and he accepted infinitesimals although with misgivings. He denoted derivatives by $dx/dt, d^2x/dt^2$ and so on, and integrals by $\int x\,dt$. His dt notation made the connection between differentiation and integration more transparent, and proved adaptable to multiple dimensions (partial derivatives, multiple integrals), variational calculus (functional derivatives) and probability (measure theory).

Leibniz's notation was adopted by a long line of continental mathematicians from the Bernoullis and Euler, through Lagrange, Gauss, Cauchy, Riemann, Weierstrass et al. to Cantor, Lebesgue and Borel at the turn of the 20th century, see Gray (2015) (Bell (1953) is still an entertaining account, if sometimes romanticized). Their legacy was to turn calculus into modern analysis, founded on a theory of the real line and the infinite, with Lebesgue measure and integral a pivotal achievement (Hawkins 1970).

Meanwhile in Britain, whether for reasons of national pride or otherwise, mathematicians adhered to Newton's fluxional notation for the better part of a century and played no

[1] We use the term 'Leibnizian' to refer only to the mathematics descending from Leibniz's calculus, and not to Leibniz's contributions to philosophy.

[2] Notably, when financial mathematicians needed a theory of credit risk, which is essentially the survivorship over time of corporations in various states of creditworthiness, they had no need whatever of any existing actuarial theory of insuring biometric risks (Bielecki & Rutkowski 2002).

[3] Newton's viewpoint was not consistent in his writings on fluxions, and he did not always eschew infinitesimals. The geometrical roots of fluxions became firmer in the work of his successors (Guicciardini 1989, Section 2.3).

[4] Dotted fluxional notation is still used in mechanics. Whatever its merits, it never contributed to the elucidation of probability.

great part in the creation of analysis[5]. Writing of two outstanding exponents of fluxions, historian Niccolò Guicciardini said:

> "However neither Maclaurin nor Simpson, notwithstanding the value of their researches, played an important role in the development of the eighteenth-century calculus. What they did was to extend the range of application of the original Newtonian calculus to its utmost limits. The British were unable to follow the developments of the continental calculus which was being profoundly modified." (Guicciardini 1989, p.68)

Newton's other legacy of physics did flourish in Britain, even while calculus did not, exemplified by such luminaries as Hamilton, Maxwell and Kelvin[6]. In the words of Michael Atiyah[7]:

> "Newton's influence on English mathematics was in a way a negative one. Nationalist controversies, fanned by the Newton-Leibniz controversy, meant that Newton's successors turned their backs on continental developments. On the other hand, Newton's mathematical treatment of physics did establish a great tradition that produced figures like James Clerk Maxwell 150 years later." (Atiyah 2014, p.286)

For mathematics, recovery took generations. The most significant centre was Cambridge (Craik 2008, Heard 2019). Writing mainly of Robert Woodhouse and the 'Analytical Society of Cambridge' which existed from 1812 to about 1814, Rouse Ball said:

> "Towards the beginning of the last century the more thoughtful members of the Cambridge school of mathematics began to recognize that their isolation from their continental contemporaries was a serious evil" (Rouse Ball 1908, p.439)

and: "... analytical methods spread from Cambridge over the rest of Britain, and by 1830 these methods had come into general use there" (p.443). Guicciardini (1989) shows that this overlooked the one Irish and four Scottish universities and several military schools[8], especially work by Playfair (pp.99–103) and Wallace (pp.103–104, 120–121), whose article of 1815 was "... the first complete English treatise on the calculus written in differential notation"; see also Ackerberg-Hastings (2008) and Craik (2016). Cambridge (unlike

[5] We cannot say that Leibniz's notation was necessary for the development of analysis, or that analysis could not have been developed using fluxional notation. Leibniz's notation was used by continental mathematicians who succeeded, while Guicciardini (1989, Chapter 6) describes some failed attempts by fluxionists. We shall not pursue counterfactuals any further.

