Mathematical modelling, finance, and the recession

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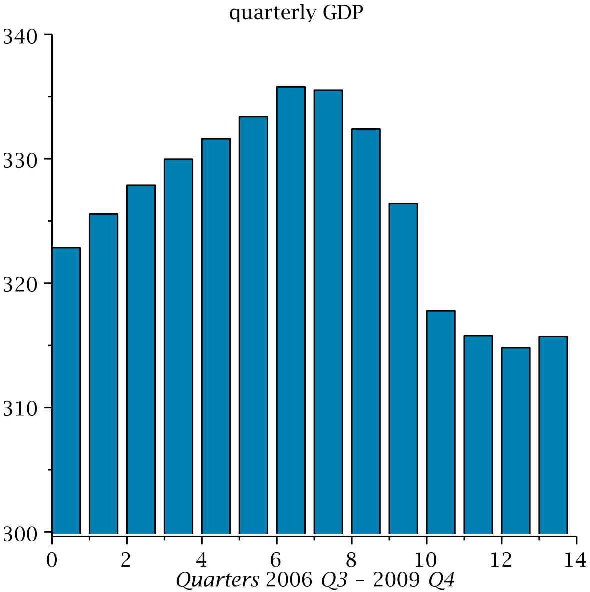
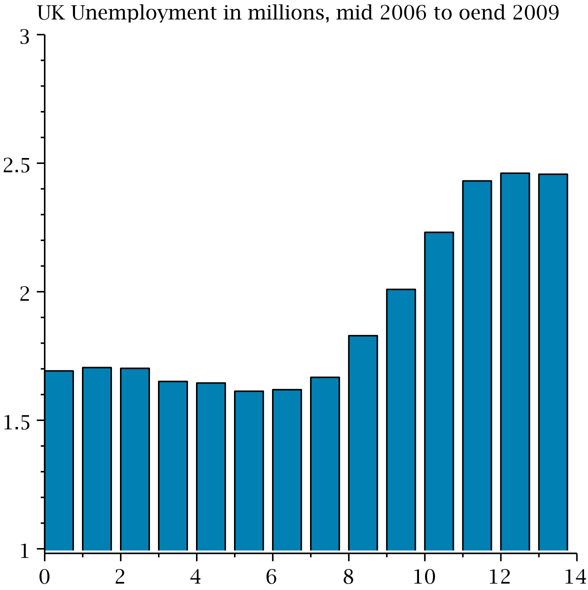
15:40 - 16:00, room 2.14

UK GDP (at 2005 prices) mid-2006 to end-2009



UK unemployment mid-2006 to end-2009



Some ingredients for a model

- Gross Domestic Product
  - GDP as expenditure
  - GDP as income
- Money, finance
  - The demand for money
  - The supply of money
- International trade
  - The exchange rate
  - The balance of payments

Gross Domestic Product evaluated as expenditure

The GDP Y can be defined as the rate at which money is being spent for providing goods and services

The UK Office of National Statistics breaks down this total into various sub-totals:

- Y := C + I + G + E - M
- C := spending by consumers (households and non-profit institutions) on goods and services
- I := private sector new investment (capital formation plus changes in inventories)
- G := government spending on goods and services (excludes interest on Gov't bonds, pensions, unemployment benefit)
- E := exports
- M := imports

GDP as expenditure: UK data /£10^9 per annum

year	2007	2008	2009
Consumption by households and NPIs	893	928	911
Investment: capital formation, inventory changes	255	244	194
Gov't spending on goods and services	295	314	330
Exports	372	422	387
iMports	416	460	421
$Y = C + I + G + E - M$	1399	1448	1401

GDP evaluated as income

Alternatively, the GDP Y can be defined as the rate at which money is being *received* in exchange for providing goods and services

- $Y = W + P + T$
- $W :=$ wages and salaries (less taxes)
- $P :=$ profits and rents
- $T :=$ taxes less subsidies

GDP as income: UK data / £10^9 per annum

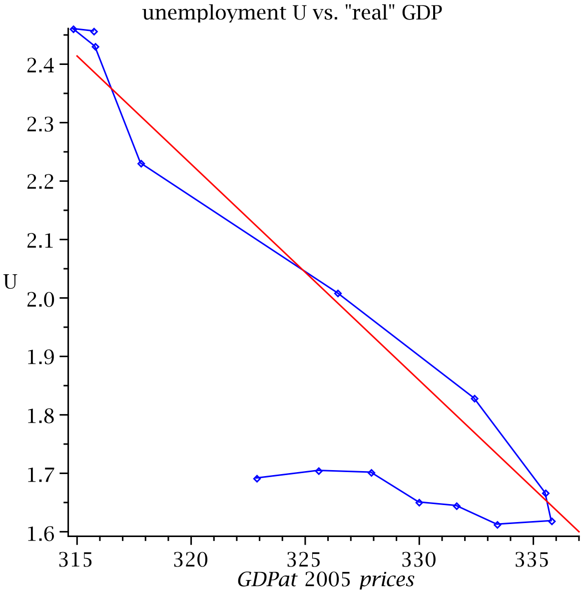
year	2007	2008	2009
Wages and salaries of employees	746	772	764
Profits of corporations and businesses	484	510	480
Taxes less subsidies	168	166	152
$Y = W + P + T$	1399	1448	1396

