Intersection types and topologies in lambda calculus

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Abstract

The known topologies in the lambda calculus are the tree topology and the filter topology. The *tree topology* on the set Λ of lambda terms is the smallest that makes the mapping from lambda terms to Böhm-like trees continuous w.r.t. the Scott topology on Böhm-like trees (Barendregt [1984]). The *filter topology* on the set of lambda terms is introduced in a similar indirect way. Namely, it is the smallest that makes the mapping from closed lambda terms to filters (Barendregt et al. [1983]) continuous w.r.t. the Scott topology on filters.

The topologies here considered on the set Λ (the *fit topologies*) are introduced directly, using basic open sets related to typing statements in the full intersection type assignment system. These topologies give rise to simple proofs of some fundamental theorems of the untyped lambda calculus such as the Continuity Theorem and the Genericity Lemma. Applying some well known properties of intersection type assignment systems we show that application is continuous and unsolvable terms are bottoms (Barendregt [1984]). In order to show that normal forms are isolated points with respect to these topologies we use properties of principal typings (Krivine [1990]).

As soon as we have topologized Λ, we get the specialization preorder and the corresponding equivalence relation on Λ (Vickers [1989]). We give a characterization of these relations in terms of β- and η-reductions.

The restriction of these topologies to the set Λ^0 of closed lambda terms appears to be unique. We compare the fit topology and the filter topology and show that they coincide on the set Λ^0 of closed lambda terms, and that for every fit topology on the set Λ of all lambda terms there is a coincident filter topology on Λ, and vice versa.

References

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