[6] Respectively Irish working in Ireland, Scottish working in England and (Northern) Irish working in Scotland, although thanks to various unions and dissolutions, Hamilton was born and died a Briton, and Kelvin died before Northern Ireland existed. To add to any confusion, then as now many people used 'English' to mean 'British', as was almost certainly the case with the words of Atiyah and Hardy cited in this Section 2. In this note we use 'British' unless greater precision is needed.

[7] Sir Michael Atiyah was a Fields medallist, Abel prizewinner, former Master of Trinity College, Cambridge (Newton's alma mater) and Newton's successor as President of the Royal Society.

[8] Oxford University contributed little, except for the work of H.J.S. Smith, who was a rare 19th century British analyst of genuine international standing (Heard 2019, pp.129–134).

Gompertz[9]) was largely won over to the 'continental notation' by 1850 (Grattan-Guinness 2011). Alas, this meant an algebraical approach due to Lagrange, which proved abortive and which continental analysts were already dropping in favour of Cauchy's limits, so "[the] shift from the fluxional calculus to the Lagrangian calculus, which marks the definitive death of the Newtonian tradition, once again left the British isolated" (Guicciardini (1989, p.138), see also Guicciardini (2004)). Historian Jeremy Gray surveyed the state of mathematics in Victorian Britain and (in polemical mood) said: "[what] strikes me too often about these people is their belief that they are the equals of their continental peers when no such comparison can be entertained" (Gray 2006, p.179).

Teaching at Cambridge was another brake on British mathematics. The Mathematical Tripos was a severe test of expertise specializing in Newtonian mechanics and, while it arguably honed many great minds, it did nothing to bring young mathematicians up to date, see Guicciardini (1989, Section 9.1) and Craik (2008). Heard (2019) noted, even of the later 19th century:

> "... Cambridge's lengthy domination of the British mathematical scene and the closed nature of the community, which together resulted in one generation of Cantabrigians teaching the next, with very little influx of new blood." (Heard 2019, p.117)

Not until the 20th century did G.H. Hardy succeed in reforming the Tripos, saying it had "... effectively ruined serious mathematics in England for a hundred years" (Snow 1967, p.34). Just as important, Hardy's textbook (Hardy 1908) at last introduced British undergraduates to rigorous analysis of Leibnizian stamp.

So it happened, that when actuarial science was called into being by practical necessity in Britain in the early 19th century, mathematics had split into a continental, Leibnizian mainstream, and a British, post-Newtonian backwater. This was the world in which Gompertz introduced his mortality law in 1825, wrote the words quoted at the start of this paper in 1862, and would be described as "one of the greatest mathematicians of Europe" (Woolhouse 1862), "the last of the learned Newtonians" (de Morgan (1865) cited in Hooker (1965)) and "one of the best mathematicians of the day" (Schooling 1924), see Hooker (1965) for further references.

3. The Growth of the (Respectable) British Actuarial Profession

Section 2 set the scene for 18th and 19th century British mathematics. We now turn to probability and long-term life insurance. Much has been written on their early

[9] Thanks to his Jewish religion, Gompertz had no formal links with Cambridge, but he remained faithful to Newton and fluxions all his life. He justified his view quoted at the start of this paper on the moral ground that the furtive differential notation: "... appears to give Leibnitz a greater claim to originality, to the prejudice of Newton, than I think he is justly entitled to" (Gompertz 1862). In fairness, Gompertz's dislike was based on genuine doubts about infinitesimals, shared by others (Guicciardini 1989), though these doubts were largely cleared up by Cauchy before Gompertz's death (Gray 2015, Chapter 5). In the same paper Gompertz wrote: "But whilst I am endeavouring to clear away the shadowing clouds which may obstruct the brilliant light of Newton's lamp from being duly perceived by the scientific eye, I am willing to acknowledge that Leibnitz's differential d has, in many instances, done great service in his own hands, in the hands of Euler, Lagrange, Laplace, and of a host of scientific men whose names cannot be pronounced without gratitude and reverence".

history, and the rôle of the life table in both; Ogborn (1962), Cox & Storr-Best (1962) and Daston (1988) before modern financial mathematics came to dominate practice, Dennett (2004), Johnson (2017) and Turnbull (2017) afterwards. These describe the origins of probability theory in the 17th century work of Fermat, Pascal, Huygens, and others, with proto-actuarial rôles for de Witt on annuities and Halley (1693) on life tables.