The accounting identity

The expenditure and income GDPs are equal:

$$Y = C + I + G + E - M = W + P + T$$

GDP and unemployment



A 'phase transition'

The two phases:

- Full employment phase: GDP increases and unemployment stays constant at its upper bound
  If GDP increases too rapidly, the result is an increase in wages and prices
- Partial employment phase: unemployment is below the upper bound and varies with GDP. Wages and prices do not increase

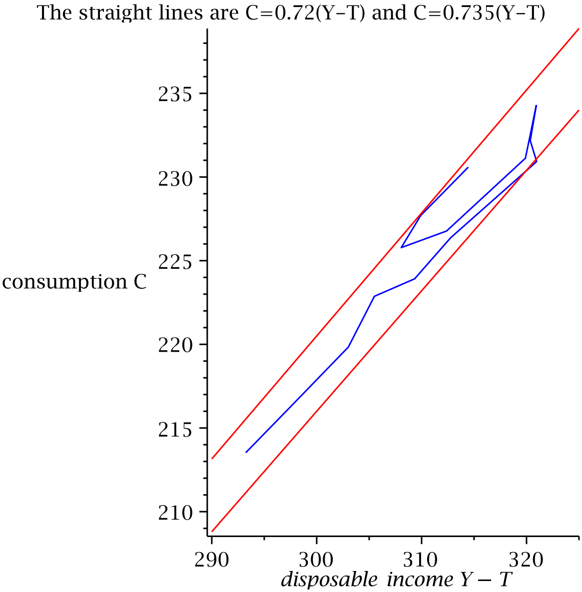
$$Y = wN$$

$w :=$ average wage, $N :=$ number employed, $N \leq N_{max}$.
Prices assumed proportional (in the short term) to w

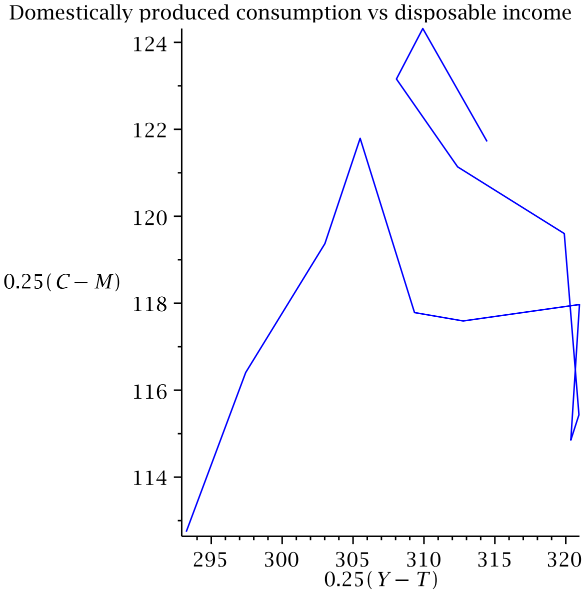
Ways to influence the GDP

- $Y = C + I + G + E - M$
- increase C : persuade householders to spend more
  (reduce taxes : 'fiscal stimulus')
- increase I : persuade firms to invest more
  (reduce interest rate)
- increase G : spend more on goods and services
- increase $E - M$: export more and/or import less

"Keynes' hypothesis": that C depends mainly on $Y - T$



Whereas $C - M$ does not depend mainly on $Y - T$



Using Keynes' hypothesis: the 'fiscal simulus'

The data on C vs $Y - T$ can be summarized by the formula

$$C \approx k(Y - T)$$

where k is about $3/4$. i.e. people spend about $3/4$ of their income and save about $1/4$.

Using this in the expenditure equation,

$$Y \approx 0.75(Y - T) + I + G + E - M$$

i.e.

$$0.25Y \approx -0.75T + I + G + E - M$$

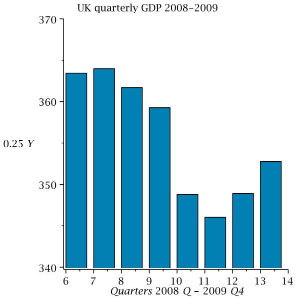
Whence, if I, E and M don't change,

$$\partial Y / \partial T = -3 \quad \text{and} \quad \partial Y / \partial G = 4$$

So a tax cut should increase Y by 3 times as much;
an increase of G should increase Y by 4 times as much.