The conventional story of British actuarial science then begins with Equitable Life in 1762 and proceeds through the pragmatic invention of sound reserving, surplus measurement and bonus distribution[10] in which Dr Richard Price was a key figure (Ogborn 1962, Cox & Storr-Best 1962, Turnbull 2017). These principles of sound management became the raison d'être of the profession. Although British universities were not directly involved, there is no suggestion that the insurance pioneers worked in a different mathematical ethos, not descended from Newton. Universities are absent too from accounts of founding the Institute of Actuaries in 1848 (Simmonds 1948) and the Faculty of Actuaries in 1856 (Davidson 1956). Graduate-level knowledge was certainly not expected[11].

As well as mathematical pedigree we should consider social context. British society enjoyed stable laws and civilized politics, relatively speaking, and after Waterloo in 1815 would not face another convulsive war for 99 years. Britain was open to commercial enterprise, about to be transformed by the twin motors of manufacturing and railways, and had a growing middle class with wealth (and female relatives) to protect. New expertise and skills were needed, and at the highest levels (as universities stood aside) new professions dignified by Royal Charters emerged to meet the demand.[12]

A more critical assessment is that professions emerged chiefly to claim rights over well-remunerated activities (Porter 1995). For example, describing evidence given to the Parliamentary Select Committee on Assurance Associations in 1853:

> "Still, several witnesses allowed that the government might reasonably require publication of financial records. This, the actuaries argued, should take the form of a straightforward factual presentation. It should not include summary numbers standing for assets and liabilities, which could imply that solvency could be determined by anyone capable of noticing which was the greater." (Porter 1995, p.112)[13]

A desire for respectability and status was surely also present. From Porter again:

[10] This short note necessarily omits many interesting topics such as, in the Anglophone world, earlier activity around individual annuities and reversions (Bellhouse 2017), the prior claim to actuarial eminence of the Scottish Ministers' Widows' Fund and its connection with Colin Maclaurin (Dunlop 1992, Milevsky 2024) and the turn towards contribution bonus in the USA (Homans 1863). Similarly in Germany, actuarial activity around pensions and widows' funds was strong in the mid-19th century independently of any life insurance industry, see footnote 24 and the references in Seal (1977) for example.

[11] Minimal knowledge of calculus would be required even up to the time that King's textbook (King 1887) was revised in 1902. Menzler (1960, p.1) described King (1887) as "a monumental tome ... in which the calculus of finite differences was still paramount" and noted (p.31, in a chapter entitled 'The Stony Path to Textbooks') that "... in later days the calculus was embarked on at school at an earlier age than was usual in 1900", clearly indicating the expected level of entry to the profession.

[12] Alongside actuaries were civil engineers (1828), accountants (Scotland 1854, England and Wales 1880) and statisticians (1887), dated by their Royal Charters. Obtaining a Royal Charter remains a mark of professional status in Britain, more recent examples including electrical engineers (1921), mechanical engineers (1930), physicists (1970) and mathematicians (1990). Since 2024 qualified members of the Institute and Faculty of Actuaries have been entitled to call themselves 'Chartered Actuaries'.

[13] The actuaries had a point, despite Porter's scepticism. In a later age, unease about just comparing

> "The main object of the new institute of actuaries [sic] was to secure the recognition befitting a profession. This entailed more than a demonstration of technical competence, and the oft-expressed disdain for mere calculation must be understood as part of a strategy of legitimation in a society that gave little respect to mere technical experts" (Porter 1995, p.110)

As Paxman (1991, p.106) put it: "... the professional tag was so sought after that the rush to respectabilize was unstoppable. By the end of the century, architects, accountants and engineers had all created their own professional institutions."

With awareness of professional status (as opposed to professionalism) came an inbuilt conservatism, guarding against intruders[14]. A few examples are given below, to add to Gompertz's words at the start of this paper.

- The Institute of Actuaries' efforts to obtain a Royal Charter were frustrated until 1884 by the opposition of many of the most senior actuaries, who objected to certificated learning and refused to join the new body[15] (Simmonds 1948).
- An early sign of universities awakening in England was the development of mathematical statistics, centred not on Cambridge but on University College London (Porter 1986, Lehmann 2011, Magnello 2011). Its tortuous passage into the actuarial syllabus is described with dry humour by Menzler (1960), including 'l'affaire Seal' and the mock trial Rex versus Seal[16].
- When Sidney Benjamin presented a paper to an Institute of Actuaries meeting in 1971 showing computer simulations of a maturity guarantee, the reception was such that the chair "... closed the meeting and classed the discussion as confidential so that its content would never be published" (Turnbull 2017, pp.195–196). A version of the paper later appeared as Benjamin (1976) and today looks mainstream, largely thanks to the later influence of 'the Wilkie model' (Wilkie 1986, 1995).
- The further reaction of the British actuarial profession to modern financial mathematics, from about 1980–2000, is related in Dennett (2004), Turnbull (2017) and van der Heide (2023). Its significance here, we will argue, is that this was the first real incursion of the Leibnizian branch of mathematical history onto territory regarded by the profession as its own[17]. The key event was probability theory being made part of 'proper' mathematics by Kolmogorov (1933). That is a topic of Section 4.

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two numbers in a balance sheet, exemplified by Redington (1952), has led to increasingly elaborate statutory solvency regimes for life insurers.

[14] Other bodies of actuaries were just as capable of fierce disputation, see for example Seal (1977, p.434) on the arguments among 19th century German actuaries over multiple decrement theory.

[15] The Faculty of Actuaries in Scotland was granted its Royal Charter in 1868.

[16] Hilary Seal (1911–1984) was a statistician and Fellow of the Faculty of Actuaries. As editor of Journal of the Institute of Actuaries Students' Society in 1947 he was 'tried' on the charge of using Society funds to "... edit and publish a highly abstruse mathematical and statistical periodical which was of no interest or value to the members of the said Society" (Menzler 1960, pp.82–83). He was found guilty.

[17] One can argue about when and where the Leibnizian branch announced itself to traditional actuaries. An early example might be Thiele's differential equation, formulated about 1875 but first documented in Gram (1910). A much later candidate would be Hoem's introduction of Markov processes to problems of

Such conservatism is not necessarily to be deprecated; actuaries are not primarily research scientists and enthusiasm often rings alarm bells; a 'wait and see' approach may often be wise. Of course, this brings with it a risk of being overtaken. Modern actuaries can claim partial ownership at best of major techniques such as mortality modelling, stochastic modelling, asset-liability matching, and risk-neutral pricing, see Smith (1996) for example.

The conclusion of this section is that the higher purpose of the British actuarial professional bodies, to apply mathematical skills to benefit society, need not be doubted, and Royal Charters bestowed the ultimate seals of approval. Behind that shield, though, professional prerogatives could be defended against trespasses including, from time to time, new mathematics. Part of the conjecture advanced here is that because of the time and place of the profession's birth, those defenders were the intellectual heirs of Newton, as represented (rather extremely) by Gompertz. When Hardy and others brought modern analysis to British universities, from 1908 onwards (see Section 2) there was no strong link through which the profession might have been kept abreast.

4. From Leibniz to Kolmogorov's (Respectable) Model of Probability

We must briefly consider the social situation in Continental Europe in the 19th century, following the end of the Napoleonic Wars in 1815. The scientific Great Powers were France and (collectively) the German states and 'science' came to mean the new Grandes Écoles in one, and the Humboldtian research universities in the other (Ruano-Borbalan 2022, Section 4). Both kinds of institution were, to an extent unknown in Britain, organs of the state regardless of the state's current position between the extremes of monarchy and republic. Official bodies knew where to find mathematical experts as needed, and there was no vacuum for professional bodies to fill. When industrialization and conditions conducive to a life insurance market followed along, some years behind Britain (Thomson 1966), there was a supply of mathematicians to meet the demand.