Efficacy of a tax cut in increasing Y: observations

quarter	$Y/4$
2008 Q1	363.4
2008 Q2	364.0
2008 Q3	361.7
2008 Q4	359.3
2009 Q1	348.8
2009 Q2	346.0
2009 Q3	348.9
2009 Q4	352.8



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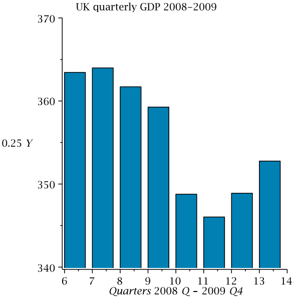


During 2009, VAT was cut by 12 £10^9, i.e. $dT = -12$ whence the "fiscal stimulus" theory would predict

$$dY = 3 \times 12 = 36, \quad d(Y/4) = 9$$

The data show that $d(Y/4)$ did not even have the hoped-for sign. This is because, although T decreased, the other variables I, E, M did not stay constant as assumed in the simple theory.

fiscal stimulus and budget deficit D

$$D := G - T$$

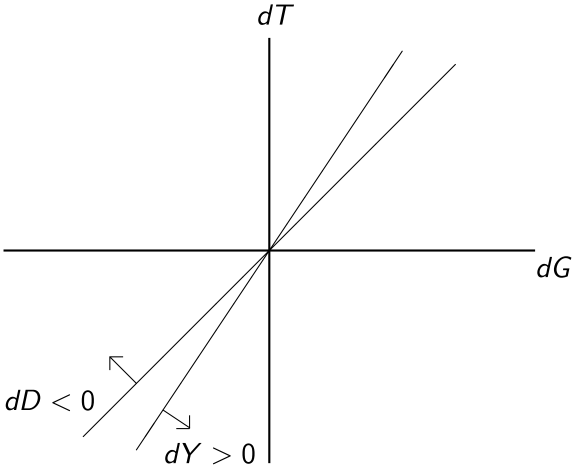
$$\begin{bmatrix} dY \\ dD \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} dG \\ dT \end{bmatrix}$$

whence

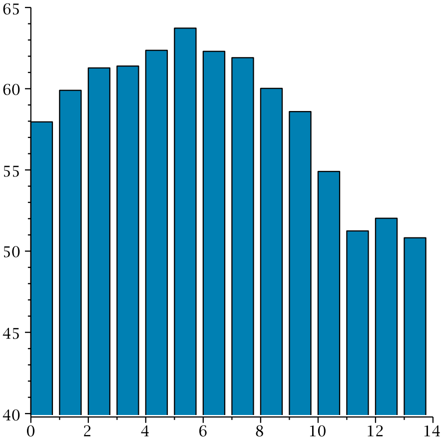
$$\begin{bmatrix} dG \\ dT \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} dY \\ dD \end{bmatrix}$$

To increase GDP without changing deficit (i.e. $dY > 0, dF = 0$) make equal increases in G and T
To reduce deficit without changing GDP (i.e. $dF < 0, dG = 0$) increase T and increase G by $3/4$ times as much.

How to reduce the deficit and increase GDP — provided that I and $E - M$ don't change.



investment



The demand for money

Definition

The quantity of money, Q, can be defined as
the sum of all cash + readily-available deposits in banks (including Bank of England) and similar institutions.

A simple model for the amount of money needed to support a given GDP is Irving Fisher's (1911) equation

$$Y = QV$$

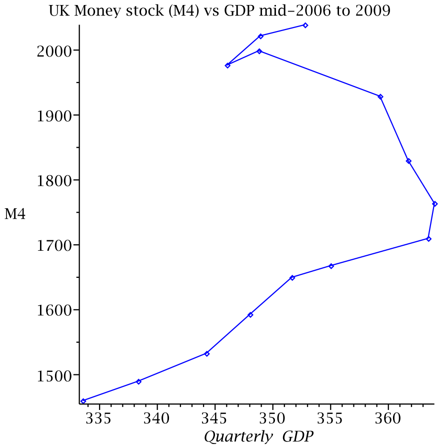
where V is the velocity of money (average ratio of people's spending rate to the amount of money they hold).

The presumption is that V is a constant or at least varies much mores slowly than Y and Q. So a simple model of the demand for money is

$$Q_{demand} = Y/V$$

In principle the demand depends on the 'price' of money, which is the interest rate r but for simplicity I'll ignore this dependence.

Does Irving Fisher's equation describe the recent UK data?



The supply of money

To meet the demand for money, banks can supply money in the form of loans. Indeed, banks (including but not only the central bank) can create money. They do this by opening new accounts or expanding credit limits on existing ones. In effect they buy financial assets (transferrable promises of future payments) in exchange for newly created money. The amount of money created by the banking system depends on the rate of interest r: the more money that is asked from them, the higher price they will charge. Denote by $Q_{supply}(r)$ the amount of money the banking system is willing to provide at interest rate r. It is an increasing function of r.

Equality of supply and demand

$$Q_{supply}(r) = Q_{demand}(r)$$

provides an equation from which Q and r could be determined — if we knew either of the two functions involved.