Just as mainstream mathematics had been neglected in Britain, probability and statistics stood outside mainstream mathematics. From its origins in games of chance in the 17th century, reducing such a game to its expectation as a 'fair' outcome was a moral action (Johnson 2017, Chapter 8). Computation was tedious, and applications were directed towards a search for expected values, not the study of random variables (Daston 1988). Even in the later 19th century, the idea of randomness as natural variation rather than measurement error was not universally accepted (Porter 1986); rather it was something best reduced to a Gaussian distribution or, later, the fewest possible moments (Magnello 2011), and the only results of real mathematical interest were the Law of Large Numbers and the Central Limit Theorem. Just such pre-modern probability theory was exemplified by the motto of the Institute of Actuaries, certum ex incertis, and by Hans Bühlmann's description of life insurance actuaries as 'essentially deterministic'[18].

life and health insurance (Hoem 1969, 1988), following Sverdrup (1965). However, the Markov assumption preserved the emphasis on expected values (Section 4), via Kolmogorov's and Thiele's differential equations.

[18]In an editorial in ASTIN Bulletin, Bühlmann wrote: "Contrary to his colleague of the First Kind in life assurance, whose methods were essentially deterministic, he [the Actuary of the Second Kind, in non-life insurance] had to master the skills of probabilistic thinking" (Bühlmann 1987).

This state of affairs was to be transformed by Kolmogorov. In 1900, David Hilbert set out his famous problems, the sixth of which included finding satisfactory axioms for probability theory (Yandell 2002). Kolmogorov (1933) did this, based on measure theory[19], adding among other things a model of information and a definition of conditional expectation, therefore allowing the future to be conditioned on the past, and giving solid ground to the stochastic processes begun by Markov and the Swedish actuary Filip Lundberg. Above all, Kolmogorov made probability a respectable subject[20] in which talented young mathematicians could thereafter make a career.

Over the next few decades, theories of Markov processes, martingales[21], stochastic calculus and more yielded many applications. Among these, perhaps inevitably, were some of an actuarial nature, but often not with actuaries in the lead. Most important would be the Itô calculus that, in that hands of Black & Scholes (1973) and later Harrison & Kreps (1979) and Harrison & Pliska (1981) spawned modern financial mathematics, and the theory of point processes (Brémaud 1981) that led to reformulations of survival models (Andersen et al. 1993) and life insurance mathematics[22].

Key for our conjecture, Kolmogorov's model of probability was the lineal descendent of the analysis begun by Leibniz's calculus (Gray 2015). Also, it was arguably the first fundamental advance in probability theory since the 17th century, imbuing it with a modernist spirit based on axioms and rigour[23] (Gray 2008, Section 4.4). One significant difference, though, was the inverted position of calculus. Classical probability was combinatorical and calculus was applied to it later. Kolmogorov's probability could hardly even be defined without the Lebesgue integral. It was not a theory for the gifted amateur.

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[19]Readers may judge for themselves the place of probability in mathematics from the account in Yandell (2002), quoted here in its entirety: "Axiomatizing the theory of probabilities was a realistic goal; Kolmogorov accomplished this in 1933" (p.160). Probability is treated mainly as an application of integration in Bourbaki (1994), and is not mentioned at all in Temple (1981).

[20]Respectability did not come easily. Meyer (2009) reported resistance to the Lebesgue integral in some quarters and "a certain bad mood, quite noticeable particularly in the United States" (p.12). Of a 1957–58 paper by Hunt he said: "This result unifying analysis and probability contributed to making the latter a respectable field" (p.9). Doob recalled that "many probabilists were complaining that measure theory was killing the charm of their subject without contributing anything new" (Snell 1997, p.306); in 1984 he received the Career Prize of the American Mathematical Society for "his fundamental work in establishing probability as a branch of mathematics . . ." (Snell 1997, p.301).

[21]The idea of a filtration and much martingale theory originated with Joe Doob (Meyer 2009). Of his own student days in the 1920s Doob said: "In those days there were very few advanced mathematics books in English, and those in French and German were too expensive for most students" (Snell 1997, p.302). Paul Halmos recalled beginning his PhD studies with Doob in 1938: "What he originally had in mind was that I apply modern, high-powered measure-theoretic methods to the statistical ideas that R.A. Fisher was promulgating. He had already started to 'clean up Fisher' with his work on the method of maximum likelihood, but there was a lot left to do." (Halmos 1985, p.61)

[22]The growth of modern probability is sufficiently recent that it is barely yet a matter for historians. Meyer (2009) and Mazliak & Shafer (2022) are notably helpful sources.

[23]Alternatives to Kolmogorov's axiomatic approach combined philosophy and mathematics. von Plato (1994) identifies two other major lines of thought, mainly concerning epistemic questions of inference, by von Mises and de Finetti. The latter was an Italian actuary.

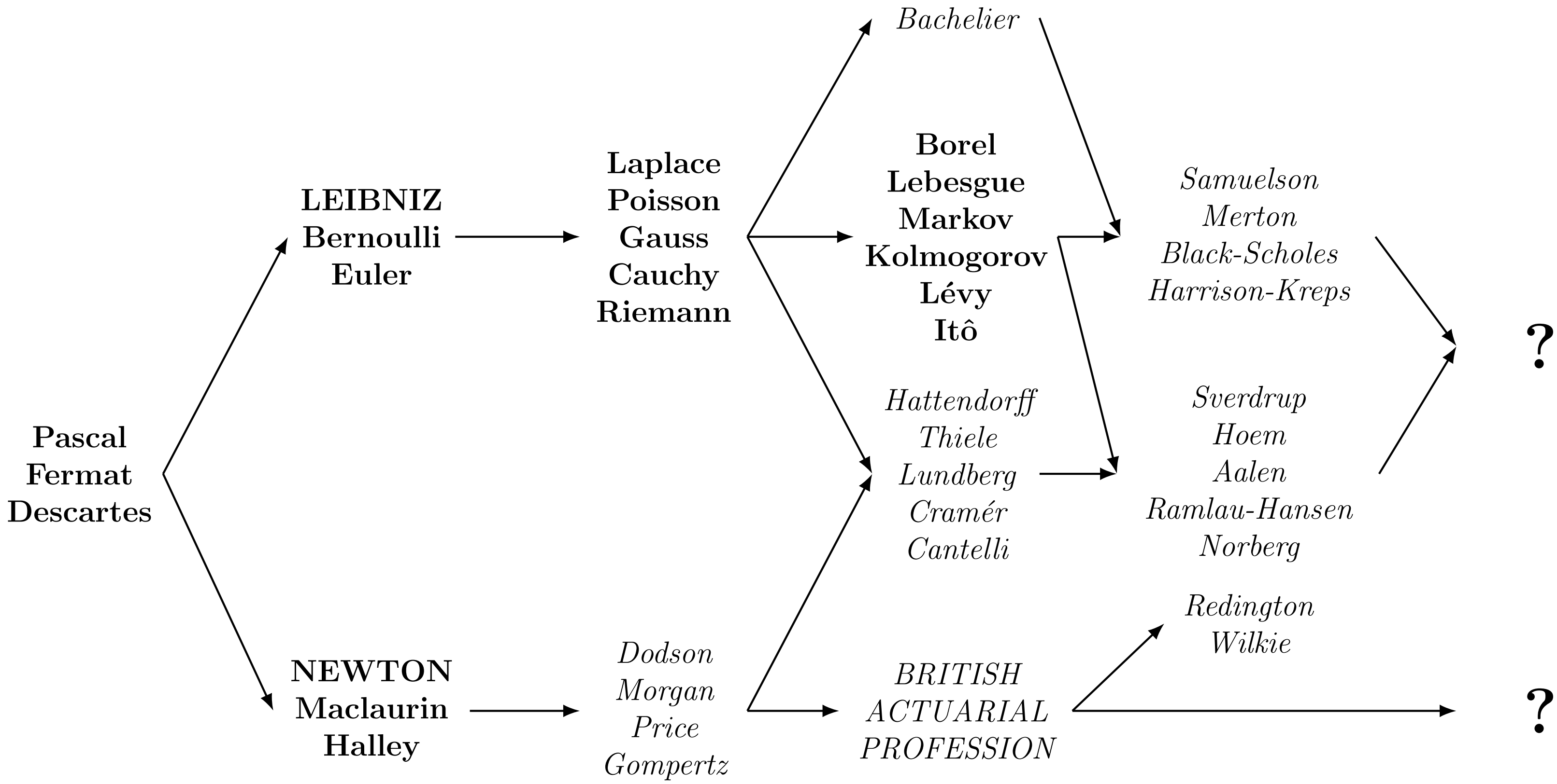

Figure 1: Indicative representation of the Leibnizian and Newtonian branches of history, as they are relevant to the development of life insurance mathematics. Arrows indicate major lines of influence, not chronology. Excepting the British actuarial profession, nodes contain selected names associated with key ideas or stages. **Bold** indicates the progress of theory, *italics* the application of theory to actuarial and financial problems.

A century after the foundation of the Institute of Actuaries, Frank Redington, the outstanding British actuary of his age, wrote a deservedly famous paper including his theory of immunization (Redington 1952). With hindsight, it might seem that he was equipped with the wrong (non-stochastic) calculus. However, it would be absurd to suggest that Redington, an actuary working in the senior echelons of a British insurance company, should have known about a stochastic calculus still confined to research journals. Consider the Leibnizian pedigree of stochastic 'immunization' in Black & Scholes (1973), two decades after Redington. Financial economists trained in US universities (heirs to Humboldt's vision) rediscovered an old Parisian thesis (Bachelier (1900), examined by Poincaré no less), connected it with a Japanese paper on stochastic analysis (Itô 1944) directly descended from Lebesgue and Borel (France) <u>via</u> Kolmogorov (USSR), and linked everything up with recent non-arbitrage ideas.

Of the histories mentioned at the start of Section 3, Daston (1988, p.3) states clearly the achievement of Kolmogorov, but only Johnson (2017, pp.206–208) follows up its importance for modern financial mathematics, and none explores the actuarial legacies of the Newton-Leibniz controversy.

Figure 1 shows, indicatively and subjectively, the historiography underlying the conjecture. Arrows indicate major lines of influence, not chronology. Excepting the British actuarial profession, nodes contain selected names associated with key ideas or stages. The figure shows the great calculus split and its two branches, one containing the British actuarial profession and the other the development of mathematical analysis. The earliest node shows Pascal and Fermat, and therefore represents also the origin of probability theory; thus we see the significance of the British profession being in one branch and Kolmogorov's reworking of probability theory being in the other.

The figure does not show the 20th century reunion of British and continental mathematics effected by Hardy and others (see Section 2). Nor does it say anything about non-life insurance, actuarial science beyond Europe, credit risk modelling or economics (in which respect the spread of the Humboldtian university to the USA (Ruano-Borbalan 2022) and the rise of business schools there (Khurana 2007) would be of great interest). Also, it highlights life insurance <u>mathematics</u>, and if the Newtonian arm looks rather bare, it might be full of bulging muscle in a version highlighting actuarial practice. (In such a version, the Leibnizian arm would lack professional bodies like Britain's, but would still include mathematicians of the calibre of Gauss[24] and Klein[25] (Germany), Thiele (Denmark), Cramér (Sweden), and Cantelli and de Finetti (Italy)[26], see Figure 1.)

[24] Gauss stabilized the Göttingen Professors' Widows' Fund in 1845–1851, saying of part of the work that took him a month: "In itself such a work is demanding, over 100,000 figures (according to my method of writing, where half or more are calculated in the head) ..." (Dunnington 2004, pp.233–234).

[25] Klein made Göttingen, the university of Gauss and Riemann, into the world's leading centre of mathematics, a status that lasted until 1933. In 1895, with support from the Prussian Parliament, Klein launched an actuarial science seminar at Göttingen "intended to train mathematicians and upper-level administrators for careers in the public and private insurance industry" (Tobies 2021, Section 7.6).

[26] The only comparable figures in Britain might be Maclaurin (footnote 10) and J.J. Sylvester, though the latter's contributions were to algebra more than to analysis. Sylvester was a founding Vice-President of the Institute of Actuaries, but for much of his life he was excluded from British universities by his Jewish religion, as Gompertz had been (Simmonds 1948, Bell 1953, Gray 2006). Of his employment as an actuary Bell said: "Such work for a creative mathematician is poisonous drudgery, and Sylvester

The Scandinavian influence on modelling biometric risks, including survival models, is present in Figure 1, in the node headed by Sverdrup. Here, mainly, life insurance mathematics has been given new foundations consistent with modern financial mathematics (Hoem & Aalen 1978, Ramlau-Hansen 1988a,b, Norberg 1991, 1992) and in that sense the two branches may be called convergent. However, nobody has yet truly conjoined the ancient law-of-large-numbers paradigm ruling independent risks and the newer hedging paradigm ruling singular financial risk, see Asmussen & Steffensen (2020, Chapter VI) for example. Since that is a frontier of research, it is a suitable point at which to draw this historical essay to a conclusion.

5. Conclusions

The conjecture outlined here is that the split triggered by the dispute between Newton and Leibniz over the invention of the calculus exerted an influence that has been overlooked in histories of actuarial science. It contributed to British mathematics being cut off from the mainstream, just when other social forces in 19th century Britain, including moribund universities, led actuarial science down the path of professionalization. Thus British actuaries took officially sanctioned 'ownership' of an intellectual corpus isolated from academia locally and mathematics globally. The basis of this expertise was the contemporary certum ex incertis model of probability and statistics in which randomness was largely reduced to the expectation.

In continental Europe, the same social purpose was served by university-trained mathematicians, including some of eminence. Thus it was accommodated alongside the flourishing Leibnizian branch of mathematics, at home in Humboldtian universities.

When the profession in Britain met new ideas from mathematical statistics in the early 20th century, its response was to assimilate them, against some resistance. Nevertheless, Bühlmann (1987) could still describe life assurance actuaries as 'essentially deterministic'. Later in the 20th century, Kolmogorov's model of probability burst out of academia and into finance, and this further challenge to professionalized expertise (now worldwide) has also been met by assimilation, with varying degrees of local difficulty. The conjecture, we have argued, is consistent with an intriguing parallel, concerning the rather human search for status and respectability; British actuaries sought it in a profession, probabilists felt its absence until being rescued by Kolmogorov.

The Newtonian and Leibnizian branches of mathematics itself were reunited in Britain in the early 20th century. A challenge for 21st century actuaries, most easily framed within their Leibnizian branch, is to reunite valuation paradigms launched respectively by 17th century probability and by Kolmogorov's reworking of the subject; in other words to marry the law of large numbers and dynamic hedging in a joint theory of valuation.

Acknowledgements

This paper is based on a presentation given at the meeting 'Gompertz 200' in Amsterdam, May 2025. I am grateful to Dr Timothy Johnson, Prof Moshe Milevsky, Dr Stephen

almost ceased to be a mathematician" (pp.426–427). Gray's opinion was less positive: "Well known to his contemporaries for his brilliance, his boundless enthusiasm, and his total failure to communicate to lesser mortals, his work was even more scattershot and error prone than Cayley's" (p.179).

Richards, Craig Turnbull, an anonymous referee and an editor for valuable comments on an earlier version of this paper.

Funding Statement

ASM's attendance at the 'Gompertz 200' meeting was supported by Longevitas Ltd..

Conflicts of Interest

None.

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